MA 113 **Name:** ______________________

Quiz 7 - October 31, 2013

1. Highly rigorous research has shown that the number of ghosts haunting the average neighborhood on Halloween night between midnight and 5 a.m can be represented by the function $g(x) = \frac{1}{4}x^4 - 2x^3 + 4x^2 + 5$ (where $x = 0$ corresponds to midnight, $x = 1$ to 1:00 a.m., and so on.)

   (a) Find all local maxima and minima of $g(x)$ strictly between midnight and 5 a.m. so you will know the best/worst times to walk through your neighborhood on Halloween night.

   (b) Use part (a) to show there exist a time between midnight and 5 a.m. when the number of ghosts per average neighborhood is decreasing at a rate of 2 ghosts per hour (i.e. show there exist c in $(0, 5)$ such that $g'(c) = -2$).

Solution:

(a) We calculate $g'(x) = x^3 - 6x^2 + 8x$. Then since all local minimums and maximums correspond to roots of $g'(x)$, we set $x^3 - 6x^2 + 8x = 0$. Factoring gives $x^3 - 6x^2 + 8x = x(x - 2)(x - 4) = 0$. Since the problem asks for times strictly between 0 and 5, $x = 0$ is not a solution. Therefore we need only consider $x = 2$ and $x = 4$. We check and see that $g'(1) = 3$, $g'(3) = -3$, and $g'(5) = 15$. Therefore $x = 2$ gives us a local maximum of $g(2) = 9$ and $x = 4$ gives us a local minimum of $g(4) = 5$.

(b) Since $g(x)$ is differentiable on $(0, 5)$, the Mean Value Theorem tells us that there exist c in $(2, 4)$ such that

$$g'(c) = \frac{g(4) - g(2)}{4 - 2} = \frac{5 - 9}{4 - 2} = -2$$

2. On October 31, 2013, a zombie apocalypse breaks out, putting the fate of the human race in peril. The number of zombies (in millions) on Earth x days after the initial infection is given by $Z(x) = 500\sqrt{x}$.

   (a) Use the linear approximation to $Z(x)$ at 25 to estimate the number of zombies wandering the world 26 days after the initial infection. Furthermore, find the error of this approximation.

   (b) **Challenge:** Use the Mean Value Theorem to show there exists a day when approximately 33.3 million new zombies are created. It may be useful to know that the current world population is estimated to be about 7.046 billion so it will take the zombies approximately 200 days to eradicate the human race. (Hint: $\frac{500 \cdot 10 - 500 \cdot 5}{100 - 25} \approx 33.3$)

Solution:

(a) Since $Z(x)$ is differentiable on $(0, 150)$ and $Z'(x) = \frac{250}{\sqrt{x}}$, we can estimate

$$L(26) \approx Z(25) + Z'(25)(26 - 25) = 500\sqrt{25} + \frac{250}{\sqrt{25}}(26 - 25) = 2550 \text{ million}$$

The actual value is $Z(26) = 500\sqrt{26} = 2549.5098$ million after rounding. Thus

$$Error = |Z(26) - L(26)| = |2550 - 2549.5098| = 0.4902 \text{ million} = 490,200.$$

(b) Note that the number of zombies created on the n^{th} day is $Z(n) - Z(n-1)$. However, linear approximation tells us that $Z(n) - Z(n-1) \approx Z'(n-1)$. Since $Z(x)$ is differentiable on $(0, 150)$, we may apply the Mean Value Theorem using $a = 25$ and $b = 100$ to conclude that there exist c in $(25, 100)$ such that

$$Z'(c) = \frac{500\sqrt{100} - 500\sqrt{25}}{100 - 25} = \frac{2500}{75} = \frac{100}{3} \approx 33.3 \text{ million}$$

Rounding c to the nearest integer and adding 1 gives us the day on which approximately 33.3 million new zombies are created.