MA 113 **Name: ANSWER KEY**
Quiz 6 - October 17, 2013

1. Suppose that we have two variable resistors connected in parallel with resistances R_1 and R_2 and measured in ohms (Ω). The total resistance is given by

$$\frac{1}{R(t)} = \frac{1}{R_1(t)} + \frac{1}{R_2(t)}.$$

   (a) Find $R(0)$ if $R_1(0) = 30\ \Omega$ and $R_2(0) = 20\ \Omega$.

   (b) Suppose that the resistance R_1 is increasing at a rate of $0.25\ \Omega/min$ and R_2 is increasing at a rate $0.5\ \Omega/min$ at $t = 0$. What is the rate of change in R at $t = 0$?

SOLUTION:

   (a) The first part just asks to find R, so use the given formula at $t = 0$, $\frac{1}{R} = \frac{1}{30} + \frac{1}{20} = \frac{2}{60} + \frac{3}{60} = \frac{5}{60}$. Now $R = \frac{60}{5} = 12\Omega$. (1 point)

   (b) Part (b) uses related rates. We are looking for $\frac{dR}{dt}$ at $t = 0$. Take the derivative of the formula using implicit differentiation. This results in

$$\frac{-1}{[R(t)]^2}\frac{dR}{dt} = \frac{-1}{[R_1(t)]^2}\frac{dR_1}{dt} + \frac{-1}{[R_2(t)]^2}\frac{dR_2}{dt},\ \text{(3 points)}$$

   and solving for $\frac{dR}{dt}$ we get

$$\frac{dR}{dt} = \frac{[R(t)]^2}{[R_1(t)]^2}\frac{dR_1}{dt} + \frac{[R(t)]^2}{[R_2(t)]^2}\frac{dR_2}{dt}.\ \text{(1 point)}$$

   Using information for $t = 0$, we obtain $\frac{[R(0)]^2}{[R_1(0)]^2}\frac{dR_1}{dt} + \frac{[R(0)]^2}{[R_2(0)]^2}\frac{dR_2}{dt} = \frac{12^2}{30^2}(.25) + \frac{12^2}{20^2}(.5) = \frac{4}{100} + \frac{18}{100} = \frac{22}{100} = 0.22\Omega$. (1 point)

2. Let the function $f(x) = 2^{x^2}$ be as given.

   (a) Find the x value(s) where the tangent line to the function is horizontal.

   (b) Write an equation for each horizontal tangent line in point-slope form.

SOLUTION:

   (a) Horizontal tangent lines imply slope zero of the function. Take f' and set it to zero. Now, $f'(x) = \frac{d}{dx}e^{\ln(2)x^2} = e^{\ln(2)x^2}\ln(2)(2x) = 2^{x^2}\ln(2)(2x)$ by the chain rule and differentiation of exponential functions, and $2^{x^2}\ln(2)(2x) = 0$ when $(2x) = 0$. Thus, $x = 0$ is the only value of x so that the tangent line is horizontal. (2 point)

   (b) Writing this tangent line in point slope form requires a point, (x_0, y_0) and a slope m, and the general equation $y - y_0 = m(x - x_0)$. In our problem $x_0 = 0, y_0 = 2^{0^2} = 2^0 = 1$, and $m = 0$ because its a horizontal line. Thus, $y - 1 = 0(x - 1)$, which implies $y = 1$. (2 points)