**MA 114 — Calculus II** Spring 2010
**Exam 1** February 9, 2010

This is a closed book exam. No books or notes are to be used during the exam. You may use a graphing calculator if it does not have symbolic manipulation capabilities. However, any device capable of electronic communication (cell phone, pager, etc.) must be turned off and out of sight during the exam.

Each question is followed by space to write your answer. Please write your solutions neatly in the space below the question. *Show your work.* Answers without justification will receive no credit. Partial credit for a problem will be given only when there is coherent written evidence that you have solved part of the problem. In particular, answers that are obtained simply as the output of calculator routines will receive no credit.

**Name:** ______________________________

**Section:** __________

**Last four digits of student identification number:** __________

| Problem | Score | Total |
|---|---|---|
| 1 | | 15 |
| 2 | | 15 |
| 3 | | 10 |
| 4 | | 10 |
| 5 | | 15 |
| 6 | | 15 |
| 7 | | 15 |
| 8 | | 15 |
| | | 110 |

**1)** (15 pts) This problem concerns the infinite series $2 + \frac{4}{3} + \frac{8}{9} + \frac{16}{27} + \dots$ .

(a) (7 pts) Write the series in summation notation where summation starts at $n = 1$.

(b) (8 pts) Find the sum of this series or show that the series diverges.

**2)** (15 pts) This problem concerns the series $\sum_{n=1}^{\infty} \frac{1}{(n+1)(n+2)}$.

(a) (8 pts) Find as simple a formula as possible for the $n$th partial sum of the series.
Hint: $\frac{1}{(n+1)(n+2)} = \frac{1}{n+1} - \frac{1}{n+2}$.

(b) (7 pts) Using your result from part (a), determine whether or not the series converges and, if so, find its sum.

**3)** (10 pts) Does the series $\sum_{n=1}^{\infty} \frac{n^2+2}{(n+1)^3}$ converge? Use the Comparison Test or the Limit Comparison Test to justify your answer. Explain carefully how you apply it.

**4)** (10 pts) Using a convergence test, determine whether or not the series $\sum_{n=1}^{\infty} \frac{3^n n^2}{n!}$ converges. State the name of the convergence test you use and explain carefully how you apply it.

**5)** (15 pts) This problem concerns the alternating series $\sum_{n=1}^{\infty} \frac{(-1)^n}{n5^n}$.

(a) (8 pts) Verify in detail that the series satisfies the hypotheses of the alternating series test.

(b) (7 pts) Show how to apply the Alternating Series Estimation Theorem to determine the least number of terms that need to be added to be sure that the value obtained is within 0.001 of the sum of the series.

**6)** (15 pts) Does the series $\sum_{n=1}^{\infty} \frac{\sin(n)}{n+2^n}$ converge? Use one or more convergence tests to justify your answer. State the name of each convergence test you use and explain carefully how you apply it.

**7)** (15 pts) This problem concerns the power series

$$\sum_{n=1}^{\infty} \frac{x^n}{4^n n^2}.$$

(a) (7 pts) Find a number $R$ so that this series converges for $|x| < R$ and diverges for $|x| > R$. Justify your answer by applying a convergence test. State clearly which convergence test you use and explain carefully how you apply it.

(b) (8 pts) Find the interval of convergence for this series. Be sure to test the endpoints $x = R$ and $x = -R$ and justify each answer with a convergence test. State clearly which convergence test you use and explain carefully how you apply it.

**8)** (15 pts)

(a) (5 pts) Find a power series representation for the function $f(x) = \frac{3x}{4 - x^2}$ using summation notation.

(b) (5 pts) Find the radius of convergence for the power series in part (a). Be sure to justify your answer using a convergence test or theorem. State the name of the convergence test or theorem you use and explain carefully how you apply it.

(c) (5 pts) A certain function $g(x)$ is known to satisfy $g(x) = \sum_{n=1}^{\infty} \frac{n!}{n^n} x^{2n}$ for all $x$ with $|x| < R$, where $R = \sqrt{e}$ is the radius of convergence of the power series. Use this fact to find a power series representation for $g'(x)$ and give its radius of convergence.