

Quiz 5 - October 10 2013

1. Let $f(x) = x^5 + 2x^4 - x^3 + 6x^2 - 7x + 3$. Find the following derivatives:

(a) $f'(x) = 5x^4 + 8x^3 - 3x^2 + 12x - 7$

(b) $f^{(2)}(x) = 5(4)x^3 + 8(3)x^2 - 3(2)x + 12 = 20x^3 + 24x^2 - 6x + 12$

(c) $f^{(3)}(x) = 20(3)x^2 + 24(2)x - 6 = 60x^2 + 48x - 6$

(d) $f^{(4)}(x) = 60(2)x + 48 = 120x + 48$

(e) $f^{(5)}(x) = 120$

(f) $f^{(6)}(x) = 0$

What is a formula for $f^{(n)}(x)$ when $n \geq 6$?

When $n \geq 6$, $f^{(n)}(x) = 0$.

2. Find the derivative of the following functions

(a) $F(x) = \cos(x) \tan(x)$

Using the Product Rule we have

$$F'(x) = \cos(x)(\tan(x))' + (\cos(x))' \tan(x) = \cos(x) \sec^2(x) - \sin(x) \tan(x)$$

We can further simplify: $F'(x) = \sec(x)(1 - \sin^2(x)) = \sec(x) \cos^2(x) = \cos(x)$

OR We can first simplify to find $F(x) = \sin(x)$, so we can immediately see $F'(x) = \cos(x)$

(b) $G(x) = \cos(\tan(x))$

Using the Chain Rule, we have

$$G'(x) = -\sin(\tan(x)) \sec^2(x)$$

(c) $H(x) = \frac{\cos(x)}{\tan(x)}$

Using the Quotient Rule we have

$$H'(x) = \frac{\tan(x)(\cos(x))' - \cos(x)(\tan(x))'}{\tan^2(x)} = \frac{-\tan(x) \sin(x) - \cos(x) \sec^2(x)}{\tan^2(x)}$$

Which simplifies to give: $H'(x) = \frac{-\tan(x) \sin(x) - \sec(x)}{\tan^2(x)}$