Exam 1

Name: ______________________ Section and/or TA: ______________

Do not remove this answer page — you will return the whole exam. You will be allowed two hours to complete this test. No books or notes may be used. You may use a graphing calculator during the exam, but NO calculator with a Computer Algebra System (CAS) or a QWERTY keyboard is permitted. Absolutely no cell phone use during the exam is allowed.

The exam consists of 10 multiple choice questions and 6 free response questions. Record your answers to the multiple choice questions on this page by filling in the circle corresponding to the correct answer.

Show all work to receive full credit on the free response problems. The wise student will show work for the multiple choice problems as well.

Multiple Choice Questions

1 (A) (B) (C) (D) (E)

2 (A) (B) (C) (D) (E)

3 (A) (B) (C) (D) (E)

4 (A) (B) (C) (D) (E)

5 (A) (B) (C) (D) (E)

6 (A) (B) (C) (D) (E)

7 (A) (B) (C) (D) (E)

8 (A) (B) (C) (D) (E)

9 (A) (B) (C) (D) (E)

10 (A) (B) (C) (D) (E)

SCORE

Multiple Choice	11	12	13	14	15	16	Total Score
40	10	10	10	10	10	10	100

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Multiple Choice Questions

1. Find all k for which the integral $\int_1^\infty \frac{1}{x^{5k/4}} dx$ diverges.

   A. $k \leq 1$
   B. $k \leq 4$
   C. $k \leq 4/5$
   D. $k \leq 5$
   E. $k \leq 5/4$

2. What substitution should we make in order to evaluate

$$\int \sqrt{4 - 9x^2}\, dx?$$

   A. $x = \frac{2}{3}\sin(u)$
   B. $x = \frac{3}{2}\sin(u)$
   C. $x = 2\sin(u)$
   D. $u = \frac{2}{3}\sin(x)$
   E. $u = \frac{3}{2}\sin(x)$

3. Let $f(x) = e^{-x^2}$. What is the smallest N that we should take so that the Trapezoid Rule approximation with N subintervals for $\int_0^3 f(x)\,dx$ is accurate to within 0.0001. Hint: $|f'(x)| \leq 1$ and $|f''(x)| \leq 2$ on $[0,3]$.

   A. 13

   B. 100

   C. 150

   D. 213

   E. 45000

4. Determine whether the improper integral converges or diverges, and if it converges, find its value.

$$\int_3^\infty \frac{1}{x^3} dx$$

   A. It diverges.

   B. It converges to 0.

   C. It converges to $1/18$.

   D. It converges to $1/9$.

   E. It converges to $1/2$.

5. If $x = \cos(u)$ and $0 \le u \le \pi$, find $\tan(u)$.

   A. $\sqrt{1-x^2}$

   B. $1/\sqrt{1-x^2}$

   C. $x/\sqrt{1-x^2}$

   D. $\sqrt{1-x^2}/x$

   E. $1/x$

6. Determine whether the improper integral converges or diverges, and if it converges, find its value.

$$\int_0^\infty \frac{3e^x}{1+e^{2x}}\,dx$$

   A. It diverges.

   B. $\frac{3\pi}{2}$

   C. $\frac{3\pi}{4}$

   D. $\frac{\pi}{2}$

   E. $\frac{\pi}{4}$

7. If we substitute $x = 2\cos(u)$ with $0 \le u \le \pi$ in the integral

$$\int \sqrt{4 - x^2}\, dx$$

we obtain

A. $-4 \int \sin^2(u)\, du$

B. $4 \int \sin^2(u)\, du$

C. $2 \int \cos^2(u)\, du$

D. $-2 \int \cos^2(u)\, du$

E. $2 \int \sin^2(u)\, du$

8. Evaluate

$$\int \frac{1}{(x-3)(x+1)}\, dx$$

A. $\ln|x-3| + \ln|x+1| + C$

B. $\frac{1}{4}\ln|x+1| - \frac{1}{4}\ln|x-3| + C$

C. $\frac{1}{4}\ln|x-3| - \frac{1}{4}\ln|x+1| + C$

D. $\frac{2x-2}{(x-3)^2(x+1)^2} + C$

E. $\ln|x-3|\ln|x+1| + C$

9. The partial fraction expansion of $\dfrac{x^2+4}{x^2(x-4)}$ is of the form:

A. $\dfrac{Ax+B}{x^2}+\dfrac{C}{x-4}$

B. $\dfrac{A}{x}+\dfrac{B}{x^2}+\dfrac{C}{x-4}$

C. $\dfrac{A}{x^2}+\dfrac{B}{x-4}$

D. $\dfrac{A}{x}+\dfrac{B}{x}+\dfrac{C}{x-4}$

E. $\dfrac{Ax^2+Bx+C}{x^2}+\dfrac{D}{x-4}$

10. Evaluate

$$\int x\cos(2x)\,dx$$

A. $\dfrac{1}{4}\sin(2x)+\dfrac{x}{2}\cos(2x)+C$

B. $\dfrac{1}{2}\cos(2x)+\dfrac{x}{2}\sin(2x)+C$

C. $\dfrac{1}{4}\cos(2x)+\dfrac{x}{2}\sin(2x)+C$

D. $\dfrac{x}{4}\cos(2x)+\dfrac{x}{2}\sin(2x)+C$

E. None of these.

Free Response Questions

11. Evaluate

$$\int \sin x \ln|\cos x|\, dx$$

Solution: Let $w = \cos x$, then $dw = -\sin x\, dx$ and

$$\int \sin x \ln|\cos x|\, dx = -\int \ln|w|\, dw$$

which we will do by parts.

$$u = \ln|w| \qquad dv = dw$$

$$du = \frac{1}{w}\, dw \qquad v = w$$

$$-\int \ln|w|\, dw = -w\ln|w| + \int dw$$

$$= -w\ln|w| + w + C$$

$$\int \sin x \ln|\cos x|\, dx = -\cos x \ln|\cos x| + \cos x + C.$$

12. The values for the function $f(x)$ are shown in the following table.

x	1	1.25	1.5	1.75	2	2.25	2.5	2.75	3
$f(x)$	2.72	2.23	1.95	1.77	1.65	1.56	1.49	1.44	1.40

Use Simpson's Rule with $n = 8$ to estimate the integral $\int_1^3 f(x)\, dx$. Round to two decimal places.

Solution:

$$S_8 = \frac{\Delta x}{3}(f(1) + 4f(1.25) + 2f(1.5) + 4f(1.75) + 2f(2) + 4f(2.25) + 2C(2.5) + 4C(2.75) + C(3))$$

$$= \frac{1}{12}(2.72 + 8.92 + 3.90 + 7.08 + 3.30 + 6.24 + 2.98 + 5.76 + 1.4)$$

$$= 3.525$$

13. Evaluate
$$\int \frac{1-2x-x^2}{x(x+1)^2}\,dx.$$

Solution: First break it into partial fractions:
$$\frac{1-2x-x^2}{x(x+1)^2} = \frac{A}{x} + \frac{B}{x+1} + \frac{C}{(x+1)^2}$$
$$1-2x-x^2 = A(x+1)^2 + Bx(x+1) + Cx$$

We get that $A = 1$, $B = -2$ and $C = -2$.
$$\int \frac{1-2x-x^2}{x(x+1)^2}\,dx = \int 1x - 2\int 1x+1 - 2\int 1x+1^2$$
$$= \ln|x| - 2\ln|x+1| + \frac{2}{x+1} + C$$

14. Evaluate
$$\int 4x^3 e^x\,dx.$$

Solution: Use integration by parts several times. In tabular form it looks like:

	u	dv	
		e^x	
$+$	$4x^3$	e^x	$4x^3e^x$
$-$	$12x^2$	e^x	$-12x^2e^x$
$+$	$24x$	e^x	$+24xe^x$
$-$	24	e^x	$-24e^x$
$+$	0	e^x	0

So the integral is $\int 4x^3e^x\,dx = (4x^3 - 12x^2 + 24x - 24)e^x + C$.

15. Determine whether the improper integral

$$\int_2^\infty \frac{4}{x^2+2x-3}\,dx$$

converges or diverges, and if it converges, find its value.

Solution: First, recognize this as an improper integral:

$$\int_2^\infty \frac{4}{x^2+2x-3}\,dx = \lim_{M\to\infty}\int_2^M \frac{4}{x^2+2x-3}\,dx.$$

Now, use partial fractions to do the antiderivative.

$$\int \frac{4}{x^2+2x-3}dx = \int \frac{1}{x-1} - \frac{1}{x+3}dx = \ln|x-1| - \ln|x+3| + C = \ln\left|\frac{x-1}{x+3}\right| + C$$

Then,

$$\begin{aligned}\int_2^\infty \frac{4}{x^2+2x-3}\,dx &= \lim_{M\to\infty}\int_2^M \frac{4}{x^2+2x-3}\,dx \\ &= \lim_{M\to\infty}\left(\ln\left|\frac{x-1}{x+3}\right|\right)_2^M \\ &= \lim_{M\to\infty} \ln\frac{M-1}{M+3} - \ln\frac{1}{5} \\ &= \ln 5\end{aligned}$$

16. Determine whether the improper integral

$$\int_9^{25} \frac{dx}{\sqrt{x-9}}$$

converges or diverges, and if it converges, find its value.

Solution:

$$\begin{aligned}\int_9^{25} \frac{dx}{\sqrt{x-9}} &= \lim_{a\to 9^+}\int_a^{25} \frac{dx}{\sqrt{x-9}} \\ &= \lim_{a\to 9^+} 2\sqrt{x-9}\Big|_a^{25} \\ &= 8\end{aligned}$$