

Quiz 4 - 3 October 2013

Make sure to show all your work.

1. The equation of the tangent line to $f(x)$ at $x = -3$ is $y = -13(x + 29)$. Determine a , $f(a)$, and $f'(a)$.

Solution: We first observe that $a = -3$. Then we note that $f'(a) = -13$, because the derivative of the function at a point a is the slope of the tangent line to the graph of that function at $x = a$. Finally, we observe that

$$f(-3) = -13(-3 + 29) = -13(26) = -338,$$

because the point $(a, f(a))$ is the intersection of the function and the tangent line.

Suggested: Score out of 4 points.

2. Consider the function $g(x) = \frac{(5x-2)(7x+1)}{2x-4}$. Find the value(s) of a where the equation of the tangent line to the graph of $g(x)$ is perpendicular to the line with equation $4x + 10y = 17$. Then determine the equation(s) of the tangent line(s) at the a value(s).

Solution: To find a , we need to find $g'(x)$ as follows:

$$g(x) = \frac{35x^2 - 9x - 2}{2x - 4} \quad (1)$$

$$g'(x) = \frac{(2x - 4)(70x - 9) - (35x^2 - 9x - 2)2}{(2x - 4)^2} \quad (2)$$

$$= \frac{(140x^2 - 298x + 36) - (70x^2 - 18x - 4)}{(2x - 4)^2} \quad (3)$$

$$= \frac{70x^2 - 280x + 40}{(2x - 4)^2} \quad (4)$$

$$= \frac{10(7x^2 - 28x + 4)}{(2x - 4)^2}. \quad (5)$$

Then we need to determine the slope of the given line by solving for y as follows:

$$10y = 17 - 4xy = \frac{17}{10} - \frac{4}{10}x. \quad (6)$$

So the slope is $-\frac{4}{10}$. Since we want the tangent line to be perpendicular to this line, we want our tangent line to have a slope of $\frac{10}{4}$. Now we solve $g'(a) = \frac{10}{4}$ for a .

$$\frac{10(7a^2 - 28a + 4)}{(2a - 4)^2} = \frac{10}{4} \quad (7)$$

$$7a^2 - 28a + 4 = \frac{(2a - 4)^2}{4} = \frac{4a^2 - 16a + 16}{4} = a^2 - 4a + 4 \quad (8)$$

$$0 = 6a^2 - 24a = 6a(a - 4) \quad (9)$$

$$6a = 0 \quad a - 4 = 0 \quad (10)$$

$$a = 0 \quad a = 4 \quad (11)$$

Notice that $g'(0) = g'(4) = \frac{10}{4}$, $g(0) = \frac{1}{2}$, and $g(4) = \frac{261}{2}$. So the equation of the tangent line at $x = 0$ is $y - \frac{1}{2} = \frac{10}{4}x$ and the equation of the tangent line at $x = 4$ is $y - \frac{261}{2} = \frac{10}{4}(x - 4)$.

Suggested: Score out of 6 points.