MA 113 Calculus I, Fall 2023
Written Assignment #1

Instructions: The purpose of this assignment is to develop your ability to formulate and communicate mathematical arguments. *Unreadable work may receive no credit.*

You should provide well-written, complete answers to each of the questions. We will look for correct mathematical arguments, well-written explanations, and correct use of English. Your solution should be formulated in complete sentences. As appropriate, you may want to include diagrams or equations written out on a separate line. You may read your textbook to find examples of how we communicate mathematics. You should also look at the section on "Expectations for student work" in the syllabus web page.

Students may use word processing software, a writing app on a tablet or pencil and paper to prepare their solutions. It may be simpler to draw graphs and mathematical expressions by hand. The final solution *must* be prepared as a single pdf and uploaded to Canvas. For those that write their solutions on paper, a tablet or phone can be used to scan the work into a pdf file. Scanning functionality is built into Google Drive and the Files app on Apple products. Since you are submitting this work to Canvas, there is no need to put your name on your work. We suggest that you not include your name so that the instructors have the option Canvas to grade anonymously.

---

1. Let $f(x) = 2x + 3$.
   (a) Find an expression for the inverse function $f^{-1}(x)$.
   (b) The graphs of f and f^{-1} are lines. Give the slope of these lines. What is the relationship between the slopes of these lines?
   (c) Suppose we have a general linear function $f(x) = mx + b$. What do you expect the slope of the inverse function f^{-1} to be?
   (d) Find the inverse of the linear function $f(x) = mx + b$ (assume $m \neq 0$) and determine if your answer to part c) is correct.

2. Draw the graph of a function for which the following hold:

$$\lim_{x\to 2^-} f(x) = 1, \lim_{x\to 2^+} f(x) = 3, f(2) = 2.$$

`Solution:` 1. a) To find the inverse function, we write $y = 2x + 3$ and interchange x and y to obtain $x = 2y + 3 = f(y)$. We now solve for y in this equation and obtain

$$y = \frac{1}{2}x - \frac{3}{2}.$$

Thus if $x = f(y)$, then the inverse function is given by

$$y = f^{-1}(x) = \frac{1}{2}x - \frac{3}{2}.$$

b) The slope of the line for f, $y = 2x + 3$ is 2 and the slope for $f^{-1}(x) = \frac{1}{2}x - \frac{3}{2}$ is 1/2.

The two slopes are reciprocals of each other.

c) In general, we might expect that for the linear function $f(x) = mx + b$, the inverse function will be a linear function with slope $1/m$ as long as m is not zero.

We know that we can transform the graph of f to the graph of f^{-1} by reflecting in the line $y = x$ which interchanges the "rise" and the "run" along the graph. This will result in changing the slope to its reciprocal.

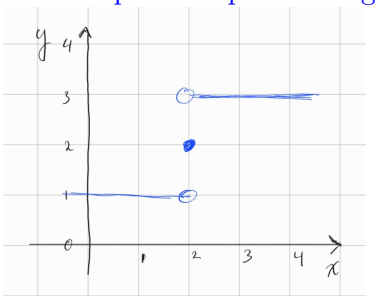
d) To determine if our answer to part c) is correct, we proceed as in part a). We write $y = mx + b$ and interchange x and y to obtain $x = my + b$. Solving for y gives

$$y = \frac{1}{m}x - \frac{b}{m}$$

where we need $m \neq 0$ to be able to divide by m.

Thus the inverse function is the linear function $f^{-1}(x) = \frac{1}{m}x - \frac{b}{m}$ with slope $1/m$. We have confirmed that our guess is part c) is correct.

2. A simple example showing the requested behavior is below.



Grading: Part 1a) method for finding inverse (1 point), correct formula for inverse function (1 point).

Part b) slopes (1 point)

Part c) Accept any reasonable answer for relations on parts b) and c) (1 point) Do not award points if no answer is given.

Part d) Find inverse function (1 point), give slope and compare with answer to part c) (1 point).

Part 2. Graph with all requested behavior (1 point).

Mark and correct unlabelled axes, but do not deduct.

Give one additional point if majority of responses are written as complete sentences.