MA 114 — Calculus II Fall 2014
Exam 2 Oct. 21, 2014

Name: ______________________

Section: ______________________

Last 4 digits of student ID #: ________

- No books or notes may be used.
- Turn off all your electronic devices and do not wear ear-plugs during the exam.
- You may use a calculator, but not one which has symbolic manipulation capabilities or a QWERTY keyboard.
- Additional blank sheets for scratch work are available upon request.
- **Multiple Choice Questions:** Record your answers on the right of this cover page by marking the box corresponding to the correct answer.
- **Free Response Questions:** Show all your work on the page of the problem. Clearly indicate your answer and the reasoning used to arrive at that answer.

Multiple Choice Answers

Question					
1	~~A~~	B	C	D	E
2	A	B	~~C~~	D	E
3	A	B	C	~~D~~	E
4	A	B	~~C~~	D	E

Exam Scores

Question	Score	Total
MC		20
5		16
6		15
7		18
8		16
9		15
Total		100

Unsupported answers for the free response questions may not receive credit!

Record the correct answer to the following problems on the front page of this exam.

1. Let f be a differentiable function. Which of the following expressions is the same as the integral

$$\int \underbrace{\frac{1}{x^2}}_{v'} \underbrace{f(x)}_{u} dx. \qquad -\frac{f(x)}{x} + \int \frac{f'(x)}{x} dx$$

$u' = f'(x)$

$v = -x^{-1}$

A. $-\dfrac{f(x)}{x} + \displaystyle\int \frac{f'(x)}{x} dx.$

B. $-\dfrac{2f(x)}{x^3} + \displaystyle\int \frac{2f'(x)}{x^3} dx.$

C. $\dfrac{f'(x)}{x} - \displaystyle\int \frac{f(x)}{x} dx.$

D. $\dfrac{f(x)}{x^2} - \dfrac{f'(x)}{x}.$

E. $\dfrac{f'(x)}{x^3} - \displaystyle\int \frac{f(x)}{x^3} dx.$

2. A solid has a circular base with radius 3 centered at 0. Its cross sections perpendicular to the x-axis are squares. Which of the following expresses the volume of the solid?

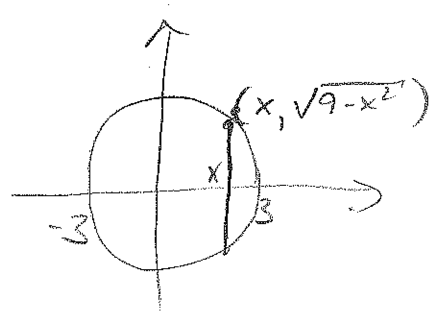
A. $\displaystyle\int_{-3}^{3} (9 - x^2) dx.$

B. $\displaystyle\pi \int_{-3}^{3} (9 - x^2)^2 dx.$

C. $\displaystyle 4 \int_{-3}^{3} (9 - x^2) dx.$

D. $\displaystyle\pi \int_{-3}^{3} \sqrt{9 - x^2} dx.$

E. $\displaystyle 2\pi \int_{-3}^{3} x(9 - x^2) dx.$



Side length $= 2\sqrt{9-x^2}$

$$V = \int_{-3}^{3} \left(2\sqrt{9-x^2}\right)^2 dx$$

$$= 4\int_{-3}^{3} (9-x^2)\, dx$$

Record the correct answer to the following problems on the front page of this exam.

3. Compute the average value of the function $f(x) = \sin(x)$ over the interval $[0, \pi]$.

A. $\frac{16}{25}$.

B. $\frac{-2}{\pi}$.

C. 2.

D. $\frac{2}{\pi}$.

E. 0.

$$\frac{1}{\pi}\int_0^{\pi} \sin x\, dx = \frac{1}{\pi}\left(-\cos x\right)\Big|_0^{\pi}$$

$$= \frac{1}{\pi}(1+1) = \underline{\underline{\frac{2}{\pi}}}$$

4. Consider the region in the first quadrant bounded by the graph of $f(x) = x^2$ and the line $x = 2$. Compute the volume of the solid obtained by rotating the region about the y-axis.

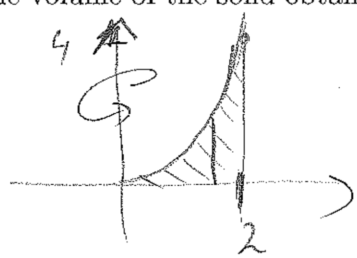
A. 4π.

B. $\frac{32\pi}{5}$.

C. 8π.

D. 25.

E. $\frac{16\pi}{5}$.



Shell method:

$$V = 2\pi\int_0^2 x\cdot x^2\, dx = 2\pi\frac{1}{4}x^4 = \frac{\pi}{2}\cdot 16$$

$$= \underline{\underline{8\pi}}$$

Free Response Questions: Show your work!

5. (a) Let a, b be real numbers. Use calculus to compute the integral $\int (ax+b)\sin(x)dx$.

$$\int \underbrace{(ax+b)}_{u}\underbrace{\sin x}_{v'}\,dx \underset{\substack{u'=a\\ v=-\cos x}}{=\!=\!=} -(ax+b)\cos x + a\int \cos x\,dx$$

$$= -(ax+b)\cos x + a\sin x + C$$

(b) Use calculus to compute the integral $\int_0^{\pi/2} \sin^2(x)\cos^3(x)dx$.

$$\int_0^{\frac{\pi}{2}} \sin^2(x)\cos^2(x)\cos(x)\,dx$$

$$= \int_0^{\frac{\pi}{2}} \sin^2(x)\left(1-\sin^2(x)\right)\cos x\,dx$$

$$\underset{\substack{u=\sin x\\ du=\cos x\,dx}}{=\!=\!=} \int_0^1 u^2(1-u^2)\,du = \left(\frac{1}{3}u^3-\frac{1}{5}u^5\right)\Big|_0^1$$

$$= \frac{1}{3}-\frac{1}{5} = \frac{2}{15}$$

Free Response Questions: Show your work!

6. In a particular neighborhood in Lexington the density of the squirrel population is given by

$$\rho(r) = \frac{160}{1+r^2} \text{ squirrels per square kilometer,}$$

where r is the distance (in km) from the backyard of squirrel lover Dr. Nuts.

(a) Write an integral that expresses the total population of squirrels within a 2-km radius of that backyard.

$$P = 2\pi \int_0^2 r\rho(r)\,dr = 2\pi \int_0^2 \frac{160r}{1+r^2}\,dr$$

(b) Evaluate the integral in (a). Give the exact answer.

$$P \underset{\substack{u=1+r^2 \\ du = 2r\,dr}}{=} 2\pi \int_1^5 \frac{80}{u}\,du = 160\pi \ln(u)\Big|_1^5$$

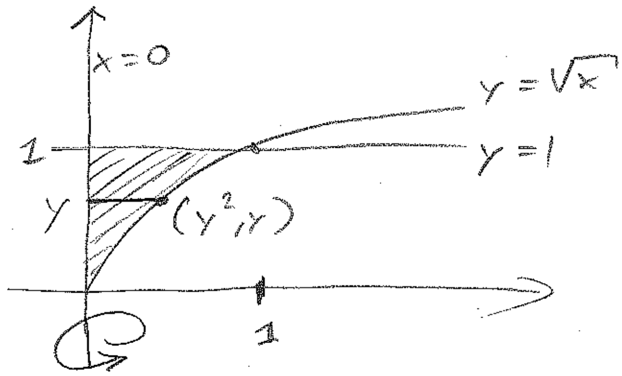
$$= \underline{160\pi \ln(5)} \approx 809$$

7. The region bounded by

$$y = \sqrt{x}, \quad y = 1, \quad \text{and } x = 0$$

is rotated about the y-axis to form a solid.

(a) Draw a CLEAR picture of the region described by the three equations.



(b) Write an integral that uses the *disk method* to compute the volume.

$$V = \pi \int_0^1 (y^2)^2 \, dy$$

(c) Write an integral that uses the *shell method* to compute the volume.

$$V = 2\pi \int_0^1 x(1 - \sqrt{x}) \, dx$$

(d) Select one of the above integrals and evaluate it to compute the volume.

Computing (b): $V = \frac{\pi}{5} y^5 \Big|_0^1 = \underline{\underline{\frac{\pi}{5}}}$

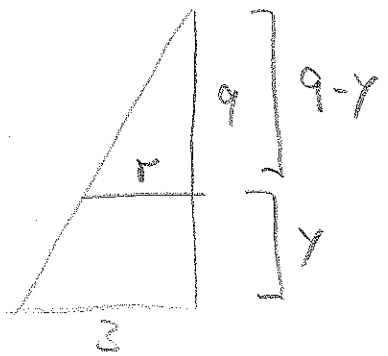
Computing (c): $V = 2\pi \int_0^1 x - x^{\frac{3}{2}} \, dx$

$= 2\pi \left(\frac{1}{2} x^2 - \frac{2}{5} x^{\frac{5}{2}} \right) \Big|_0^1 = 2\pi \left(\frac{1}{2} - \frac{2}{5} \right) = \underline{\underline{\frac{\pi}{5}}}$

Free Response Questions: Show your work!

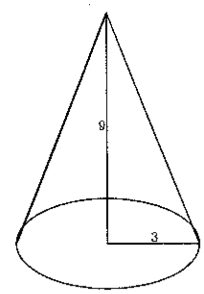
8. A right cone with a circular base of radius 3m and height 9m is to be built with a material having a density of 30 kg per m^3. See the figure for such a cone.

(a) Compute the area of the cross section at height y above the base. Also give the unit with your answer.



$$\frac{9}{3} = \frac{9-y}{r},$$

$$\text{so } r = 3 - \frac{y}{3}.$$



Area at height y is

$$\underline{A(y) = \pi\left(3-\frac{y}{3}\right)^2 \quad m^2}.$$

(b) Present the integral that expresses the work against gravity to build the cone. Use that the acceleration due to gravity is 9.8 m/s^2 and that all the material to be used is lying on the ground. Give the unit with your answer. **Do not evaluate the integral.**

$$W = \int_0^9 \underbrace{\pi\left(3-\frac{y}{3}\right)^2}_{\substack{\text{area at } y \\ m^2}} \cdot \underbrace{30}_{\substack{\text{density} \\ \frac{kg}{m^3}}} \cdot \underbrace{9.8}_{\substack{\text{gravity} \\ \frac{m}{s^2}}} \cdot \underbrace{y}_{\substack{\text{vertical} \\ \text{distance} \\ m}} \underbrace{dy}_{m}, \qquad \frac{kg\text{-}m^2}{s^2}$$

Free Response Questions: Show your work!

9. (a) Give the Taylor series centered at 0 of the function

$$f(x) = \frac{1}{1-x}.$$

$$f(x) = \sum_{n=0}^{\infty} x^n$$ (geometric series; converges for $|x|<1$)

(b) Use the result from (a) to find the Taylor series centered at 0 for

$$g(x) = \frac{1}{(1-x)^2}.$$

$$g(x) = f'(x) = \sum_{n=1}^{\infty} n x^{n-1}$$

(c) Find the sum of the series

$$\frac{1}{2} + 2\left(\frac{1}{2}\right)^2 + 3\left(\frac{1}{2}\right)^3 + 4\left(\frac{1}{2}\right)^4 + \dots$$

and express your answer as a rational number a/b.
[Hint: Use the result from part (b) and multiply by x.]

the series can be written as $\sum_{n=1}^{\infty} n\left(\frac{1}{2}\right)^n$

Note that $xg(x) = \sum_{n=1}^{\infty} n x^n$. Thus

$$\sum_{n=1}^{\infty} n\left(\frac{1}{2}\right)^n = \frac{1}{2}\cdot g\left(\frac{1}{2}\right) = \frac{1}{2}\cdot 4 = \underline{2}$$