Exam 2

Name: ______________________________ Section: ______________

Do not remove this answer page — you will return the whole exam. You will be allowed two hours to complete this test. If you find you need scratch paper during the exam, please ask. You may not use any of your own notes, paper or anything else not provided. You may use a graphing calculator during the exam, but NO calculator with a Computer Algebra System (CAS). Absolutely no communication device use during the exam is allowed.

The exam consists of 10 multiple choice questions and 5 free response questions. Record your answers to the multiple choice questions on this page by filling in the circle corresponding to the correct answer.

Show all work to receive full credit on the free response problems. It will also help you check your answers to show work on multiple choice problems.

Multiple Choice Questions

1	A	B	C	D	E	**6**	A	B	C	D	E
2	A	B	C	D	E	**7**	A	B	C	D	E
3	A	B	C	D	E	**8**	A	B	C	D	E
4	A	B	C	D	E	**9**	A	B	C	D	E
5	A	B	C	D	E	**10**	A	B	C	D	E

Multiple Choice	11	12	13	14	15	Total Score
50	10	10	10	10	10	100

Multiple Choice Questions

1. (5 points) Give the first four terms of the sequence $\{a_1, a_2, a_3, a_4\}$ defined by

$$a_n = \frac{3n}{\sqrt{n^2+4}}.$$

A. $\{1, \frac{3}{2}, \frac{9}{5}, 2\}$
B. $\{\frac{3}{\sqrt{5}}, \frac{6}{\sqrt{8}}, \frac{9}{\sqrt{13}}, \frac{12}{\sqrt{20}}\}$
C. $\{\frac{3}{\sqrt{6}}, \frac{6}{\sqrt{8}}, \frac{9}{\sqrt{10}}, \frac{12}{\sqrt{12}}\}$
D. $\{3, 3, 3, 3\}$
E. $\{1, \frac{5}{2}, \frac{43}{10}, \frac{63}{10}\}$

2. (5 points) Find the limit of the **sequence** $\{a_1, a_2, a_3, \dots\}$ defined by

$$a_n = \ln(2n+9) - \ln(3n+5).$$

A. $\ln(\frac{2}{3})$
B. $\ln(\frac{11}{8})$
C. 0
D. ∞
E. $-\infty$

3. (5 points) For the series $\displaystyle\sum_{n=1}^{\infty} \frac{3}{5^n}$, find the **partial sum** S_3.

A. $S_3 = \frac{3}{125}$
B. $S_3 = \frac{147}{125}$
C. $S_3 = \frac{93}{125}$
D. $S_3 = \frac{3}{4}$
E. $S_3 = \frac{3}{2}$

4. (5 points) Find the sum $\sum_{n=1}^{\infty} \frac{1+2^n}{5^n}$.

   A. $\frac{3}{2}$
   B. $\frac{7}{5}$
   C. $\frac{11}{12}$
   D. $\frac{35}{12}$
   E. this sum diverges to ∞

5. (5 points) Does the series $\sum_{n=1}^{\infty} \frac{2^n}{n!}$ converge or diverge?

   A. Converges by the alternating series test.
   B. Diverges by limit comparison test with $\sum_{n=1}^{\infty} \frac{1}{2^n}$.
   C. Diverges because $\lim_{n\to\infty} \frac{2^n}{n!} = \infty$.
   D. Converges because $\lim_{n\to\infty} \frac{2^n}{n!} = 0$.
   E. Converges by the ratio test since $\lim_{n\to\infty} \frac{2}{n+1} = 0$.

6. (5 points) What should we compare $\sum_{n=3}^{\infty} \frac{\sqrt{n^3-5n}}{n^4+7}$ to for a conclusive limit comparison test?

   A. $\sum_{n=3}^{\infty} \frac{1}{n^4}$
   B. $\sum_{n=3}^{\infty} \frac{1}{n}$
   C. $\sum_{n=3}^{\infty} \frac{1}{n^{3/2}}$
   D. $\sum_{n=3}^{\infty} \frac{1}{n^{5/2}}$
   E. $\sum_{n=3}^{\infty} \frac{1}{n^{10/3}}$

7. (5 points) Which of the following is always true for a series $\sum_{n=1}^{\infty} a_n$? There is only one correct answer.

A. If the series is convergent, then it is also absolutely convergent.

B. If the series is alternating, then it is convergent.

C. If $\lim_{n\to\infty} a_n = L$ then $\sum_{n=1}^{\infty} a_n = L$.

D. If $\lim_{n\to\infty} a_n = 0$ then the series converges.

E. If $\lim_{n\to\infty} a_n \neq 0$ then the series diverges.

8. (5 points) Find the first four terms of the Maclaurin series of $f(x)$ if

$$f(0) = 7, f'(0) = 2, f''(0) = 13, f'''(0) = -1.$$

A. $f(x) = 7 + 2x + \frac{13x^2}{2} - \frac{x^3}{3} + \ldots$

B. $f(x) = 7 + 2x + \frac{13x^2}{2} - \frac{x^3}{6} + \ldots$

C. $f(x) = \frac{1}{7} + \frac{x}{2} + \frac{x^2}{13} - x^3 + \ldots$

D. $f(x) = 7x + \frac{2x^2}{2} + \frac{13x^3}{3} - \frac{x^4}{4} + \ldots$

E. $f(x) = 7 - \frac{(x-2)}{2!} + \frac{(x-13)}{3!} + \frac{(x+1)}{4!} + \ldots$

9. (5 points) Find the interval of convergence of the power series $\sum_{n=1}^{\infty} \frac{(x-2)^n}{n!}$.

A. $\{2\}$
B. $\{0\}$
C. $[1, 3]$
D. $(-3, -1)$
E. $(-\infty, \infty)$

10. (5 points) Given that the power series for $\sqrt{x}$ centered at $a = 1$ is

$$\sqrt{x} = 1 + \frac{1}{2}(x-1) - \frac{1}{8}(x-1)^2 + \frac{1}{16}(x-1)^3 + \ldots$$

what is the power series centered at $a = 1$ for $\frac{1}{\sqrt{x}}$? Hint: first recall that $\frac{d}{dx}\sqrt{x} = \frac{1}{2\sqrt{x}}$.

A. $1 + (x-1) + \frac{1}{4}(x-1)^2 - \frac{1}{24}(x-1)^3 + \ldots$
B. $1 + \frac{1}{2!}(x-1) - \frac{3}{8 \cdot 3!}(x-1)^2 + \ldots$
C. $2 - (x-1) + \frac{3}{4}(x-1)^2 + \ldots$
D. $\frac{1}{2} - \frac{1}{4}(x-1) + \frac{3}{16}(x-1)^2 + \ldots$
E. $1 - \frac{1}{2}(x-1) + \frac{3}{8}(x-1)^2 + \ldots$

Free Response Questions

11. (a) (5 points) Find the sum of the series. Clearly show steps to justify your answer.

$$\sum_{n=1}^{\infty}\left(\frac{1}{2n+5}-\frac{1}{2n+7}\right)$$

(b) (5 points) Use the limit comparison test to show that $\displaystyle\sum_{n=2}^{\infty}\frac{n^3-2n}{7n^4+1}$ diverges. Show steps clearly with proper notation.

12. Decide if the series converges or diverges. Clearly state which test(s) are used, and show all steps.

(a) (5 points) $\displaystyle\sum_{n=1}^{\infty} \frac{\sin^2(n)}{n^4+2}$

(b) (5 points) $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{5n-1}{9n+2}$

13. Are the series absolutely convergent, conditionally convergent, or divergent? Clearly state which test(s) are used, and show all steps.

    (a) (5 points) $\displaystyle\sum_{n=1}^{\infty} \left(\frac{n-1}{3n+5}\right)^n$

    (b) (5 points) $\displaystyle\sum_{n=1}^{\infty} \frac{n!}{n^{35}}$

14. (10 points) Find the interval of convergence for the series. Hint: show clearly where you test the endpoints of your interval.

$$\sum_{n=1}^{\infty} \frac{(-1)^n (x+5)^n}{n \cdot 7^n}$$

15. (a) (5 points) Write a Taylor series centered at $a = 0$ (i.e., a Maclaurin series) for $f(x) = x^3 \cos(5x)$.

(b) (5 points) Write a Taylor series centered at $a = 0$ (i.e., a Maclaurin series) for $f(x) = \arctan(5x)$ and state the Interval of Convergence.