MA 113 Calculus I, Fall 2013
Written Assignment #3
Due Wednesday, October 2, 2013, at beginning of lecture

Instructions: The purpose of this assignment is to develop your ability to formulate and communicate mathematical arguments. Your complete assignment should have your name and section number on each page, be stapled, and be neat and legible. *Unreadable work will receive no credit.*

You should provide well-written, complete answers to each of the questions. We will look for correct mathematical arguments, complete explanations, and correct use of English. Your solution should be formulated in complete sentences. As appropriate, you may want to include diagrams or equations written out on a separate line. You may read your textbook to find examples of how we communicate mathematics.

Students are encouraged to use word-processing software to produce high quality solutions. However, you may find that it is simpler to add graphs and equations using pen or pencil.

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1. (4 points) Let f be a continuous function with domain the closed interval $[0, 2]$ and taking the following values: $f(0) = 2$, $f(1) = -1$, and $f(2) = 1$.
   (a) What is the minimum number of zeros of the function f on $[0, 2]$? (A zero of f is a point x_0 so that $f(x_0) = 0$.)
   (b) Does f have an inverse on the interval $[0, 2]$?

2. (6 points) The goal of this assignment is to derive the power law for the derivative of x^n, for positive integers $n = 1, 2, \ldots$, using the Binomial Theorem.
   (a) The Binomial Theorem states that for real numbers a and b and a positive integer n we have:
   $$(a+b)^n = \sum_{k=0}^{n} \binom{n}{k} a^{n-k} b^k = \binom{n}{0} a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \cdots + \binom{n}{n} b^n.$$
   The binomial coefficient is defined by
   $$\binom{n}{k} = \frac{n!}{k!(n-k)!},$$
   where $n!$ is the factorial of n. This is a number given by the product $n! = n \cdot (n-1) \cdot (n-2) \cdots 3 \cdot 2 \cdot 1$, and $0! = 1$. Using the Binomial Theorem compute $(a+b)^n$ for $n = 2, 3,$ and 4. Compare the results with the results obtained by expanding the expression $(a+b)^n$. For help with the summation notation, see section 5.1 of Rogawski, page 289.
   (b) Find a simple expression for the binomial coefficient $\binom{n}{1}$.
   (c) Use the definition of the derivative and the Binomial Theorem to show that the derivative of x^n is nx^{n-1} for any positive integer n.