Name: Answers Section: ____________

Do not remove this answer page — you will return the whole exam. You will be allowed two hours to complete this test. No books or notes may be used. You may use a graphing calculator during the exam, but NO calculator with a Computer Algebra System (CAS) or a QWERTY keyboard is permitted. Absolutely no cell phone use during the exam is allowed.

The exam consists of 10 multiple choice questions and 5 free response questions. Record your answers to the multiple choice questions on this page by filling in the circle corresponding to the correct answer.

Show all work to receive full credit on the free response problems. It will also help you check your answers to show work on multiple choice problems.

Multiple Choice Questions

1 (A) (B) (C) (D) ~~(E)~~ 6 (A) (B) ~~(C)~~ (D) (E)

2 (A) ~~(B)~~ (C) (D) (E) 7 (A) ~~(B)~~ (C) (D) (E)

3 (A) (B) (C) ~~(D)~~ (E) 8 (A) ~~(B)~~ (C) (D) (E)

4 ~~(A)~~ (B) (C) (D) (E) 9 (A) (B) (C) ~~(D)~~ (E)

5 (A) (B) (C) (D) ~~(E)~~ 10 (A) (B) ~~(C)~~ (D) (E)

Multiple Choice	11	12	13	14	15	Total Score
50	10	10	10	10	10	100

Multiple Choice Questions

1. (5 points) Select the answer that is equal to $\int x f(x)\, dx$.

   A. $x f(x) + \int x f'(x)\, dx$
   B. $x f(x) - \int x f'(x)\, dx$
   C. $\frac{x^2}{2} f(x) + \int x f'(x)\, dx$
   D. $x f(x) - \int \frac{x^2}{2} f(x)\, dx$
   E. $\frac{x^2}{2} f(x) - \int \frac{x^2}{2} f'(x)\, dx$

2. (5 points) What substitution would be most useful to evaluate the integral $\int \sqrt{x^2+4}\, dx$?

   A. $x = 2\sec(u)$
   B. $x = 2\tan(u)$
   C. $x = 2\sin(u)$
   D. $2u = \tan(x)$
   E. $u = 2\sin(x)$

3. (5 points) Four of the options below might appear in the partial fractions decomposition of

$$f(x) = \frac{x^3}{(x^2-1)(x^2+1)^2(x+1)}.$$

Select the option that does NOT appear in the partial fractions decomposition of f.

A. $\dfrac{A}{x-1}$

B. $\dfrac{B}{x+1}$

C. $\dfrac{C}{(x+1)^2}$

D. $\dfrac{D}{(x-1)^2}$

E. $\dfrac{E_1x+E_2}{x^2+1}$

4. (5 points) Let L_n and R_n be left and right endpoint approximations to the integral $I = \displaystyle\int_{\pi/2}^{\pi} \sin(x)\,dx$. Which of the following is true?

A. $R_n < I < L_n$

B. $I < R_n < L_n$

C. $R_n < L_n < I$

D. $L_n < I < R_n$

E. $L_n < R_n < I$

5. (5 points) Find the value of A in the partial fractions decomposition

$$\frac{2x}{(x+1)(x+3)} = \frac{A}{x+1} + \frac{B}{x+3}.$$

A. $A = 1$

B. $A = 3$

C. $A = 2$

D. $A = -3$

E. $A = -1$

6. (5 points) Find the anti-derivative $\int \sin(x)\cos^5(x)\,dx$.

A. $\frac{1}{6}\cos^6(x) + C$

B. $\frac{1}{6}\sin^6(x) + C$

C. $-\frac{1}{6}\cos^6(x) + C$

D. $-\frac{1}{6}\sin^6(x) + C$

E. $\frac{1}{2}\sin^2(x)\ \cos^5(x) + C$

7. (5 points) If $x = 2\sin(t)$ and $-\pi/2 < t < \pi/2$, find $\sec(t)$.

A. $\frac{1}{2}\sqrt{4-x^2}$

B. $2/\sqrt{4-x^2}$

C. $2\sqrt{1-4x^2}$

D. $1/(2\sqrt{1-4x^2})$

E. $x/\sqrt{4-x^2}$

8. (5 points) For which values of a does the integral $\int_0^\infty e^{ax}\,dx$ converge?

A. $(0, \infty)$

B. $(-\infty, 0)$

C. $(-\infty, 0]$

D. $[1, \infty)$

E. $(-\infty, -1)$

9. (5 points) A car is driving west along a straight road in west Texas. The table below gives its velocity every fifteen minutes over the course of an hour.

Time (minutes)	0	15	30	45	60
Velocity (miles/hour)	60	80	85	80	75

Use Simpson's rule with 4 subintervals to estimate the distance travelled in the hour and select the option below that is closest.

A. 76

B. 77

C. 78

D. 79

E. 80

10. (5 points) All of the integrals below except one are improper integrals. Select the answer that is NOT an improper integral.

A. $\displaystyle\int_0^\infty \frac{1}{1+x^2}\,dx$

B. $\displaystyle\int_0^1 \frac{1}{x^2}\,dx$

C. $\displaystyle\int_1^2 \frac{1}{x^2}\,dx$

D. $\displaystyle\int_1^\infty e^{-x}\,dx$

E. $\displaystyle\int_1^\infty x\,dx$

Free Response Questions

11. (a) (7 points) Find the anti-derivative

$$I = \int \sin(x)e^{2x}\,dx.$$

Solution: We integrate by parts twice.

let $u = e^{2x}$ $\quad v = -\cos(x)$

$du = 2e^{2x}\,dx$ $\quad dv = \sin(x)\,dx$

$$I = -e^{2x}\cos(x) + 2\int \cos(x)e^{2x}\,dx$$

let $u = e^{2x}$ $\quad v = \sin(x)$

$du = 2e^{2x}\,dx$ $\quad dv = \cos(x)\,dx$

$$I = -e^{2x}\cos(x) + 2\left(e^{2x}\sin(x) - 2\int \sin(x)e^{2x}\,dx\right).$$

$$\therefore \quad 5\int \sin(x)e^{2x}\,dx = -e^{2x}\cos(x) + 2e^{2x}\sin(x)$$

$$\int \sin(x)e^{2x}\,dx = -\frac{1}{5}e^{2x}\cos(x) + \frac{2}{5}e^{2x}\sin(x) + C.$$

(b) (3 points) Check your answer in part a) by differentiating.

Solution: Use product rule (1 point). Simplify (1 point). Obtain $\sin(x)e^{2x}$ (1 point).

Let $f(x) = -\frac{1}{5}e^{2x}\cos(x) + \frac{2}{5}e^{2x}\sin(x)$

Then $f'(x) = -\frac{2}{5}e^{2x}\cos(x) + \frac{1}{5}e^{2x}\sin(x) + \frac{4}{5}e^{2x}\sin(x) + \frac{2}{5}e^{2x}\cos(x)$

$= e^{2x}\sin(x).$

12. (10 points) Find the anti-derivative

$$\int \sqrt{9-x^2}\,dx.$$

Simplify your answer.

Let $x = 3\sin\theta$

$dx = 3\cos\theta\, d\theta$

$$\int \sqrt{9-x^2}\,dx = \int \sqrt{9-9\sin^2\theta}\,(3\cos\theta)\,d\theta$$

$$= \int \sqrt{9(1-\sin^2\theta)}\,(3\cos\theta)\,d\theta$$

$$= \int \sqrt{9\cos^2\theta}\,(3\cos\theta)\,d\theta$$

$$= \int 9\cos^2\theta\,d\theta$$

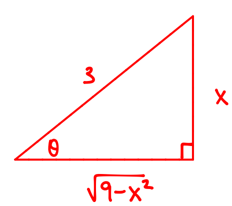
$$= \int \frac{9}{2}(1+\cos(2\theta))\,d\theta$$

$$= \frac{9}{2}\theta + \frac{9}{4}\sin(2\theta)$$

$$= \frac{9}{2}\theta + \frac{9}{2}\sin\theta\cos\theta$$

$$= \frac{9}{2}\arcsin\left(\frac{x}{3}\right) + \frac{9}{2}\,\frac{x}{3}\,\frac{\sqrt{9-x^2}}{3}$$

$$= \frac{9}{2}\arcsin\left(\frac{x}{3}\right) + \frac{1}{2}x\sqrt{9-x^2} + C.$$



13. If we use the Trapezoidal Rule T_n with n subintervals to approximate the integral $I = \int_a^b f(x)\,dx$, then the error $E_T = I - T_n$ satisfies the estimate

$$|E_T| \le \frac{K(b-a)^3}{12\,n^2}.$$

Here K is an upper bound for $|f''(x)|$ and thus $|f''(x)| \le K$ for $a \le x \le b$.

Consider the integral

$$I = \int_2^4 \frac{1}{x}\,dx.$$

(a) (4 points) Use the trapezoid rule with 4 subdivisions to compute an approximation of the value I. Round your answer to three decimal places.

$T_4 = \frac{0.5}{2}\left(\frac{1}{2} + 2\frac{1}{2.5} + 2\frac{1}{3} + 2\frac{1}{3.5} + \frac{1}{4}\right)$

$\approx 0.697.$

(b) (6 points) Find the smallest possible value for K that may be used when applying the error estimate to study the integral used to define I. Use the error estimate for the trapezoid rule to find the a value of n for which the error $|I - T_n|$ is at most 0.001.

Let $f(x) = x^{-1}$.

Then $f'(x) = -x^{-2}$, $f''(x) = 2x^{-3}$.

For $x \in [2,4]$, $|f''(x)| = \frac{2}{x^3} \le \frac{2}{2^3} = \frac{1}{4}$.

The smallest possible value for K is $\frac{1}{4}$.

$|E_T| \le \frac{\frac{1}{4}(2^3)}{12\,n^2} \le 0.001$ implies $n^2 \ge \frac{\frac{1}{4}(2^3)}{12(0.001)} = \frac{500}{3}$

or $n \ge 12.9\ldots$

When $n \ge 13$, the error $|E_T|$ is at most 0.001.

14. (10 points) Find the anti-derivative

$$\int \frac{9-x}{(x+1)(x^2+9)}\,dx.$$

The partial fraction decomposition is of the form

$$\frac{9-x}{(x+1)(x^2+9)} = \frac{A}{x+1} + \frac{Bx+C}{x^2+9} = \frac{A(x^2+9)+(Bx+C)(x+1)}{(x+1)(x^2+9)}.$$

$\therefore \quad 9-x = Ax^2 + 9A + Bx^2 + Cx + Bx + C.$

Solve
$$\begin{aligned} A + B &= 0 & (1)\\ B + C &= -1 & (2)\\ 9A + C &= 9. & (3) \end{aligned}$$

(1) implies $A = -B$. (2) implies $C = -1-B$.

Substitute into (3): $-9B - 1 - B = 9$ implies $B = -1$. So $A = 1$ and $C = 0$.

$$\int \frac{9-x}{(x+1)(x^2+9)}\,dx = \int \frac{1}{x+1} - \frac{x}{x^2+9}\,dx$$

$$= \ln|x+1| - \frac{1}{2}\ln(x^2+9) + C.$$

15. (a) (4 points) Use the definition to write the improper integral

$$\int_0^3 \frac{1}{\sqrt{x}}\,dx$$

as a limit of proper integrals.

$$\int_0^3 \frac{1}{\sqrt{x}}\,dx = \lim_{a\to 0^+} \int_a^3 \frac{1}{\sqrt{x}}\,dx$$

(b) (6 points) Evaluate the expression you found in part a) and determine if the improper integral is convergent. If it is convergent, give the value.

$$\int_0^3 \frac{1}{\sqrt{x}}\,dx = \lim_{a\to 0^+} \int_a^3 \frac{1}{\sqrt{x}}\,dx$$

$$= \lim_{a\to 0^+} 2x^{1/2}\Big|_a^3$$

$$= \lim_{a\to 0^+} 2\sqrt{3} - 2\sqrt{a}$$

$$= 2\sqrt{3}.$$

The improper integral is convergent and is equal to $2\sqrt{3}$.