Exam 1

Name: _____   Section: _____

Do not remove this answer page — you will return the whole exam. You will be allowed two hours to complete this test. If you find you need scratch paper during the exam, please ask. You may not use any of your own notes, paper, or anything else not provided. You may use a graphing calculator during the exam, but NO calculator with a Computer Algebra System (CAS). Absolutely no communication or use of a communication device is allowed during the exam.

The exam consists of 10 multiple choice questions and 5 free response questions. Record your answers to the multiple choice questions on this page by filling in the circle corresponding to the correct answer.

Show **all work** using proper notation to receive full credit on the free response problems. It will also help you check your answers to show work on multiple choice problems.

Multiple Choice Questions

1 (A) (B) (C) (D) (E)   **6** (A) (B) (C) (D) (E)

2 (A) (B) (C) (D) (E)   **7** (A) (B) (C) (D) (E)

3 (A) (B) (C) (D) (E)   **8** (A) (B) (C) (D) (E)

4 (A) (B) (C) (D) (E)   **9** (A) (B) (C) (D) (E)

5 (A) (B) (C) (D) (E)   **10** (A) (B) (C) (D) (E)

Multiple Choice	11	12	13	14	15	Total Score
50	10	10	10	10	10	100

This page may be used for scratch work.

Multiple Choice Questions

1. (5 points) Find $\displaystyle\int (5x + 4)e^{2x}\ dx$.

     A. $\dfrac{5}{4}(5x + 4)e^{2x} + \dfrac{1}{4}e^{2x} + C$

     B. $2(5x + 4)e^{2x} - 20e^{2x} + C$

     C. $2(5x + 4)e^{2x} + 20e^{2x} + C$

     D. $\dfrac{1}{2}(5x + 4)e^{2x} - \dfrac{5}{4}e^{2x} + C$

     E. $2(5x + 4)e^{2x} - 4e^{2x} + C$

2. (5 points) If $f(-1) = 5$, $f(2) = 3$, $f'(-1) = -7$ and $f'(2) = 12$, and $f''(x)$ is continuous, what is $\displaystyle\int_{-1}^{2}(2x - 1)f''(x)\ dx$?

     A. 41

     B. 73

     C. 19

     D. 11

     E. 61

3. (5 points) Find $\int \sin(2x)\sec(x)\,dx$.

     A. $-2\cos(x) + C$

     B. $\frac{1}{2}\cos(2x)\cdot\ln|\sec(x)+\tan(x)| + C$

     C. $\ln|\sec(x)+\tan(x)| + C$

     D. $2\cos(2x) + C$

     E. $-2\cos(2x)\cdot\ln|\sec(x)+\tan(x)| + C$

4. (5 points) Find $\int 2\sin^3(4x)\,dx$.

     A. $-\dfrac{1}{2}\left(\cos(4x) - \dfrac{\cos^3(4x)}{3}\right) + C$

     B. $2\left(\cos(4x) + \dfrac{\cos^3(4x)}{3}\right) + C$

     C. $\dfrac{1}{2}\left(\cos(4x) + \dfrac{\cos^3(4x)}{3}\right) + C$

     D. $\dfrac{1}{2}\left(\cos(4x) - \dfrac{\cos^3(4x)}{3}\right) + C$

     E. $-2\left(\cos(4x) - \dfrac{\cos^3(4x)}{3}\right) + C$

5. (5 points) If $x = 3\sin(\theta)$, which of the following is equal to $\tan(\theta)$?

    A. $\dfrac{3}{\sqrt{9-x^2}}$

    B. $\dfrac{x}{\sqrt{9-x^2}}$

    C. $\dfrac{3}{x}$

    D. $\dfrac{\sqrt{9-x^2}}{x}$

    E. $\dfrac{\sqrt{9-x^2}}{3}$

6. (5 points) Which of the following is equal to $\displaystyle\int \sqrt{4-x^2}\,dx$ after making the substitution $x = 2\sin(\theta)$?

    A. $\displaystyle\int \cos^2(\theta)\,d\theta$

    B. $\displaystyle\int 2\cos(\theta)\,d\theta$

    C. $\displaystyle\int 4\sin^2(\theta)\,d\theta$

    D. $\displaystyle\int \sin(\theta)\,d\theta$

    E. $\displaystyle\int 4\cos^2(\theta)\,d\theta$

7. (5 points) Choose the correct Partial Fraction Decomposition form for $\dfrac{2x^2 - 4x + 1}{(x^2 + 16)(x^2 - 2x + 1)}$.

A. $\dfrac{A}{x^2 + 16} + \dfrac{B}{x^2 - 2x + 1}$

B. $\dfrac{Ax + B}{x^2 + 16} + \dfrac{C}{x^2 - 2x + 1}$

C. $\dfrac{Ax + B}{x^2 + 16} + \dfrac{C}{x - 1} + \dfrac{D}{(x - 1)^2}$

D. $\dfrac{A}{x^2} + \dfrac{B}{16} + \dfrac{C}{x^2} + \dfrac{D}{-2x} + \dfrac{E}{1}$

E. $\dfrac{Ax + B}{x^2 + 16} + \dfrac{C}{x - 1} + \dfrac{D}{x - 1}$

8. (5 points) In the Partial Fraction Decomposition $\dfrac{3x + 1}{(x + 3)(x - 1)} = \dfrac{A}{x + 3} + \dfrac{B}{x - 1}$, what are the values of A and B?

A. $A = 3$ and $B = 1$

B. $A = 2$ and $B = 1$

C. $A = -3$ and $B = 1$

D. $A = 1$ and $B = 2$

E. $A = 1$ and $B = -3$

9. (5 points) Evaluate $\int_0^\infty \dfrac{1}{(x+2)^2}\, dx$.

     A. ∞

     B. $\dfrac{3}{2}$

     C. $\dfrac{1}{5}$

     D. $\dfrac{1}{2}$

     E. $\dfrac{1}{4}$

10. (5 points) Find the Trapezoidal rule estimate of $\int_2^5 f(x)\, dx$ with $n = 6$.

     A. $\dfrac{3}{2}\big(f(2) + 4f(2.5) + 2f(3) + 4f(3.5) + 2f(4) + 4f(4.5) + f(5)\big)$

     B. $\dfrac{1}{2}\big(f(2.25) + f(2.75) + f(3.75) + f(4.25) + f(4.75)\big)$

     C. $\dfrac{1}{6}\big(f(2) + 2f(2.5) + 2f(3) + 2f(3.5) + 2f(4) + 2f(4.5) + f(5)\big)$

     D. $\dfrac{1}{2}\big(f(2) + f(2.5) + f(3) + f(3.5) + f(4) + f(4.5)\big)$

     E. $\dfrac{1}{4}\big(f(2) + 2f(2.5) + 2f(3) + 2f(3.5) + 2f(4) + 2f(4.5) + f(5)\big)$

Free Response Questions: Show all steps clearly to receive full credit.

11. (a) (5 points) Compute $\int x^2\, e^x\, dx$.

(b) (5 points) Compute $\int \dfrac{1}{x^2 + 10x + 40}\, dx$.

12. (10 points) Evaluate $\displaystyle\int \frac{1}{\sqrt{9x^2 - 16}}\, dx$ using trigonometric substitution.
    You **must** simplify your answer such that it does not include any inverse trigonometric functions.

13. (10 points) Determine whether the improper integral converges, and if so, evaluate it. Justify your answer by showing your work.

$$\int_0^\infty x\, e^{-x^2}\, dx$$

14. (10 points) Using the method of partial fractions, compute

$$\int \frac{4x^2 + 5x - 1}{(x - 2)(x + 3)^2} \, dx.$$

15. (a) (5 points) Let $f(x) = e^{-x^2}$. Use the **Trapezoid Rule** to approximate $\int_1^3 f(x)\, dx$ with $n = 4$. If the form of your final answer is a decimal, it must be accurate to two decimal places.

(b) (5 points) Use the fact that $|f''(x)| \leq 1$ on $[1, 3]$ to estimate the error in your approximation from part (a). If the form of your final answer is a decimal, it must be accurate to two decimal places.