MA 114 — Calculus II Fall 2014
Sections [redacted] and 401, 402
5-8
Exam 3 Nov. 18, 2014

Name: ______________________________

Section: ______________________________

Last 4 digits of student ID #: ________

- No books or notes may be used.
- Turn off all your electronic devices and do not wear ear-plugs during the exam.
- You may use a calculator, but not one which has symbolic manipulation capabilities or a QWERTY keyboard.
- Additional blank sheets for scratch work are available upon request.
- **Multiple Choice Questions:**
  Record your answers on the right of this cover page by marking the box corresponding to the correct answer.
- **Free Response Questions:**
  Show all your work on the page of the problem. Clearly indicate your answer and the reasoning used to arrive at that answer.

Multiple Choice Answers

Question					
1	A	~~B~~	C	D	E
2	A	B	C	D	~~E~~
3	A	B	C	D	~~E~~
4	A	B	~~C~~	D	E

Exam Scores

Question	Score	Total
MC		20
5		16
6		15
7		18
8		16
9		15
Total		100

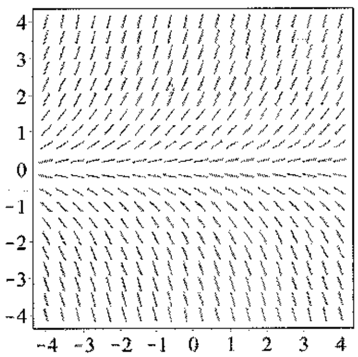
Unsupported answers for the free response questions may not receive credit!

Record the correct answer to the following problems on the front page of this exam.

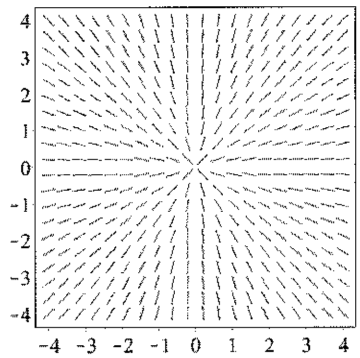
1. Which of the following is the slope field for $y' = \dfrac{y}{x}$?

$y = x \neq 0$: slope 1

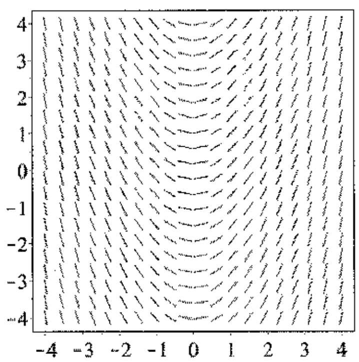
$y = 0 \neq x$: slope 0



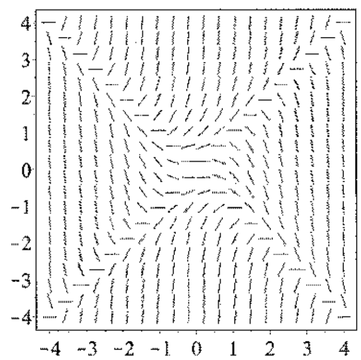
A



B



C



D

E - None of these

2. Which of the following expressions is the Simpson's rule approximation with 4 intervals for $\int_1^9 \sqrt{x}\,dx$?

A. $2\left(\sqrt{2}+\sqrt{4}+\sqrt{6}+\sqrt{8}\right)$.

B. $\left(\sqrt{1}+2\sqrt{3}+2\sqrt{5}+2\sqrt{7}+\sqrt{9}\right)$.

C. $\frac{4}{3}\left(\sqrt{1}+2\sqrt{3}+4\sqrt{5}+2\sqrt{7}+\sqrt{9}\right)$.

D. $2\left(\sqrt{1}+\sqrt{3}+\sqrt{5}+\sqrt{7}\right)$.

E. $\frac{2}{3}\left(\sqrt{1}+4\sqrt{3}+2\sqrt{5}+4\sqrt{7}+\sqrt{9}\right)$.

Record the correct answer to the following problems on the front page of this exam.

3. Which of the following integrals represents the arclength of $y = 3\sin(x^2)$ over $[0,2]$?

A. $2\pi \int_0^2 \sqrt{1+9\sin^2(x^2)}\,dx.$

B. $2\pi \int_0^2 3\sin(x^2)\sqrt{1+36x^2\cos^2(x^2)}\,dx.$

C. $\int_0^2 \sqrt{1+9\sin^2(x^2)}\,dx.$

D. $\pi \int_0^2 9\sin^2(x^2)\,dx.$

E. $\int_0^2 \sqrt{1+36x^2\cos^2(x^2)}\,dx.$

$$\int_0^2 \sqrt{1+y'(x)^2}\,dx = \int_0^2 \sqrt{1+(6x\cos(x^2))^2}\,dx$$

4. Which of the following is the correct form for the partial fraction decomposition of

$$\frac{x^2+5}{(x+1)^2(x+3)}?$$

A. $\dfrac{Ax^2+B}{(x+1)^2} + \dfrac{Cx^2+D}{(x+1)} + \dfrac{Ex^2+F}{(x+3)}.$

B. $\dfrac{Ax+B}{(x+1)^2} + \dfrac{C}{(x+1)} + \dfrac{D}{(x+3)}.$

C. $\dfrac{A}{(x+1)^2} + \dfrac{B}{(x+1)} + \dfrac{C}{(x+3)}.$

D. $\dfrac{A}{(x+1)^2} + \dfrac{B}{(x+3)}.$

E. $\dfrac{A}{(x+1)^2(x+3)} + \dfrac{B}{(x+1)(x+3)} + \dfrac{C}{(x+3)}.$

Free Response Questions: Show your work!

5. Use calculus to compute the integral $\int \frac{1}{(16-x^2)^{3/2}} dx$.

$x = 4\sin\theta$

$dx = 4\cos\theta \, d\theta$

$\sqrt{16-x^2} = 4\cos\theta$

$$\int \frac{4\cos\theta}{(4\cos\theta)^3} d\theta = \frac{1}{16}\int \frac{1}{\cos^2\theta} d\theta$$

$$= \frac{1}{16}\int \sec^2\theta \, d\theta + C = \frac{1}{16}\tan\theta + C$$

$$= \frac{1}{16}\frac{\sin\theta}{\cos\theta} + C = \frac{1}{16}\frac{x/4}{\sqrt{16-x^2}/4} + C$$

$$= \frac{1}{16}\frac{x}{\sqrt{16-x^2}} + C$$

Alternatively,

$$\frac{1}{16}\tan\theta + C = \frac{1}{16}\tan\left(\arcsin\left(\frac{x}{4}\right)\right) + C$$

Free Response Questions: Show your work!

6. (a) Use calculus to compute the integral $\int \frac{3}{x^2 - 5x + 6}\, dx$.

$$\frac{3}{x^2-5x+6} = \frac{3}{(x-2)(x-3)} = \frac{A}{x-2} + \frac{B}{x-3}$$

So $3 = A(x-3) + B(x-2)$. $x=2$ yields $3 = -A$, so $\boxed{A=-3}$. $x=3$ yields $\boxed{3=B}$.

Hence

$$\int \frac{3}{x^2-5x+6}\,dx = \int \frac{-3}{x-2}\,dx + \int \frac{3}{x-3}\,dx$$

$$= \underline{-3\ln|x-2| + 3\ln|x-3| + C} = \underline{3\ln\left|\frac{x-3}{x-2}\right| + C}$$

(b) Use calculus to compute the integral $\int \frac{3x+1}{x^2+4}\, dx$. $= \int \frac{3x}{x^2+4}\,dx + \int \frac{1}{x^2+4}\,dx$

$$\int \frac{3x}{x^2+4}\,dx \underset{\substack{u=x^2+4 \\ du = 2x\,dx}}{=} \frac{3}{2}\int \frac{du}{u} = \frac{3}{2}\ln|u| = \frac{3}{2}\ln|x^2+4|$$

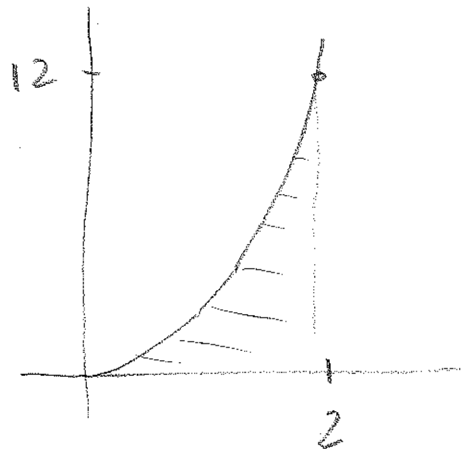
$$\int \frac{1}{x^2+4}\,dx \underset{\substack{x = 2\tan\theta \\ dx = 2\sec^2\theta\, d\theta \\ x^2+4 = 4\sec^2\theta}}{=} \int \frac{2\sec^2\theta}{4\sec^2\theta}\,d\theta = \int \frac{1}{2}\,d\theta$$

$$= \frac{1}{2}\theta = \frac{1}{2}\arctan\left(\frac{x}{2}\right)$$

So

$$\int \frac{3x+1}{x^2+4}\,dx = \underline{\frac{3}{2}\ln|x^2+4| + \frac{1}{2}\arctan\left(\frac{x}{2}\right) + C}$$

7. Find the centroid of the region below $f(x) = 3x^2$ and above the x-axis with $0 \le x \le 2$.



$$M = \int_0^2 3x^2\,dx = x^3\Big|_0^2 = \underline{\underline{8}}$$

$$M_y = \int_0^2 x\,f(x)\,dx = \int_0^2 3x^3\,dx = \frac{3}{4}x^4\Big|_0^2 = \underline{\underline{12}}$$

$$M_x = \frac{1}{2}\int_0^2 f(x)^2\,dx = \frac{1}{2}\int_0^2 9x^4\,dx = \frac{9}{10}x^5\Big|_0^2 = \underline{\underline{\frac{144}{5}}}$$

So, centroid is

$$\left(\frac{M_y}{M}, \frac{M_x}{M}\right) = \left(\frac{12}{8}, \frac{144}{5\cdot 8}\right) = \underline{\underline{\left(\frac{3}{2}, \frac{18}{5}\right)}}$$

Free Response Questions: Show your work!

8. (a) Find the general solution to the differential equation $y' = -2xy$.

$$\int \frac{1}{y}\,dy = \int (-2x)\,dx$$

$$\ln|y| = -x^2 + C$$

$$|y| = e^{-x^2+C} = e^C \cdot e^{-x^2}$$

$$y = \pm e^C e^{-x^2}$$

Since e^C takes all nonzero values and $y=0$ is also a solution, we obtain $\underline{y = Ce^{-x^2}}$ as the general solution.

(b) Solve the initial value problem $\dfrac{dy}{dt} = 4y^2\cos(t) + y^2$ where $y(0) = \dfrac{1}{2}$.

$$y' = (4\cos(t) + 1)\,y^2$$

$$\int \frac{1}{y^2}\,dy = \int (4\cos(t) + 1)\,dt$$

$$-\frac{1}{y} = 4\sin(t) + t + C$$

$$y = \frac{1}{C - 4\sin(t) - t}$$

$y(0) = \dfrac{1}{C} = \dfrac{1}{2}$, hence $C = 2$.

So, the solution is

$$\underline{y = \frac{1}{2 - 4\sin(t) - t}}$$

9. A bowl of soup starts at 90° C. After an hour, the soup has cooled to 40° C. The temperature in the room is 25° C. Let $y(t)$ be the temperature of the soup after t hours. According to Newton's law of cooling, the differential equation satisfied by the temperature is $y' = -k(y - T_0)$ where T_0 is the temperature of the room and k is some constant.

(a) Find the solution $y(t)$. Don't compute the constant k but all other constants that may appear.

$$y' = -k(y-25)$$

So $y(t) = Ce^{-kt} + 25$.

$y(0) = C + 25 = 90$, so $C = 65$.

$$\underline{y(t) = 65e^{-kt} + 25}$$

(b) Determine the constant k. **Give the exact value.**

$$y(1) = 65e^{-k} + 25 = 40$$

$$e^{-k} = \frac{15}{65} = \frac{3}{13}$$

$$-k = \ln\left(\frac{3}{13}\right)$$

$$\boxed{k = -\ln\left(\frac{3}{13}\right) = \ln\left(\frac{13}{3}\right) \approx 1.466}$$