# *Exam 1*

Name: ______________________________ Section: ______________

Do not remove this answer page — you will return the whole exam. You will be allowed two hours to complete this test. No books or notes may be used. You may use a graphing calculator during the exam, but NO calculator with a Computer Algebra System (CAS) or a QWERTY keyboard is permitted. Absolutely no cell phone use during the exam is allowed.

The exam consists of 10 multiple choice questions and 5 free response questions. Record your answers to the multiple choice questions on this page by filling in the circle corresponding to the correct answer.

Show all work to receive full credit on the free response problems. It will also help you check your answers to show work on multiple choice problems.

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Multiple Choice Questions

**1** (A) (B) (C) (D) (E)

**2** (A) (B) (C) (D) (E)

**3** (A) (B) (C) (D) (E)

**4** (A) (B) (C) (D) (E)

**5** (A) (B) (C) (D) (E)

**6** (A) (B) (C) (D) (E)

**7** (A) (B) (C) (D) (E)

**8** (A) (B) (C) (D) (E)

**9** (A) (B) (C) (D) (E)

**10** (A) (B) (C) (D) (E)

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| Multiple Choice | 11 | 12 | 13 | 14 | 15 | Total Score |
|---|---|---|---|---|---|---|
| 50 | 10 | 10 | 10 | 10 | 10 | 100 |
| | | | | | | |

Trig identities

- $\sin^2(x) + \cos^2(x) = 1$,
- $\sin^2(x) = \frac{1}{2}(1 - \cos(2x))$ and $\cos^2(x) = \frac{1}{2}(1 + \cos(2x))$
- $\sin(x + y) = \sin(x)\cos(y) + \cos(x)\sin(y)$ and $\cos(x + y) = \cos(x)\cos(y) - \sin(x)\sin(y)$

Multiple Choice Questions

1. (5 points) Find $\int w^2 \ln(w) dw$.

   A. $w^3 \ln(w) + C$

   B. $\ln(w)\frac{w^3}{3} - \frac{w^2}{3} + C$

   C. $w + C$

   **D.** $\ln(w)\frac{w^3}{3} - \frac{w^3}{9} + C$

   E. $\ln(w)\frac{w^2}{2} - w^2 + C$

2. (5 points) If $f(0) = 2$, $f(1) = 3$, $f'(0) = 5$ and $f'(1) = -4$, what is $\int_0^1 x f''(x) dx$?

   A. 5

   **B. -5**

   C. 2

   D. -2

   E. 0

3. (5 points) Find $\int \tan^2(x)\cos^3(x)dx$.

   **A.** $\frac{1}{3}\sin^3(x) + C$
   B. $\frac{1}{3}\cos^3(x) + C$
   C. $\frac{1}{3}\tan^3(x) + C$
   D. $\frac{1}{4}\sin^4(x) + C$
   E. $\frac{1}{4}\cos^4(x) + C$

4. (5 points) Find $\int_0^\pi \sin^5(x)dx$.

   **A.** $\frac{16}{15}$
   B. $\frac{6}{15}$
   C. $\frac{16}{5}$
   D. $\frac{5}{6}$
   E. $\frac{15}{16}$

5. (5 points) What substitution would you make to transform

$$\int x^2\sqrt{a^2-(bx)^2}dx \text{ to } -\frac{a^4}{b^3}\int \cos^2(u)\sin^2(u)du?$$

   A. $x = \frac{b}{a}\cos(u)$
   B. $x = \frac{b}{a}\sin(u)$
   C. $x = \cos(u)$
   D. $x = \sin(u)$
   **E.** $x = \frac{a}{b}\cos(u)$

6. (5 points) Find $\int \frac{x^2}{\sqrt{9-x^2}}dx$.

A. $\frac{9}{2}\left(\sin^{-1}\left(\frac{x}{3}\right)\right)+C$

B. $\frac{9}{2}\left(\frac{x}{3}\frac{\sqrt{3^2-x^2}}{3}\right)+C$

**C.** $\frac{9}{2}\left(\sin^{-1}\left(\frac{x}{3}\right)-\frac{x}{3}\frac{\sqrt{3^2-x^2}}{3}\right)+C$

D. $\frac{9}{2}\left(\cos^{-1}\left(\frac{x}{3}\right)-\frac{x}{3}\frac{\sqrt{3^2-x^2}}{3}\right)+C$

E. $\frac{9}{2}\left(\cos^{-1}\left(\frac{x}{3}\right)-\frac{x}{3}\frac{x}{3}\right)+C$

7. (5 points) What is the form of the partial fraction decomposition of

$$\frac{x^5+3x-2}{(x-1)(x-5)^3(x^2+1)^2}?$$

A. $\frac{A}{(x-1)}+\frac{D}{(x-5)^3}+\frac{Gx+H}{(x^2+1)^2}$

**B.** $\frac{A}{(x-1)}+\frac{B}{(x-5)}+\frac{C}{(x-5)^2}+\frac{D}{(x-5)^3}+\frac{Ex+F}{(x^2+1)}+\frac{Gx+H}{(x^2+1)^2}$

C. $\frac{A}{(x-1)}+\frac{B}{(x-5)}+\frac{C}{(x-5)^2}+\frac{D}{(x-5)^3}+\frac{E}{(x^2+1)}+\frac{F}{(x^2+1)^2}$

D. $\frac{A}{(x-1)}+\frac{B}{(x-5)}+\frac{D}{(x-5)^3}+\frac{Ex+F}{(x^2+1)}+\frac{Gx+H}{(x^2+1)^2}$

E. $\frac{A}{(x-1)}+\frac{B}{(x-5)}+\frac{Ex+F}{(x^2+1)}$

8. (5 points) Find the coefficient $C$ in the partial fraction decomposition

$$\frac{x}{(x-1)(x+1)^2}=\frac{A}{x-1}+\frac{B}{x+1}+\frac{C}{(x+1)^2}$$

**A.** $\frac{1}{2}$

B. $\frac{1}{4}$

C. $\frac{2}{3}$

D. 1

E. $-1$

9. (5 points) Each of the integrals below can be estimated using the midpoint rule. If we divide into 3 regions, which of the integrals below has an estimate of zero?

A. $\int_0^{\pi} \sin(x)dx$

B. $\int_0^2 x^2 dx$

C. $\int_{-1}^2 x^2 dx$

D. $\int_0^3 2dx$

**E.** $\int_{\frac{-\pi}{2}}^{\frac{5\pi}{2}} \sin(x)dx$

For E).

$M_3 = \pi(\sin(0) + \sin(\pi) + \sin(2\pi))$

$= 0.$

10. (5 points) Which of the following is the estimate of $\int_1^5 \frac{1}{x}dx$ using Simpson's rule with $n = 4$?

A. $\frac{17}{3}$

**B.** $\frac{17}{9}$

C. $\frac{4}{3}$

D. $\frac{1}{4}\ln(5)$

E. 3

$S_4 = \frac{4}{3\cdot 4}\cdot\left(1 + 4\cdot\frac{1}{2} + 2\cdot\frac{1}{3} + 4\cdot\frac{1}{4} + \frac{1}{5}\right).$

$= \frac{1}{3}\left(\frac{15}{15} + \frac{30}{15} + \frac{10}{15} + \frac{15}{15} + \frac{3}{15}\right)$

$= \frac{1}{3}\cdot\frac{1}{15}(73) = 73/15.$

Free Response Questions

11. (a) (8 points) Compute $\int (3t+5)\cos\left(\frac{t}{2}\right)\,dt$

**Solution:** Let $u = 3t+5$ and $dv = \cos\left(\frac{t}{2}\right)$. Then $u = 3dt$ and $v = 2\sin\left(\frac{t}{2}\right)$

$$\begin{aligned}\int (3t+5)\cos\left(\frac{t}{2}\right)dt &= (3t+5)2\sin\left(\frac{t}{2}\right) - \int 2\sin\left(\frac{t}{2}\right)3dt \\ &= (3t+5)2\sin\left(\frac{t}{2}\right) + 12\cos\left(\frac{t}{2}\right) + C\end{aligned}$$

(b) (2 points) Compute $\int_0^{\pi} (3t+5)\cos\left(\frac{t}{2}\right)\,dt$

12. (a) (8 points) Compute $\int \sin^6(x)\cos^3(x)dx$

**Solution:** $\sin^6 x \cos^3 x = \sin^6 x(1-\sin^2 x)\cos x = \sin^6 x \cos x - \sin^8 x \cos x$ So

$$\int \sin^6 x \cos^3 x dx = \int \sin^6 x \cos x dx - \int \sin^8 x \cos x dx$$

Then use $u$-substitution with $u = \sin x$ to git

$$\begin{aligned}\int \sin^6 x \cos x dx - \int \sin^8 x \cos x dx &= \int u^6 du - \int u^8 du \\ &= \frac{1}{7}u^7 - \frac{1}{9}u^9 + C \\ &= \frac{1}{7}\sin^7 x - \frac{1}{9}\sin^9 x + C\end{aligned}$$

(b) (2 points) Compute $\int_0^\pi \sin^6(x)\cos^3(x)dx$

13. (a) (8 points) Compute $\int \frac{1}{(1+x^2)^2}dx$

**Solution:** The identity $\sin^2 x + \cos^2 x = 1$ gives $\tan^2 x + 1 = \sec^2 x$. Use the substitution $x = \tan u$ and $dx = \sec^2 u du$ to get

$$\begin{aligned}
\int \frac{1}{(1+x^2)^2}dx &= \int \frac{1}{(1+\tan^2 u)^2}\sec^2 u du \\
&= \int \frac{1}{\sec^4 u}\sec^2 u du = \int \cos^2 u du = \frac{1}{2}\int (1+\cos(2u))du \\
&= \frac{1}{2}\left(u + \frac{1}{2}\sin(2u)\right) + C = \frac{1}{2}(u + \cos u \sin(u)) + C \\
&= \frac{1}{2}\left(\tan^{-1} x + \frac{1}{\sqrt{1+x^2}}\frac{x}{\sqrt{1+x^2}}\right) + C \\
&= \frac{1}{2}\left(\tan^{-1} x + \frac{x}{1+x^2}\right) + C
\end{aligned}$$

(b) (2 points) Compute $\int_{-1}^{1} \frac{1}{(1+x^2)^2}dx$

**Solution:**

$$\begin{aligned}
\int_{-1}^{1} \frac{1}{(1+x^2)^2}dx &= \frac{1}{2}\left(\tan^{-1} x + \frac{x}{1+x^2}\right)\Big|_{-1}^{1} \\
&= \frac{1}{2}\left(\tan^{-1}(1) + \frac{1}{1+1^2}\right) - \frac{1}{2}\left(\tan^{-1}(-1) + \frac{-1}{1+(-1)^2}\right) \\
&= \frac{1}{2}\left(\frac{\pi}{4} + \frac{1}{2}\right) - \frac{1}{2}\left(\frac{-\pi}{4} + \frac{-1}{2}\right) \\
&= \frac{1}{2}\left(\frac{\pi}{2} + 1\right)
\end{aligned}$$

14. (10 points) Using the method of partial fractions, compute

$$\int \frac{4}{(x-1)^2(x+1)}\,dx.$$

**Solution:**

$$\begin{aligned}
\frac{4}{(x-1)^2(x+1)} &= \frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{C}{x+1} \\
4 &= A(x-1)(x+1) + B(x+1) + C(x-1)^2 \\
&= A(x^2+x-x-1) + B(x+1) + C(x^2-2x+1) \\
&= Ax^2 - A + Bx + B + Cx^2 - 2Cx + C \\
&= (A+C)x^2 + (B-2C)x - A + B + C
\end{aligned}$$

Then

$$\begin{aligned}
0 &= A + C \\
0 &= B - 2C \\
4 &= -A + B + C
\end{aligned}$$

Taking $A = -C$ and $B = 2C$, we have

$$4 = -(-C) + 2C + C$$

So $C = 1$, $A = -1$ and $B = 2$

$$\begin{aligned}
\int \frac{6}{(x-2)^2(x+1)}\,dx &= \int \frac{-1}{x-1} + \frac{2}{(x-1)^2} + \frac{1}{x+1} dx \\
&= -\ln|x-1| - \frac{2}{x-1} + \ln|x+1| + C
\end{aligned}$$

15. (a) (4 points) Use the midpoint rule to estimate the integral

$$\int_{-1}^{3} x^2 dx$$

Use four intervals.

**Solution:**

$$\begin{aligned} M_4 &= \frac{1}{1}\left(\left(-\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2 + \left(1+\frac{1}{2}\right)^2 + \left(2+\frac{1}{2}\right)^2\right) \\ &= \left(-\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2 + \left(\frac{3}{2}\right)^2 + \left(\frac{5}{2}\right)^2 \\ &= \frac{1}{4} + \frac{1}{4} + \frac{9}{4} + \frac{25}{4} = \frac{1}{4}36 = 9 \end{aligned}$$

(b) (4 points) Use the trapezoid rule to estimate the integral

$$\int_{-1}^{3} x^2 dx$$

Use four intervals.

**Solution:**

$$\begin{aligned} T_4 &= \frac{1}{1}\frac{1}{2}\left(\left((-1)^2 + 0^2\right) + \left(0^2 + 1^2\right) + \left(1^2 + 2^2\right) + \left(2^2 + 3^2\right)\right) \\ &= \frac{1}{2}(1 + 1 + 5 + 13) \\ &= \frac{20}{2} = 10 \end{aligned}$$

(c) (2 points) Which of these estimates is closer to the actual value?

**Solution:** $\int x^2 dx = \frac{1}{3}x^3 + C$ so $\int_{-1}^{3} x^2 dx = \frac{1}{3}(3^3 - (-1)^3) = \frac{27+1}{3} = \frac{28}{3}$