Exam 1

Name: _____ Section: _____

Do not remove this answer page — you will return the whole exam. You will be allowed two hours to complete this test. If you find you need scratch paper during the exam, please ask. You may not use any of your own notes, paper or anything else not provided. You may use a graphing calculator during the exam, but NO calculator with a Computer Algebra System (CAS). Absolutely no communication device use during the exam is allowed.

The exam consists of 10 multiple choice questions and 5 free response questions. Record your answers to the multiple choice questions on this page by filling in the circle corresponding to the correct answer.

Show **all work** using proper notation to receive full credit on the free response problems. It will also help you check your answers to show work on multiple choice problems.

Multiple Choice Questions

1	A	B	C	D	E	**6**	A	B	C	D	E
2	A	B	C	D	E	**7**	A	B	C	D	E
3	A	B	C	D	E	**8**	A	B	C	D	E
4	A	B	C	D	E	**9**	A	B	C	D	E
5	A	B	C	D	E	**10**	A	B	C	D	E

Multiple Choice	11	12	13	14	15	Total Score
50	10	10	10	10	10	100

This page may be used for scratch work.

Multiple Choice Questions

1. (5 points) Find $\int x\cos(2x)\,dx$.

   A. $\frac{1}{4}x^2\sin(2x)+C$

   B. $x^2\sin(2x)-x^2\cos(2x)+C$

   C. $2x\sin(2x)-x^2\sin(2x)+C$

   D. $\frac{1}{2}x\sin(2x)+\frac{1}{4}\cos(2x)+C$

   E. $\frac{1}{2}x^2\sin(2x)+\frac{1}{2}x\cos(2x)+C$

2. (5 points) If $f(0)=3$, $f(4)=2$, $f'(0)=35$ and $f'(4)=-1$, and $f''(x)$ is continuous, what is $\int_0^4 (5x+1)f''(x)\,dx$?

   A. -36

   B. -44

   C. -51

   D. -55

   E. -79

3. (5 points) Find $\int (3+\cos x)^2 \, dx$.

A. $\frac{19}{2}x + \frac{1}{2}\sin x \cos x + C$

B. $\frac{19}{2}x + 6\sin x + \frac{1}{4}\sin(2x) + C$

C. $9x + 6\sin x + \frac{1}{3}\cos^3 x + C$

D. $9x + \frac{1}{3}\cos^3 x + C$

E. $\frac{1}{3}(3+\cos x)^3 + C$

4. (5 points) Find $\int \sin^4(3x)\cos^3(3x) \, dx$.

A. $\frac{1}{15}\sin^5(3x) - \frac{1}{21}\sin^7(3x) + C$

B. $\frac{1}{9}\cos^9(3x) - \frac{1}{4}\sin^4(3x) + C$

C. $\left(\frac{1}{2}x - \frac{1}{4}\sin(6x)\right)^4 + C$

D. $\frac{1}{15}\sin^5(3x)\left(1-\sin^2(3x)\right) + C$

E. $\frac{1}{60}\sin^5(3x)\cos^4(3x) + C$

5. (5 points) Which of the following is equal to the integral

$$\int \frac{dx}{x^2\sqrt{x^2+25}}$$

after making the substitution $x = 5\tan(\theta)$?

A. $\displaystyle\int 5\sec^2\theta \; d\theta$

B. $\displaystyle\int \frac{d\theta}{125\tan^2\theta\sec\theta}$

C. $\displaystyle\int \frac{\sec^2\theta \; d\theta}{5\tan^2\theta(\tan\theta+5)}$

D. $\displaystyle\int \frac{d\theta}{25\tan^3\theta+125\tan^2\theta}$

E. $\displaystyle\int \frac{\sec\theta}{25\tan^2\theta} \; d\theta$

6. (5 points) Which expression below corresponds to $\sin\left(\arctan\left(\frac{x}{5}\right)\right)$?

A. $\dfrac{\sqrt{x^2+25}}{5}$

B. $\dfrac{x}{\sqrt{x^2+25}}$

C. $\dfrac{5}{\sqrt{x^2-25}}$

D. $\dfrac{\sqrt{25-x^2}}{5}$

E. $\dfrac{x}{\sqrt{25-x^2}}$

7. (5 points) Choose the form of the partial fraction decomposition of

$$\frac{2x+3}{x^3(x^2+16)}$$

A. $\dfrac{A}{x}+\dfrac{B}{x^2}+\dfrac{C}{x^3}+\dfrac{D}{x^2+16}$

B. $\dfrac{A}{x}+\dfrac{B}{x^2}+\dfrac{C}{x^3}+\dfrac{Dx+E}{x^2+16}$

C. $\dfrac{A}{x}+\dfrac{B}{x^2}+\dfrac{C}{x^3}+\dfrac{D}{x+4}+\dfrac{E}{(x+4)^2}$

D. $\dfrac{A}{x^3}+\dfrac{B}{x^2+16}$

E. $\dfrac{Ax+B}{x^3}+\dfrac{Cx+D}{x^2+16}$

8. (5 points) Let $f(x)$ be a function that satisfies $|f''(x)| \leq 2$ on the interval $[1,7]$. If we estimate $\displaystyle\int_1^7 f(x)\,dx$ using Trapezoidal integration with 12 subintervals, how accurate can we expect our approximation to be?

A. within .25

B. within .75

C. within .45

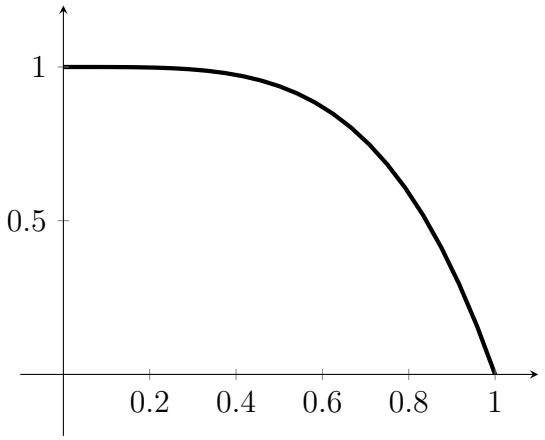
D. within .5

E. within .005

9. (5 points) Find $\displaystyle\int_5^\infty \frac{1}{x^3}\,dx$.

   A. $\dfrac{1}{50}$
   B. $\dfrac{1}{45}$
   C. $\dfrac{1}{125}$
   D. $\dfrac{1}{2500}$
   E. ∞

10. (5 points) For the graph of $y = f(x)$ shown, let I be the value of $\displaystyle\int_0^1 f(x)\,dx$, and let L_n, R_n, M_n and T_n be the approximations using left, right, midpoint and trapezoidal integration. List these in order **from smallest to largest**.



   A. L_n, T_n, I, M_n, R_n
   B. R_n, T_n, I, M_n, L_n
   C. R_n, M_n, I, T_n, L_n
   D. L_n, M_n, I, T_n, R_n
   E. L_n, T_n, M_n, R_n, I

Free Response Questions: Show all steps clearly to receive full credit.

11. (a) (5 points) Compute $\int x^5 \ln x \, dx$.

(b) (5 points) Compute $\int \frac{7-2x}{x^2+1} \, dx$.

12. (10 points) Compute $\displaystyle\int \frac{1}{(16-x^2)^{3/2}}\,dx$ using trigonometric substitution. You **must** simplify your answer.

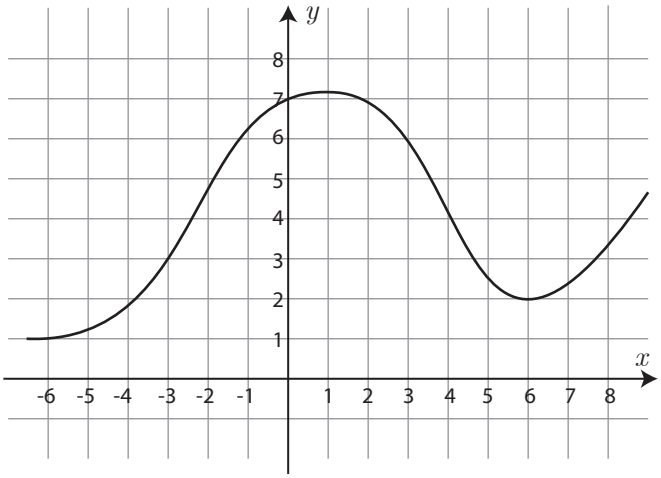
13. (10 points) Determine whether the improper integral converges, and if so, evaluate it. Justify your answer by showing your work and using proper notation.

$$\int_7^{20} \frac{dx}{(x-7)^4}$$

14. (10 points) Using the method of partial fractions, compute

$$\int \frac{3x^2 - 16x + 31}{(x-1)(x-4)^2}\, dx.$$

15. (a) (5 points) Apply the **Trapezoidal** rule to estimate the integral $\int_{-3}^{6} f(x)\,dx$ using 3 intervals (i.e, find T_3), where the graph of $f(x)$ is given below.



(b) (5 points) Use the **Midpoint** rule to estimate $\int_{1}^{9} \frac{1}{x^2}\,dx$ using 4 intervals, i.e. find M_4. You do not need to simplify your answer.