MA 113 Calculus I, Fall 2013
Written Assignment #4
Due Wednesday, 9 October 2013 , at beginning of lecture

Instructions: The purpose of this assignment is to develop your ability to formulate and communicate mathematical arguments. Your complete assignment should have your name and section number on each page, be stapled, and be neat and legible. *Unreadable work will receive no credit.*

You should provide well-written, complete answers to each of the questions. We will look for correct mathematical arguments, complete explanations, and correct use of English. Your solution should be formulated in complete sentences. As appropriate, you may want to include diagrams or equations written out on a separate line. You may read your textbook to find examples of how we communicate mathematics.

Students are encouraged to use word-processing software to produce high quality solutions. However, you may find that it is simpler to add graphs and equations using pen or pencil.

---

1. In the real world, falling objects (parachutists, raindrops, birds of prey, etc.) experience, in addition to the pull of gravity, a drag force proportional to the square of the velocity. If $h(t)$ is the height of the falling object, then dh/dt is its velocity, and d^2h/dt^2 is its acceleration. Newton's law gives

$$\frac{d^2h}{dt^2} = -g + k\left(\frac{dh}{dt}\right)^2 \tag{1}$$

where g is the acceleration due to gravity, and k is a constant that depends on the size and shape of the object (note that g and k are positive). For this reason, the velocity of a falling object does not become arbitrarily large but approaches a *terminal velocity*. Terminal velocity is defined as the velocity at which acceleration is zero.

   (a) (2 points) Using (1), find an expression for the terminal velocity in terms of g and k. Note that, with our choice of coordinates, a falling object will have negative velocity.

   (b) (2 points) A peregrine falcon is a bird of prey that hunts by tucking in its wings and dive-bombing its prey. If the peregrine tucks in its wings, the constant k is 0.00121 m^{-1} if height is measured in meters and time in seconds. The constant g is 9.8 m/sec^2. A peregrine falcon hunting a rabbit reaches terminal velocity 90 meters above the ground. At that same moment, the rabbit sees the peregrine falcon hurtling toward it. How long does it have to escape?

2. This problem concerns higher derivatives.

   (a) (2 points) A mystery polynomial $P(x)$ of degree 3 has the following value and derivatives at $x = 0$: $P(0) = 3$, $P'(0) = 1$, $P''(0) = 4$, $P'''(0) = 12$. Find $P(x)$.

   (b) (2 points) Suppose now that $P(0) = a_0$, $P'(0) = a_1$, $P''(0) = a_2$, and $P'''(0) = a_3$ where a_0, a_1, a_2, a_3 are constants. Find a formula for $P(x)$ in terms of the constants a_0, a_1, a_2, a_3.

   (c) (2 points) Conjecture a formula for finding a polynomial of degree n given the numbers $P(0) = a_0$, $P'(0) = a_1$, $\cdots$, $P^{(n)}(0) = a_n$. It may be useful to recall the definition of $n!$ from Written Assignment 3.