MA 114 — Calculus II Spring 2013
Exam 2 5 March 2013

Name: ______________________________

Section: __________

Last 4 digits of student ID #: __________

This exam has six multiple choice questions (six points each) and five free response questions with points as shown. Additional blank sheets are available if necessary for scratch work. No books or notes may be used. Turn off your cell phones and do not wear ear-plugs during the exam. You may use a calculator, but not one which has symbolic manipulation capabilities.

On the multiple choice problems:

1. You must give your *final answers* in the *multiple choice answer box* on the front page of your exam.
2. Carefully check your answers. No credit will be given for answers other than those indicated on the *multiple choice answer box*.

On the free response problems:

1. Clearly indicate your answer and the reasoning used to arrive at that answer *(unsupported answers will not receive credit)*.
2. Give exact answers, rather than decimal approximations to the answer (unless otherwise stated).

Each free response question is followed by space to write your answer. Please write your solutions neatly in the space below the question. You are not expected to write your solution next to the statement of the question.

Multiple Choice Answers

Question					
1	A	B	C	D	E
2	A	B	C	D	E
3	A	B	C	D	E
4	A	B	C	D	E
5	A	B	C	D	E
6	A	B	C	D	E

Exam Scores

Question	Score	Total
MC		36
7		12
8		12
9		14
10		14
11		12
Total		100

Record the correct answer to the following problem on the front page of this exam.

(1) The form of the partial fraction decomposition of the rational function

$$f(x) = \frac{3x+2}{(x+1)^2(x^2+3)},$$

with the parameters A, B, C, D, E being constants to be determined, is:

A) $\dfrac{A}{(x+1)^2} + \dfrac{Bx+C}{x^2+3}$

B) $\dfrac{A}{x+1} + \dfrac{Bx+C}{(x+1)^2} + \dfrac{D}{x^2+3}$

C) $\dfrac{A}{x+1} + \dfrac{Bx+C}{(x+1)^2} + \dfrac{Dx+E}{x^2+3}$

D) $\dfrac{A}{x+1} + \dfrac{B}{(x+1)^2} + \dfrac{Cx+D}{x^2+3}$

E) none of the above

(2) Which of the following statements is false?

A) $\displaystyle\int_1^\infty \frac{dx}{x^2}$ converges

B) $\displaystyle\int_0^1 \frac{dx}{x^{4/3}}$ diverges

C) $\displaystyle\int_1^2 \frac{dx}{(x-1)^2}$ diverges

D) $\displaystyle\int_2^4 \frac{dx}{x-2}$ converges

E) $\displaystyle\int_2^\infty \frac{dx}{\sqrt{x^2-1}}$ diverges

Record the correct answer to the following problem on the front page of this exam.

(3) Which of the integrals below represents the length of the curve $y = \tan x$ from $x = 0$ to $x = \pi/4$?

A) $\displaystyle\int_0^{\pi/4} \sqrt{1+\tan^2 x}\, dx$

B) $\displaystyle\int_0^{\pi/4} \sqrt{1+\tan^2 x \sec^2 x}\, dx$

C) $\displaystyle\int_0^{\pi/4} \sqrt{1+\sec^4 x}\, dx$

D) $\displaystyle 2\pi \int_0^{\pi/4} \tan x \sqrt{1+\tan^2 x}\, dx$

E) $\displaystyle\int_0^{\pi/4} x \tan x\, dx$

(4) A triangular laminar (thin plate) of uniform mass density has its vertices at the points $A = (0, 3)$, $B = (0, 0)$, and $C = (1, 0)$ in the x-y plane. Where is the center of mass of the laminar located?

A) $(0, 1)$

B) $(\frac{1}{3}, 1)$

C) $(\frac{1}{2}, \frac{3}{2})$

D) $(\frac{2}{3}, 1)$

E) $(\frac{1}{3}, \frac{3}{2})$

Record the correct answer to the following problem on the front page of this exam.

(5) Which of the following differential equations is NOT separable?

A) $xy' + y = y^2$

B) $(1 + x^2)y' = x^3 y$

C) $x(y^2 - 1) + y(x^2 - 1)y' = 0$

D) $y' = \sin y$

E) $y^2 + x^2 y' = xyy'$

(6) Which of the following statements is false? In what follows, k and b are given constants, and C stands for an arbitrary constant.

A) The general solution of the differential equation $y' = k(y - b)$ is $y = b + Ce^{kt}$.

B) The general solution of the differential equation $y' = k(y - b)$ is $y = b - Ce^{kt}$.

C) The general solution of the differential equation $y' = -k(y - b)$ is $y = b + Ce^{-kt}$.

D) If $k > 0$, then all solutions of $y' = k(y - b)$ tend to ∞ as $t \to \infty$.

E) If $k > 0$, then all solutions of $y' = -k(y - b)$ approach the same limit as $t \to \infty$.

(7) Evaluate the integral

$$\int \frac{3x}{(x-1)(x^2+2)} dx.$$

Free Response Questions: Show your work!

(8) Compute the surface area of the surface obtained by rotating the graph of $y = \sqrt{1 + 2x}$ about the x-axis over the interval $[0, 1]$.

Free Response Questions: Show your work!

(9) The following table gives the measured values of a force function $f(x)$, where x is in meters and $f(x)$ in newtons.

x	0	2	4	6	8
$f(x)$	10.0	9.5	9.3	9.1	9.2

(a) Use Simpson's Rule to estimate the work done by the force f in moving an object from $x = 0$ to $x = 8$ meters.

(b) It is known that the force function $f(x)$ satisfies the inequality $|f^{(4)}(x)| \leq 2$ on the interval $[0, 8]$. Let S_N be the Nth approximation to $\displaystyle\int_0^8 f(x)dx$ by Simpson's rule. Use the given inequality on $|f^{(4)}(x)|$ to find the smallest N that guarantees $\text{Error}(S_N) \leq 10^{-1}$. (Hint: Use the error bound for S_N given on the last page of the exam.)

Free Response Questions: Show your work!

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(10) Let $T_n(x)$ $(n = 0, 1, 2, \cdots)$ be the nth Taylor polynomial for $f(x) = e^x$ centered at $a = 0$.

(a) Find the Taylor polynomial $T_n(x)$.

(b) Find a value of n for which

$$|e^x - T_n(x)| \leq 10^{-2}$$

on the interval $[0, 1]$. (Hint: Use the error bound given on the last page of the exam.)

Free Response Questions: Show your work!

(11) Solve the initial value problem

$$\frac{dx}{dt} = x^2(1 - t^2), \qquad x(1) = 1.$$

USEFUL FORMULAS

Error bound for S_N

Let S_N be the Nth approximation to $\int_a^b f(x)\,dx$ by Simpson's rule. Let K_4 be a number such that $|f^{(4)}(x)| \leq K_4$ for all $x \in [a, b]$. Then

$$\text{Error}(S_N) \leq \frac{K_4(b-a)^5}{180N^4}$$

Error bound for using $T_n(x)$ to approximate $f(x)$

Let f be a function which has all its derivatives $f^{(k)}$ $(k = 1, 2, 3, \cdots)$ continuous on an interval I. Let a and b be in I, and let $T_n(x)$ be the nth Taylor polynomial of f centered at $x = a$. Let K be a number such that $|f^{(n+1)}(x)| \leq K$ for all x between a and b. Then

$$|f(x) - T_n(x)| \leq K \frac{|b-a|^{n+1}}{(n+1)!} \qquad \text{for all } x \text{ between } a \text{ and } b.$$