# MA 113 Calculus I, Fall 2023
## Written Assignment #6

**Instructions:** The purpose of this assignment is to develop your ability to formulate and communicate mathematical arguments. *Unreadable work may receive no credit.*

You should provide well-written, complete answers to each of the questions. We will look for correct mathematical arguments, complete explanations, and correct use of English. Your solution should be formulated in complete sentences. As appropriate, you may want to include diagrams or equations written out on a separate line. You may read your textbook to find examples of how we communicate mathematics. You should also read the page about writing mathematics in the general information section of our Canvas shell.

Students may use word processing software, a writing app on a tablet or pencil and paper to prepare their solutions. It may be simpler to draw graphs and mathematical expressions by hand. The final solution *must* be prepared as a single pdf and uploaded to Canvas. For those that write their solutions on paper, a tablet or phone can be used to produce a pdf of their work. Scanning functionality is built into Google Drive and the Files app on Apple products. Since you are submitting this paper in Canvas, there is no need to put your name on your work. We suggest that you not include your name as we may use the option within Canvas to grade anonymously.

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1. Consider a right circular cylinder with a fixed volume $V$.
   - (a) Write an expression for the surface area of the cylinder in terms of the radius of the base $r$ and the cylinder's height $h$. Include the top and the bottom when computing the surface area.
   - (b) Find the radius and height that give the cylinder with smallest surface area. (Your answers will depend on $V$.) Show that that the ratio $h/r$ is independent of $V$.
   - (c) Give a careful explanation as to why you have found the smallest surface area.
   - (d) Find a can and measure that ratio $h/r$. Is it close to what you found in part c)?
   - (e) Speculate as to why manufacturers might choose to produce cans that do not use the least amount of material, even though this increases the cost of material.

`Solution:` a) The surface area is $A = 2\pi r^2 + 2\pi rh$.

b) If we fix the volume $V = \pi r^2 h$, we have that $h = V/\pi r^2$ and we can use this relation to express the surface area in terms of $r$,

$$A(r) = 2\pi r^2 + \frac{2\pi r V}{\pi r^2} = 2\pi r^2 + \frac{2V}{r}.$$

We compute the derivative, $A'(r) = 4\pi r - 2V/r^2$. We solve to find the critical point is the solution of $r^3 = V/(2\pi)$ or $r = (V/(2\pi))^{1/3}$. For this value of $r$,

$$h = \frac{V}{\pi r^2} = \frac{V}{\pi (V/2\pi)^{2/3}} = 2^{2/3}\left(\frac{V}{\pi}\right)^{1/3}.$$

For these values of $h$ and $r$, the ratio $h/r$ is

$$\frac{h}{r} = 2^{2/3} \left(\frac{V}{\pi}\right)^{1/3} \left(\frac{2\pi}{V}\right)^{1/3} = 2.$$

c) Since $V > 0$, we have $A'(r) < 0$ for $0 < r < (V/(2\pi))^{1/3}$ and $A'(r) > 0$ for $r > (V/(2\pi))^{1/3}$. This means that $A$ is decreasing to the left of the critical point and increasing to the right of the critical point and the critical point gives a global minimum for $A(r)$ on $(0, \infty)$.

Alternate argument: The second derivative $A''(r) = 4\pi + 4V/r^3$ satisfies $A'' > 0$ if $r > 0$. Since the function $A$ is convex up on $(0, \infty)$, the function lies above the tangent line at $c$. In other words $A(x) \geq A(c)$ for $x > 0$.

d) Measuring a few cans gives the following results:

| Type | $h$ | $r$ | $h/r$ |
|---:|---|---|---|
| Beverage (Coke) | 12.5 cm | 3.1 cm | 4 |
| Vegetables | 11 cm | 3.6 cm | 3 |
| Tomato paste | 8.3 cm | 2.6 cm | 3.2 |
| Tuna | 4.3 cm | 3.6 cm | 1.2 |

These cans do not have $h/r$ close to 2. Except for the tuna, the cans tend to have the ratio larger than 2. In other words, the cans are taller than we would expect if the goal is minimize surface area.

e) Possible reasons to explain the shape of the cans.

- A taller can might be stronger.
- A taller can might be easier to manufacture.
- The taller can be more convenient to put on shelves or be more pleasing to look at.

All of these are guesses.

Grading: a) Formula for surface area (1 point)

b) Express surface area in terms of one variable (1 point), critical number (1 point), find dimensions and compute ratio (1 point)

c) Explanation (1 point) Accept graphical or explanations using calculus

d) Measurements and computed ratio (1 point)

e) Speculation, accept any plausible explanation (1 point)

Add 1 point if majority of answers are expressed in complete sentences.