Exam 1

Name: _____ Section: _____

Do not remove this answer page — you will return the whole exam. You will be allowed two hours to complete this test. If you find you need scratch paper during the exam, please ask. You may not use any of your own notes, paper, or anything else not provided. You may use a graphing calculator during the exam, but NO calculator with a Computer Algebra System (CAS). Absolutely no communication or use of a communication device is allowed during the exam.

The exam consists of 10 multiple choice questions and 5 free response questions. Record your answers to the multiple choice questions on this page by filling in the circle corresponding to the correct answer.

Show **all work** using proper notation to receive full credit on the free response problems. It will also help you check your answers to show work on multiple choice problems.

Multiple Choice Questions

1	A	B	C	D	E	**6**	A	B	C	D	E
2	A	B	C	D	E	**7**	A	B	C	D	E
3	A	B	C	D	E	**8**	A	B	C	D	E
4	A	B	C	D	E	**9**	A	B	C	D	E
5	A	B	C	D	E	**10**	A	B	C	D	E

Multiple Choice	11	12	13	14	15	Total Score
50	10	10	10	10	10	100

This page may be used for scratch work.

Multiple Choice Questions

1. (5 points) Evaluate $\int 2xe^{3x}\,dx$.

   A. $\frac{1}{3}e^{3x}(3x)+C$

   B. $\frac{1}{3}x^2e^{3x}+C$

   C. $\frac{2}{3}e^{3x}(3x-1)+C$

   D. $\frac{2}{9}e^{3x}(3x-1)+C$

   E. $\frac{2}{3}e^{3x}(3x)+C$

2. (5 points) Evaluate $\int x^3\cos(x^2)\,dx$.

   A. $\frac{1}{2}(x\sin(x)-2x\cos(x))+C$

   B. $\frac{1}{2}(x^2\sin(x^2)+\cos(x^2))+C$

   C. $\frac{1}{4}\left(x^4\sin\left(\frac{x^3}{3}\right)\right)+C$

   D. $\frac{1}{4}\left(3x^2\cos(x^2)-2x^4\sin(x^2)\right)+C$

   E. $\frac{1}{4}(3x^2\cos(x^2))+C$

3. (5 points) Evaluate $\int \sin(x)\sec^2(x)\,dx$.

A. $\cos(x) + C$

B. $\sin(x) + C$

C. $\sec(x) + C$

D. $-\cos(x) + C$

E. $-\sec(x) + C$

4. (5 points) Evaluate $\int (1 - \sin(4x))^2\,dx$.

A. $\frac{1}{16}(24x - \sin(8x) + 8\cos(4x)) + C$

B. $\frac{1}{16}(24x + \sin(4x) - 8\cos(4x)) + C$

C. $\frac{1}{16}(12x + 8\sin(8x) - \cos(4x)) + C$

D. $\frac{1}{16}(12x + 8\sin(4x) - \cos(8x)) + C$

E. $\frac{1}{16}(24x - 8\sin(8x) + \cos(4x)) + C$

5. (5 points) If $\sin(\theta) = \dfrac{x}{4}$, which of the following corresponds to $\cos(\theta)$?

A. $\dfrac{x}{\sqrt{16 - x^2}}$

B. $\dfrac{x}{\sqrt{16 + x^2}}$

C. $\dfrac{4}{\sqrt{16 + x^2}}$

D. $\dfrac{\sqrt{16 - x^2}}{4}$

E. $\dfrac{x}{4\sqrt{16 + x^2}}$

6. (5 points) Which of the following is equal to $\displaystyle\int \frac{1}{x^2\sqrt{4 - x^2}}\, dx$ after making the substitution $x = 2\sin(\theta)$?

A. $\displaystyle\int 4\cos^2(\theta)\, d\theta$

B. $\displaystyle\int 2\sin^2(\theta)\, d\theta$

C. $\displaystyle\int \frac{1}{4}\sec(\theta)\, d\theta$

D. $\displaystyle\int \frac{1}{2}\sec^2(\theta)\, d\theta$

E. $\displaystyle\int \frac{1}{4}\csc^2(\theta)\, d\theta$

7. (5 points) What is the form of the partial fraction decomposition of $\dfrac{x+1}{(x-1)(x+2)^2(x^2+4)}$?

A. $\dfrac{A}{x-1}+\dfrac{B}{x+2}+\dfrac{C}{(x+2)^2}+\dfrac{Dx+E}{x^2+4}$

B. $\dfrac{A}{x-1}+\dfrac{B}{x+2}+\dfrac{C}{x+2}+\dfrac{Dx+E}{x^2+4}$

C. $\dfrac{A}{x-1}+\dfrac{B}{(x+2)^2}+\dfrac{Cx+D}{x^2+4}$

D. $\dfrac{A}{x-1}+\dfrac{B}{(x+2)^2}+\dfrac{Cx}{x^2+4}$

E. $\dfrac{A}{x-1}+\dfrac{B}{(x+2)^2}+\dfrac{C}{x^2+4}$

8. (5 points) In the partial fraction decomposition $\dfrac{1}{(x+7)(x+6)}=\dfrac{A}{x+7}+\dfrac{B}{x+6}$, what are the values of A and B?

A. $A=1$ and $B=-1$

B. $A=-1$ **and** $B=1$

C. $A=6$ and $B=7$

D. $A=7$ and $B=6$

E. $A=1$ and $B=4$

9. (5 points) Evaluate $\displaystyle\int_{5/4}^{6} \frac{1}{(4x-5)^{3/2}}\,dx$.

A. $4/5$

B. 1

C. $5\pi/2$

D. $\pi/2$

E. The integral diverges to ∞.

10. (5 points) Evaluate $\displaystyle\int_{0}^{\infty} \frac{5}{x^2+1}\,dx$.

A. $4/5$

B. 1

C. $\pi/2$

D. $5\pi/2$

E. The integral diverges to ∞.

Free Response Questions: Show all steps clearly to receive full credit.

11. (a) (5 points) Compute $\int \sin^3(5x)\, dx$.

Solution:

$$\begin{aligned}
&= \int \sin^2(5x)\sin(5x)\, dx \\
&= \int (1-\cos^2(5x)\sin(5x)\, dx \\
&\quad [u = \cos(5x), du = -5\sin(5x)dx] \\
&= -\frac{1}{5}\int 1-u^2\, du \\
&= -\frac{1}{5}[u - \frac{1}{3}u^3] + C \\
&= -\frac{1}{5}[\cos(5x) - \frac{1}{3}\cos^3(5x)] + C
\end{aligned}$$

(b) (5 points) Compute $\displaystyle\int \frac{1}{x^2+4x+13}\, dx$.

Solution: Note that $x^2+4x+13 = (x+2)^2+9$. Therefore,

$$\int \frac{1}{x^2+4x+13}\, dx = \int \frac{1}{(x+2)^2+9}\, dx = \frac{1}{3}\arctan\left(\frac{x+2}{3}\right) + C$$

12. (10 points) Evaluate $\displaystyle\int \frac{x^2}{\sqrt{4-x^2}}\,dx$ using trigonometric substitution.

Solution: We use the substitution that $x = 2\sin(\theta)$. Therefore, $dx = 2\cos(\theta)\,d\theta$. Plugging in this substitution gives:

$$\begin{aligned}
\int \frac{x^2}{\sqrt{4-x^2}}\,dx &= \int \frac{(2\sin(\theta))^2}{\sqrt{4-(2\sin(\theta))^2}} \cdot (2\cos(\theta))\,d\theta \\
&= \int \frac{4\sin^2(\theta)\cdot(2\cos(\theta))}{2\sqrt{1-\sin^2(\theta)}}\,d\theta \\
&= \int \frac{4\sin^2(\theta)\cos(\theta)}{\sqrt{\cos^2(\theta)}}\,d\theta \\
&= \int 4\sin^2(\theta)\,d\theta \\
&= 4\int (\frac{1}{2} - \frac{1}{2}\cos(2\theta))\,d\theta \\
&= 2\theta - \sin(2\theta) + C \\
&= 2\theta - 2\sin(\theta)\cos(\theta) + C
\end{aligned}$$

We then use that $\sin(\theta) = \dfrac{x}{2}$ to write our antiderivative in terms of x. In particular, we have $\sin(\theta) = \dfrac{x}{2}$ and $\cos(\theta) = \dfrac{\sqrt{4-x^2}}{2}$ to get:

$$2\theta - 2\sin(\theta)\cos(\theta) + C = 2\sin^{-1}\left(\frac{x}{2}\right) - 2\left(\frac{x}{2}\right)\left(\frac{\sqrt{4-x^2}}{2}\right) + C$$

13. (10 points) Compute $\int x \cdot \arctan(x)\, dx$. (Hint: $\dfrac{x^2}{x^2+1} = 1 - \dfrac{1}{1+x^2}$.)

Solution: We'll use Integration by Parts. Let $u = \arctan(x)$ and $v' = x$. Then $u' = \dfrac{1}{x^2+1}$ and $v = \dfrac{x^2}{2}$. Then

$$\begin{aligned} uv - \int u'v\, dx &= \arctan(x) \cdot \left(\frac{x^2}{2}\right) - \int \frac{1}{x^2+1} \cdot \frac{x^2}{2}\, dx \\ &= \arctan(x) \cdot \left(\frac{x^2}{2}\right) - \frac{1}{2}\int \frac{x^2}{x^2+1}\, dx \\ &= \arctan(x) \cdot \left(\frac{x^2}{2}\right) - \frac{1}{2}\int \left(1 - \frac{1}{1+x^2}\right)\, dx \\ &= \arctan(x) \cdot \left(\frac{x^2}{2}\right) - \frac{1}{2}x + \frac{1}{2}\arctan(x) + C \end{aligned}$$

14. (10 points) Using the method of partial fractions, compute

$$\int \frac{2x^2+4x+1}{(x+2)^2(x+1)}\ dx.$$

Solution: Note that

$$\frac{2x^2+4x+1}{(x+2)^2(x+1)} = \frac{A}{x+2} + \frac{B}{(x+2)^2} + \frac{C}{x+1}w$$

Therefore,

$$2x^2+4x+1 = A(x+2)(x+1) + B(x+1) + C(x+2)^2$$

Plugging in $x=-1$ gives $C=-1$. Plugging in $x=-2$ gives $B=-1$. Finally, plugging in $x=0$ (or any other value for x) gives $A=3$. So

$$\begin{aligned}\int \frac{2x^2+4x+1}{(x+2)^2(x+1)}\ dx &= \int \left(\frac{3}{x+2} - \frac{1}{(x+2)^2} - \frac{1}{x+1}\right)\ dx \\ &= 3\ln|x+2| + \frac{1}{x+2} - \ln|x+1| + C\end{aligned}$$

15. (10 points) Determine the convergence of $\int_{-\infty}^{0} x\, e^x$. If the integral converges, state what it converges to.

Solution:

$$
\begin{aligned}
&= \lim_{b\to-\infty} \int_b^0 x \cdot e^x \, dx \\
&\qquad [u = x, v' = e^x \implies u' = 1, v = e^x] \\
&= \lim_{b\to-\infty} [xe^x - \int_b^0 e^x \, dx] \\
&= \lim_{b\to-\infty} [xe^x - e^x]\Big|_b^0 \\
&= \lim_{b\to-\infty} [0 - 1 - (be^b - e^b)] \\
&= -1
\end{aligned}
$$

Therefore, the integral converges to -1.