MA 113 **Name:** ________________________________

Quiz 9 - November 14, 2013

1. Suppose f is a twice differentiable function such that $f''(t) = t - \cos t$, $f'(0) = 2$, and $f(0) = -2$. Find $f'(t)$. Then find $f(t)$.

Solution: $f'(t)$ is an antiderivative of $f''(t)$. Therefore,

$$f'(t) = \frac{t^2}{2} - \sin t + C$$

From the information given, $f'(0) = 2$, yielding the equation

$$2 = \frac{0^2}{2} - \sin 0 + C$$

$$2 = C$$

Hence,

$$f'(t) = \frac{t^2}{2} - \sin t + 2$$

Similarly, $f(t)$ is an antiderivative of $f'(t)$. Therefore,

$$f(t) = \frac{t^3}{6} + \cos t + 2t + C$$

From the information given, $f(0) = -2$, yielding the equation

$$-2 = \frac{0^3}{6} + \cos 0 + 2(0) + C$$

$$-2 = 0 + 1 + 0 + C$$

$$-3 = C$$

Hence,

$$f(t) = \frac{t^3}{6} + \cos t + 2t - 3$$

2. Let $g(x) = x^2 - x + 1$.

   (a) Subdivide the interval $[1, 4]$ into three equal subintervals and compute R_3, the value of the right-endpoint approximation to the area under the graph g on the interval $[1, 4]$.

   (b) Sketch the graph of g and the rectangles that make up your approximation. Is the area under the graph larger or smaller than R_3?

Solution:

Note that $\Delta x = \dfrac{b-a}{N} = \dfrac{4-1}{3} = 1$.
The general term in the summation for R_3 is

$$g(a+j\Delta x) = g(1+j) = (1+j)^2 - (1+j) + 1 = j^2 + j + 1$$

Therefore,

$$\begin{aligned} R_3 &= \Delta x \sum_{j=1}^{N} g(a + j\Delta x) \\ &= \sum_{j=1}^{3} (j^2 + j + 1) \\ &= (3 + 7 + 13) \\ &= 23 \end{aligned}$$

Alternatively, we see that

$$\begin{aligned} \text{Height of Rectangle 1} &= g(2) = 3 \\ \text{Height of Rectangle 2} &= g(3) = 7 \\ \text{Height of Rectangle 3} &= g(4) = 13 \end{aligned}$$

and thus,

$$R_3 = 1 \cdot (3 + 7 + 13) = 23$$

We can see that the rectangles used in R_3 extend above the graph of $g(x)$ and provide an overestimation of the area under the graph.