MA 114 — Calculus II Fall 2015
Exam 2 October 20, 2015

Name: ____________________

Section: ____________________

Last 4 digits of student ID #: ________

- No books or notes may be used.
- Turn off all your electronic devices and do not wear ear-plugs during the exam.
- You may use a calculator, but not one which has symbolic manipulation capabilities or a QWERTY keyboard.
- Additional blank sheets for scratch work are available upon request.
- **Multiple Choice Questions:** Record your answers on the right of this cover page by marking the box corresponding to the correct answer.
- **Free Response Questions:** Show all your work on the page of the problem. Clearly indicate your answer and the reasoning used to arrive at that answer.

Multiple Choice Answers

Question					
1	A	B	C	D	E
2	A	B	C	D	E
3	A	B	C	D	E
4	A	B	C	D	E
5	A	B	C	D	E
6	A	B	C	D	E
7	A	B	C	D	E

Exam Scores

Question	Score	Total
MC		28
8		15
9		14
10		15
11		13
12		15
Total		100

Unsupported answers for the free response questions may not receive credit!

$$e^x = \sum_{n=0}^{\infty} \frac{1}{n!} x^n \quad \text{on } (-\infty, \infty)$$

$$\sin(x) = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} x^{2n+1} \quad \text{on } (-\infty, \infty)$$

$$\ln(1+x) = \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n} x^n \quad \text{on } (-1, 1]$$

$$\cos(x) = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} x^{2n} \quad \text{on } (-\infty, \infty)$$

Record the correct answer to the following problems on the front page of this exam.

1. Compute the average value of the function $f(x) = 3x^2 + 2x - 10$ over the interval $[1, 5]$.

A. 10.

B. 27.

C. 54.

D. 81.

E. 108.

Average =
$\frac{1}{5-1} \cdot \int_1^5 3x^2 + 2x - 10\, dx$
$= \frac{1}{4}(x^3 + x^2 - 10x)\Big|_1^5 = \underline{\underline{27}}$

2. Let g be a differentiable function. Which of the following equals the integral $\int 3x^2 g(x)dx$?

A. $3x^2g(x) - \int x^3 g'(x)dx$.

B. $x^3g(x) + \int 3x^2 g'(x)dx$.

C. $x^3g(x) - \int x^3 g'(x)dx$.

D. $6xg(x) - \int 3x^2 g'(x)dx$.

E. $6xg'(x) - \int x^3 g(x)dx$.

$\int \underbrace{3x^2}_{v'} \underbrace{g(x)}_{u}\, dx = $
$u' = g'$
$v = x^3$
$x^3 g(x) - \int x^3 g'(x)\, dx$

Record the correct answer to the following problems on the front page of this exam.

3. Suppose the function f has Taylor series expansion $f(x) = \sum_{n=0}^{\infty} \frac{1}{2^n}(x-3)^n$ and the radius of convergence is positive. Find $f^{(4)}(3)$.

A. 1.

B. $\frac{3}{16}$.

C. $\frac{3}{2}$.

D. $\frac{1}{2}$.

E. 0.

$$\frac{f^{(4)}(3)}{4!} = \frac{1}{2^4}, \text{ so}$$

$$f^{(4)}(3) = \frac{4!}{2^4} = \frac{3}{2}$$

4. Use the Taylor series given on the front page to determine the value of the sum $\sum_{n=1}^{\infty} \frac{1}{n \cdot 2^n}$.

A. $\cos(2)$.

B. $\ln(2)$.

C. $\ln\left(\frac{1}{2}\right)$.

D. $\ln(2n)$.

E. $e^{1/2}$.

$$\sum_{n=1}^{\infty} \frac{1}{n 2^n} = \sum_{n=1}^{\infty} \frac{1}{n}\left(\frac{1}{2}\right)^n$$

$$= -\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n}\left(-\frac{1}{2}\right)^n = -\ln\left(1-\frac{1}{2}\right)$$

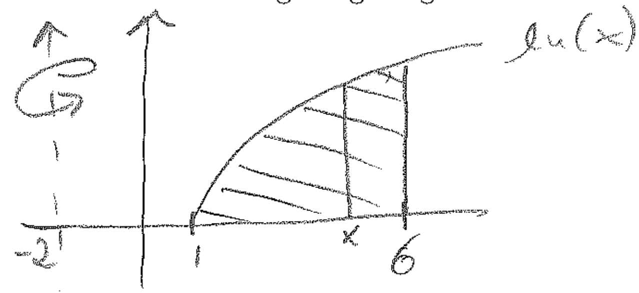
$$= -\ln\left(\frac{1}{2}\right) = \ln(2)$$

Record the correct answer to the following problems on the front page of this exam.

5. The region enclosed by the graph of $f(x) = \ln(x)$, the line $x = 6$ and the x-axis is

rotated about the line $x = -2$.

When the method of cylindrical shells is used, which of the following integrals gives the volume of the resulting solid body?



A. $2\pi \int_0^6 (x-2)\ln(x)\,dx.$

B. $2\pi \int_1^6 (x+2)\ln(x)\,dx.$

C. $\pi \int_{-2}^6 \ln(x)^2\,dx.$

D. $2\pi \int_{-2}^6 x\ln(x)^2\,dx.$

E. $2\pi \int_0^6 \ln(x)\,dx.$

$$V = 2\pi \int_1^6 \text{radius} \cdot \text{height}\, dx$$

$$= 2\pi \int_1^6 (x+2)\ln(x)\,dx$$

6. Which of the following is the correct substitution for the integral

$$\int \frac{3x^2}{\sqrt{49+x^2}}dx.$$

A. $x = 49\sec(\theta)$.

B. $x = 49\tan(\theta)$.

C. $x = 7\sin(\theta)$.

D. $x = 7\sec(\theta)$.

E. $x = 7\tan(\theta)$.

$$\tan^2\theta + 1 = \sec^2\theta$$

$$\sqrt{49 + (7\tan\theta)^2} = 7\sec\theta$$

Record the correct answer to the following problems on the front page of this exam.

7. Which of the following integrals and substitutions equals the integral $\int \cos^5(x)\sin^2(x)dx$?

A. $\int (u^6 - 2u^4 + u^2)du$, where $u = \sin(x)$.

B. $\int (u^8 + 2u^4 + u^2)du$, where $u = \sin(x)$.

C. $\int (u^2 - 2u^4)du$, where $u = \sin(x)$.

D. $\int (u^6 - 2u^4 + u^2)du$, where $u = \cos(x)$.

E. $\int (u^5 + u^7)du$, where $u = \cos(x)$.

$$\int \cos^5(x)\sin^2(x)dx = \int \cos^4(x)\sin^2(x)\cos x\,dx$$

$$= \int (1-\sin^2 x)^2 \sin^2(x)\cos x\,dx$$

$$= \int (1-u^2)^2 u^2\,du = \int u^6 - 2u^4 + u^2\,du$$

$u = \sin x$

$du = \cos x\,dx$

Free Response Questions: Show your work!

8. Find the first four terms of the Taylor series for $f(x) = \sqrt{4x-3}$ centered at 1. Your answer has to be of the form

$$a_0 + a_1(x-1) + a_2(x-1)^2 + a_3(x-1)^3,$$

where you have to determine the coefficients a_0, a_1, a_2, a_3.

$f(x) = (4x-3)^{\frac{1}{2}}$, $\quad f(1) = 1$

$f'(x) = \frac{1}{2} \cdot 4(4x-3)^{-\frac{1}{2}}$
$= 2(4x-3)^{-\frac{1}{2}}$, $\quad f'(1) = 2$

$f''(x) = 2 \cdot \left(-\frac{1}{2}\right) \cdot 4(4x-3)^{-\frac{3}{2}}$
$= -4(4x-3)^{-\frac{3}{2}}$, $\quad f''(1) = -4$

$f'''(x) = -4 \cdot \left(-\frac{3}{2}\right) \cdot 4(4x-3)^{-\frac{5}{2}}$
$= 24(4x-3)^{-\frac{5}{2}}$, $\quad f'''(1) = 24$

First 4 terms are

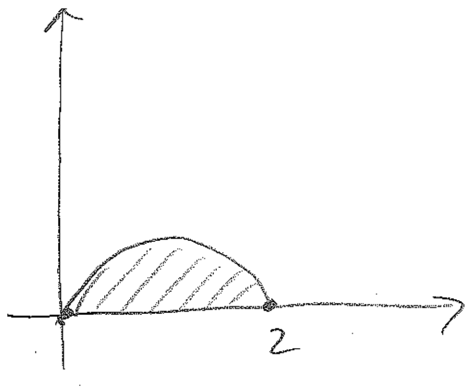
$1 + 2(x-1) + \frac{-4}{2!}(x-1)^2 + \frac{24}{3!}(x-1)^3$

$= \boxed{1 + 2(x-1) - 2(x-1)^2 + 4(x-1)^3}$

Free Response Questions: Show your work!

9. A solid is given whose base is the region enclosed by the x-axis and the graph of $f(x) = x(2-x)$. The cross sections parallel to the y-axis are rectangles of height $6x$ (where x is the point where the cross section intersects the x-axis).

(a) Sketch the base region.



$f(x) = 0$ for $x = 0, 2$.
The graph is a parabola opening downward.

(b) Compute the volume of the solid.

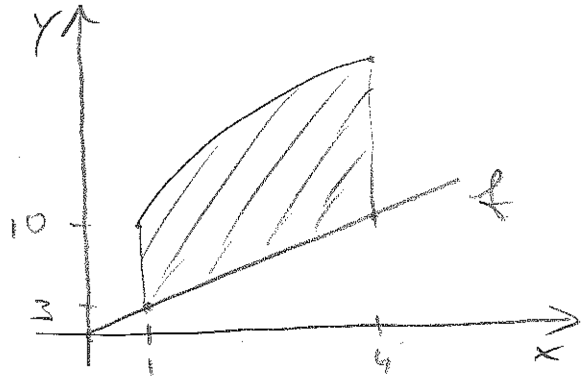
$$V = \int_0^2 6x \cdot x(2-x)\,dx = \int_0^2 12x^2 - 6x^3\,dx$$

$$= \left(4x^3 - \frac{3}{2}x^4\right)\Big|_0^2 = 32 - 24 = \boxed{8.}$$

Free Response Questions: Show your work!

10. Consider the region bounded by the graphs of the functions $f(x) = 3x$ and $g(x) = 10\sqrt{x}$ and the two lines $x = 1$ and $x = 4$.

(a) Sketch the region in a coordinate system.



(b) Find the volume of the solid given by revolving this region about the x-axis. **Give the exact answer!**

$$V = \pi \int_1^4 g^2(x) - f^2(x)\,dx = \pi \int_1^4 100x - 9x^2\,dx$$

$$= \pi\,(50x^2 - 3x^3)\Big|_1^4 = \pi\,(800 - 192 - 50 + 3)$$

$$= \boxed{561\pi}$$

(c) Find the volume of the solid given by revolving this region about the y-axis. **Give the exact answer!**

$$V = 2\pi \int_1^4 x(g(x) - f(x))\,dx = 2\pi \int_1^4 10x^{\frac{3}{2}} - 3x^2\,dx$$

$$= 2\pi\left(4x^{\frac{5}{2}} - x^3\right)\Big|_1^4 = 2\pi\,(128 - 64 - 4 + 1)$$

$$= \boxed{122\pi}$$

Free Response Questions: Show your work!

11. A conical tank of height 12m and circular base with radius 3m sits on its base.

(a) Compute the area of the cone's cross section at height y above the base.

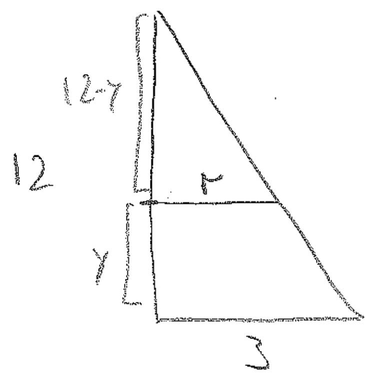
Area at height y is

$A(y) = \pi r^2$, where

$\frac{12-y}{r} = \frac{12}{3} = 4$, so

$r = 3 - \frac{y}{4}$. Thus

$A(y) = \pi \left(3 - \frac{y}{4}\right)^2$



(b) A liquid with density 640 kg/m^3 is to be pumped into the tank from a small hole in the base. Give the integral that obtains the work against gravity to pump the liquid into the tank until the tank is completely filled. The acceleration due to gravity is 9.8 m/sec^2. **Do not evaluate the integral!**

$$\int_0^{12} (\text{area at } y) \cdot (\text{density}) \cdot (\text{gravity}) \cdot \left(\begin{matrix}\text{vertical}\\ \text{distance at } y\end{matrix}\right) dy$$

$$= \boxed{\int_0^{12} \pi \left(3 - \frac{y}{4}\right)^2 \cdot 640 \cdot 9.8 \cdot y \, dy}$$

Free Response Questions: Show your work!

12. Compute the following indefinite integrals.

(a) $\int \underbrace{(4x-1)}_{u}\underbrace{e^x}_{v'}\,dx.$

$u' = 4$

$v = e^x$

$(4x-1)e^x - 4\int e^x dx$

$= (4x-1)e^x - 4e^x + C$

$= \boxed{(4x-5)e^x + C}$

(b) $\int \underbrace{3\sqrt{x}}_{v'}\,\underbrace{\ln(x)}_{u}\,dx.$

$v' = 3x^{\frac{1}{2}}$

$u' = \frac{1}{x}$

$v = 2x^{\frac{3}{2}}$

$2x^{\frac{3}{2}}\ln(x) - 2\int x^{-1}\cdot x^{\frac{3}{2}}dx$

$= 2x^{\frac{3}{2}}\ln(x) - 2\int x^{\frac{1}{2}}dx$

$= \boxed{2x^{\frac{3}{2}}\ln(x) - \frac{4}{3}x^{\frac{3}{2}} + C}$