MA 113 Calculus I, Fall 2013
Written Assignment #5
Due Wednesday, 30 October 2013, at beginning of lecture

Instructions: The purpose of this assignment is to develop your ability to formulate and communicate mathematical arguments. Your complete assignment should have your name and section number on each page, be stapled, and be neat and legible. *Unreadable work will receive no credit.*

You should provide well-written, complete answers to each of the questions. We will look for correct mathematical arguments, complete explanations, and correct use of English. Your solution should be formulated in complete sentences. As appropriate, you may want to include diagrams or equations written out on a separate line. You may read your textbook to find examples of how we communicate mathematics.

Students are encouraged to use word-processing software to produce high quality solutions. However, you may find that it is simpler to add graphs and equations using pen or pencil.

When y is an algebraic function of x described by the equation $f(x, y) = 0$, we have a simpler process to find the tangent line to the curve $f(x, y) = 0$ at a chosen point. The aim of this assignment is to develop this technique.

1. (1 point) Consider the curve C which is described by the equation $2x^2 + y^2 - xy - 4x + y = 0$. Thus C is the set of all points that satisfy the equation $2x^2 + y^2 - xy - 4x + y = 0$. Using implicit differentiation, calculate the equation of the tangent line to this curve at the origin $(0, 0)$.

2. (3 points) Let (a, b) be any point on the same curve C. Using implicit differentiation, show that the equation of the tangent line to the curve C at (a, b) is given by:

$$(4a - b - 4)(x - a) + (2b - a + 1)(y - b) = 0.$$

3. (2 points) Using the formula from the previous part, determine the equation of the tangent line at the point $(2, 1)$. Verify that the point $(2, 1)$ is on the curve and verify that you get the same equation by the usual calculation using implicit differentiation.

4. (2 points) Suppose we consider a new curve C^* which is the graph of the equation $2x^2 + y^2 + pxy - 4x + y = 0$, where p is any constant. Explain why the new curve C^* still has the same tangent line at $(0, 0)$. **Suggestion:** It is a good idea to recognize the pattern in your calculations.

5. (2 points) Suppose that a curve C is the graph of an equation $f(x, y) = 0$ where

$$f(x, y) = ax + by + pxy$$

   and p is a constant and $b \neq 0$.

   Show that the tangent line to this curve at $(0, 0)$ has equation $ax + by = 0$.

   Give an example of a term beside pxy that we can add to f without changing the tangent line at $(0, 0)$.

   Extra Credit: (1 point) In case $a \neq 0$ but $b = 0$, explain why $ax = 0$ is still the equation of the tangent line. In this case dy/dx will be undefined, but you can find dx/dy. Make a sketch of a sample curve of this type to illustrate your conclusion.