MA 113 Name ______________________

Quiz 3 Fall 2013

1. Evaluate the following limits

   (a)

$$\lim_{x\to 0} x^2 \sin(\frac{1}{x}) \qquad \text{Hint: What is the range of } \sin(\frac{1}{x})?$$

Solution: Since $-1 \leq \sin(1/x) \leq 1$, then $-x^2 \leq x^2 \sin(1/x) \leq x^2$. By the Squeeze Theorem and

$$\lim_{x->0} -x^2 = \lim_{x->0} x^2 = 0,$$

we have

$$\lim_{x\to 0} x^2 \sin(\frac{1}{x}) = 0$$

   (b)

$$\lim_{x\to 0} \frac{1-\cos(x)}{\sin^2(5x)} \qquad \text{Hint: Multiply by } \frac{1+\cos(x)}{1+\cos(x)}.$$

Solution:

$$\begin{aligned}
\lim_{x\to 0} \frac{1-\cos(x)}{\sin^2(5x)} &= \lim_{x\to 0} \frac{1-\cos(x)}{\sin^2(5x)} \cdot \frac{1+\cos(x)}{1+\cos(x)} \\
&= \lim_{x\to 0} \frac{1-\cos^2(x)}{\sin^2(5x)(1+\cos(x))} \\
&= \lim_{x\to 0} \frac{\sin^2(x)}{\sin^2(5x)} \cdot \frac{1}{1+\cos(x)} \\
&= \frac{1}{5^2} \cdot \frac{1}{2} \\
&= \frac{1}{50}
\end{aligned}$$

2. Use the Intermediate Value Theorem to show that the function

$$f(x) = 2x - x^2 \ln x$$

has a zero in the interval $[1, e]$.

Proof: The function f is continuous on $(0, \infty)$ and thus on the subinterval $[1, e]$. Since $f(1) = 2 > 0$ and $f(e) = (2 - e)e < 0$, the function must satisfy $f(c) = 0$ for some c in $(1, e)$ by the Intermediate Value Theorem.