Exam 3

Name: ____________________________ Section: ____________

Do not remove this answer page — you will return the whole exam. You will be allowed two hours to complete this test. No books or notes may be used. You may use a graphing calculator during the exam, but NO calculator with a Computer Algebra System (CAS) or a QWERTY keyboard is permitted. Absolutely no cell phone use during the exam is allowed.

The exam consists of 10 multiple choice questions and 5 free response questions. Record your answers to the multiple choice questions on this page by filling in the circle corresponding to the correct answer.

Show <u>all work</u> to receive full credit on the free response problems. Also, it helps you to show work on multiple choice problems so you can re-check your answers.

Multiple Choice Questions

1 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ **6** Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ

2 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ **7** Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ

3 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ **8** Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ

4 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ **9** Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ

5 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ **10** Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ

Multiple Choice	11	12	13	14	15	Total Score
50	10	10	10	10	10	100

Multiple Choice Questions

1. (5 points) Find the average value of the function $f(x) = \cos^4 x \sin x$ on $0 \leq x \leq \pi$.

   A. $\dfrac{2}{5\pi}$

   B. $\dfrac{5}{2\pi}$

   C. 0

   D. 4

   E. 5

2. (5 points) The base of a solid S is the circle $x^2 + y^2 = 4$ and the parallel cross-sections perpendicular to the base are equilateral triangles. Remember that if an equilateral triangle has side length s, its area is $\dfrac{\sqrt{3}}{4}s^2$. Which of the following integrals correctly computes the volume of the solid?

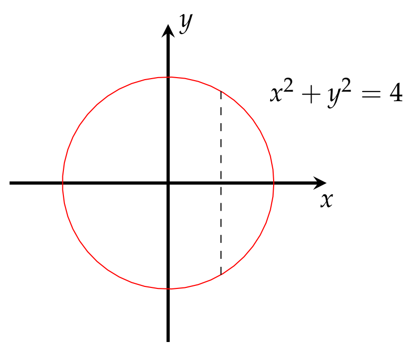
   A. $\displaystyle\int_{-2}^{2} \sqrt{4 - x^2}\, dx$

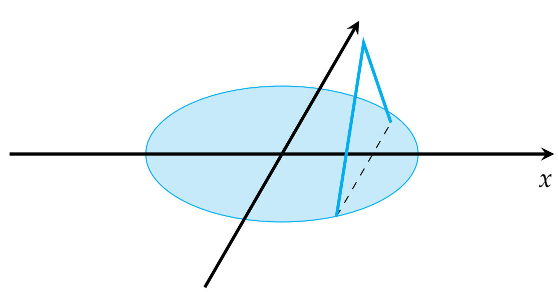
   B. $\displaystyle\int_{-2}^{2} \sqrt{3}(4 - x^2)\, dx$

   C. $\displaystyle\int_{-2}^{2} \frac{\sqrt{3}}{2}(4 - x^2)\, dx$

   D. $\displaystyle\int_{-2}^{2} (4 - x^2)^2\, dx$

   E. $\displaystyle\int_{-2}^{2} \sqrt{1 + 4x^2}\, dx$





3. (5 points) Which of the following is the Cartesian equation for the parametric curve $x(t) = \sin t, y(t) = 1 - \cos t$?

   A. $x^2 + y^2 = 1$

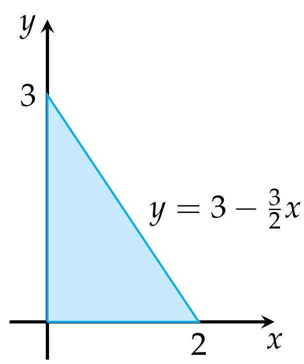
   B. $x^2 + (y+1)^2 = 1$

   C. $x^2 + (y-1)^2 = 1$

   D. $(x-1)^2 + y^2 = 1$

   E. $x^2 + y^2 = 2xy$

4. (5 points) What is the center of mass of the triangular region shown?



   A. $(1/3, 2/3)$

   B. $(3, 2)$

   C. $(2, 3)$

   D. $(2/3, 1)$

   E. $(1/3, 1/3)$

5. (5 points) Which of the following integrals computes the arc length of the curve $x = 1 + 3t^2$, $y = 4 + 2t^3$ for $0 \leq t \leq 1$?

A. $\int_0^1 \sqrt{1 + (6t)^2 + (36t^2)^2}\, dt$

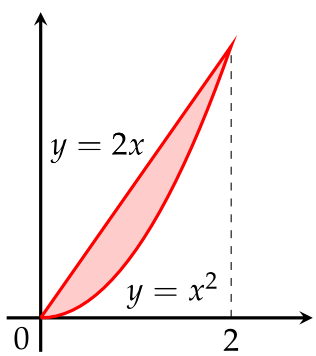
B. $\int_0^1 \sqrt{(1 + 3t^2)^2 + (4 + 2t^3)^2}\, dt$

C. $\int_0^1 \sqrt{3t^2 + 2t^3}\, dt$

D. $\int_0^1 \sqrt{2t^2 + 3t^3}\, dt$

E. $\int_0^1 \sqrt{(6t)^2 + (6t^2)^2}\, dt$

6. (5 points) Which of the following integrals computes the volume of the solid obtained by rotating the region between the curves $y = x^2$ and $y = 2x$ about the x-axis?



A. $\int_0^2 2\pi(2x - x^2)\, dx$

B. $\int_0^2 2\pi(x^2 - 2x)\, dx$

C. $\int_0^2 \pi\left(4x^2 - x^4\right)\, dx$

D. $\int_0^2 \pi\left(x^4 - 4x^2\right)\, dx$

E. $\int_0^2 \pi\left(2x - x^2\right)^2\, dx$

7. (5 points) If $x = e^t \cos t$ and $y = e^t \sin t$, then $dy/dx =$

   A. $e^t \tan t$

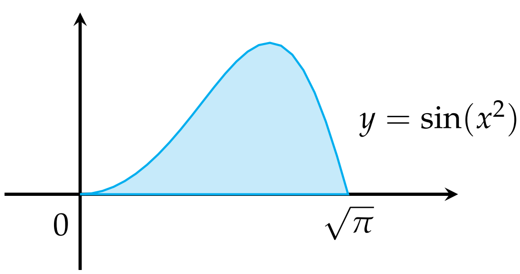
   B. $(\cos t + \sin t)/(\cos t - \sin t)$

   C. $(e^t \cos t)/(e^t \sin t)$

   D. $(\cos t - \sin t)/(\cos t + \sin t)$

   E. $(e^t \sin t)/(e^t \cos t)$

8. (5 points) Find the volume of the solid obtained by rotating the region shown in the figure about the y-axis.



   A. $\pi/2$

   B. π

   C. $3\pi/2$

   D. 2π

   E. 4π

9. (5 points) A surface S is obtained by rotating the curve

$$x = t\sin t,\ y = t\cos t, \quad 0 \le t \le \pi/2$$

about the x-axis. The area of the surface S is:

A. $S = \int_0^{\pi/2} 2\pi t\cos(t)\sqrt{1+t^2}\,dt$

B. $S = \int_0^{\pi} 2\pi t\cos(t)\sqrt{1+t^2}\,dt$

C. $S = \int_0^{\pi} 2\pi t\sin(t)\sqrt{1+t^2}\,dt$

D. $S = \int_0^{\pi} 2\pi t^2\sin(t)\,dt$

E. $S = \int_0^{\pi/2} 2\pi t^2\sin(t)\,dt$

10. (5 points) Find the center of mass of the three equal-mass particles shown in the figure at right.

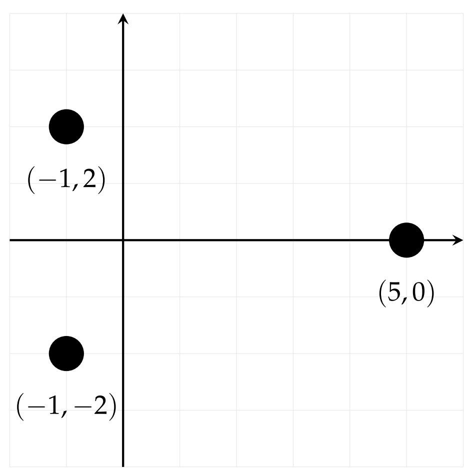
A. $(-1, 0)$

B. $(0, 0)$

C. $(1, 0)$

D. $(2, 0)$

E. $(3, 0)$



Free Response Questions

11. (10 points) Find the points (x, y) on the curve $x = \cos t$, $y = \cos 3t$, $0 \leq t \leq \pi$, where the tangent line is horizontal.

12. (a) (5 points) Set up but do not evaluate an integral for the exact arc length of the curve

$$x = t \sin t,\ y = t \cos t, \quad 0 \le t \le 1$$

Your final answer should not involve trig functions.

(b) (5 points) Find dy/dx and d^2y/dx^2 for the curve $x = t^2 + 1$, $y = t^2 + t$. For what values of t is the curve concave upward?

13. (a) (5 points) Find the area enclosed by the curve $x = a\cos^2\theta$, $y = a\sin^2\theta$ between $\theta = 0$ and $\theta = \pi/2$.

(b) (5 points) Find the center of mass of the region bounded by the circle $x^2 + y^2 = 4$ and the x-axis.

14. Using the method of cylindrical shells, find the volume generated by rotating the region bounded by the curves

$$y = e^{-x^2}, \quad y = 0, \quad x = 0, \quad x = 1$$

about the y-axis.

15. Let $\mathscr{R}$ be the region bounded by the curves $y = x^3$, $y = 0$, and $x = 1$. Find the volume obtained by rotating $\mathscr{R}$ about the line $x = 2$. You may use either the washer or shell method, but be sure to make clear which method you are using and illustrate with a careful sketch.