Name: ______________________________ Section: ______________

Do not remove this answer page — you will return the whole exam. You will be allowed two hours to complete this test. No books or notes may be used. You may use a graphing calculator during the exam, but you may not use a calculator that has symbolic manipulation capabilities of any sort. This forbids the use of TI-89, TI-Nspire CAS, HP 48, TI 92, and many others, as stated on the syllabus. Absolutely no cell phone use during the exam is allowed.

The exam consists of 10 multiple choice questions and 5 free response questions. Record your answers to the multiple choice questions on this page by filling in the circle corresponding to the correct answer.

Show all work to receive full credit on the free response problems. You should also show your work on the multiple choice questions as it will make it easier for you to check your work. You should give exact answers, rather than a decimal approximation unless the problem asks for a decimal answer. Thus, if the answer is 2π, you should not give a decimal approximation such as 6.283 as your final answer.

Multiple Choice Questions

1 (A) (B) (C) (D) (E) **6** (A) (B) (C) (D) (E)

2 (A) (B) (C) (D) (E) **7** (A) (B) (C) (D) (E)

3 (A) (B) (C) (D) (E) **8** (A) (B) (C) (D) (E)

4 (A) (B) (C) (D) (E) **9** (A) (B) (C) (D) (E)

5 (A) (B) (C) (D) (E) **10** (A) (B) (C) (D) (E)

Multiple Choice	11	12	13	14	15	Total Score
50	10	10	10	10	10	100

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Multiple Choice Questions

1. (5 points) Find the average value of the function $f(x) = \sec^2(x)$ on the interval $[-\pi/4, \pi/4]$.

   A. $\dfrac{1}{4\pi}$

   B. $\dfrac{4}{\pi}$

   C. $\dfrac{1}{2\pi}$

   D. $\dfrac{2}{\pi}$

   E. $\dfrac{1}{\pi}$

2. (5 points) Find the volume of the solid obtained by rotating the triangle

$$\{(x, y) \mid 1 \leq y \leq x + 1,\ 0 \leq x \leq 1\}$$

about the x-axis.

   A. $7\pi/3$

   B. $4\pi/3$

   C. π

   D. $\pi/2$

   E. $\pi/3$

3. (5 points) We create a solid S by rotating the region between the curve $y = \sqrt{x}$, the x-axis, and the line $x = 4$ about the y-axis. We slice S with a plane perpendicular to the y-axis and containing the line $y = 1$. Find the area of the resulting cross-section.

   A. 24π

   B. 18π

   C. 16π

   D. 15π

   E. 12π

4. (5 points) Let R be the upper half-disk $\{(x, y) \mid (x-2)^2 + y^2 \leq 4,\ y \geq 0\}$. We rotate R about the y-axis to form a solid of revolution S. If the method of cylindrical shells is used to find the volume of S, select the correct integral.

   A. $2\pi \int_0^2 x\sqrt{4 - x^2}\, dx$

   B. $2\pi \int_0^4 x\sqrt{4x - x^2}\, dx$

   C. $2\pi \int_0^2 x\sqrt{4x - x^2}\, dx$

   D. $\pi \int_0^2 x\sqrt{4 - x^2}\, dx$

   E. $\pi \int_0^2 x(4x - x^2)\, dx$

5. (5 points) The line L in the plane passes through the points $(3, 5)$ and $(5, 9)$. Which of the following parametric equations are **not** parametric equations for L?

   A. $x = 3 + 2t,\ y = 5 + 4t$

   B. $x = 3 - 4t,\ y = 5 - 8t$

   C. $x = 3 + 2t^3,\ y = 5 + 4t^3$

   D. $x = 3 + 2t,\ y = 5 - 4t$

   E. $x = 1 + 2t,\ y = 1 + 4t$

6. (5 points) Three masses are located in the plane. The first mass is 3 grams at $(-1, -2)$, the second mass is 2 grams located at $(2, 3)$ and the third mass is 5 grams located at $(3, -1)$. Find the center of mass of the system.

   A. $(3/5, 1/2)$

   B. $(8/5, 1/2)$

   C. $(3/5, -1)$

   D. $(8/5, -1/2)$

   E. $(7/10, -1/2)$

7. (5 points) Consider the curve with parametric equations $x(t) = t^3$, $y(t) = t^4$. Find the slope of the tangent line to the curve at $(-8, 16)$.

   A. $7/12$

   B. $31/12$

   C. $-31/12$

   D. $8/3$

   E. $-8/3$

8. (5 points) The parametric equations

$$x(t) = -2 + 4\cos(3t), \quad y(t) = 2 - 4\sin(3t), \quad 0 \le t < 2\pi/3$$

describe a circle. Find the center C and the radius r of this circle.

A. $C = (4, -4)$, $r = 2$

B. $C = (2, -2)$, $r = 3$

C. $C = (-2, 2)$, $r = 3$

D. $C = (2, 4)$, $r = 3$

E. $C = (-2, 2)$, $r = 4$

9. (5 points) Let C be the line segment with parametric equations $x(t) = 4t$, $y(t) = 3t$, for $2 \le t \le 4$. Find the area of the surface obtained by rotating C about the x-axis.

A. 120π

B. 140π

C. 160π

D. 180π

E. 240π

10. (5 points) The curve with parametric equations

$$x(t) = at + 3, \quad y(t) = 2at^2 - 5$$

contains the point $(1, -1)$. Find a.

A. -2

B. -1

C. 1

D. 2

E. 3

Free Response Questions

11. Let R be the region enclosed by $y = 1 - x^2$ and $y = 1 - x$ $(0 \le x \le 1)$. The region R is rotated about the x-axis to obtain a solid of revolution, S.

(a) (5 points) Use the washer method to write an integral giving the volume of S.

Solution: For $0 \le x \le 1$, we have $1 - x^2 \ge 1 - x$. Thus the volume V of S is given by

$$V = \int_0^1 \pi[(1 - x^2)^2 - (1 - x)^2]\, dx.$$

(b) (5 points) Evaluate the integral and give the volume of S. **Exact answer is required**.

Solution: We have

$$(1 - x^2)^2 - (1 - x)^2 = x^4 - 3x^2 + 2x.$$

Thus

$$V = \pi \int_0^1 [x^4 - 3x^2 + 2x]\, dx = \pi \left[\frac{x^5}{5} - x^3 + x^2 \right]_{x=0}^{x=1} = \frac{\pi}{5}.$$

12. Consider the lamina H bounded by the curve $y = 2\cos x$ $(-\pi/2 \le x \le \pi/2)$ and the segment $-\pi/2 \le x \le \pi/2$ of the x-axis. Assume that the density $\rho = 1$.

(a) (7 points) Find the total mass m and the moments M_x and M_y of H. [Hint: You may use symmetry to find one of the moments.]

Solution: We have

$$m = \int_{-\pi/2}^{\pi/2} 2\cos(x)\,dx = [2\sin(x)]_{x=-\pi/2}^{x=\pi/2} = 4.$$

Since H is symmetric with respect to the y-axis, we conclude that $M_y = 0$. For M_x we have

$$M_x = \frac{1}{2}\int_{-\pi/2}^{\pi/2} (2\cos(x))^2\,dx = \int_{-\pi/2}^{\pi/2} 2\cos^2(x)\,dx.$$

Since $2\cos^2(x) = 1 + \cos(2x)$, we get

$$M_x = \int_{-\pi/2}^{\pi/2} (1 + \cos(2x))\,dx = \left[x + \frac{\sin(2x)}{2}\right]_{x=-\pi/2}^{x=\pi/2} = \pi.$$

(b) (3 points) Find the center of mass for H.

Solution: We have

$$(\bar{x}, \bar{y}) = \left(\frac{M_y}{m}, \frac{M_x}{m}\right) = \left(0, \frac{\pi}{4}\right).$$

13. Consider the curve C given by $y = \sqrt{x+1}$ for $1 \le x \le 11$.

(a) (5 points) The curve C is rotated about the x-axis to obtain a surface of revolution S. Express the area A of S as an integral.

Solution: We have $f(x) = y = \sqrt{x+1}$ and

$$\begin{aligned} A &= \int_1^{11} 2\pi f(x)\sqrt{1+(f'(x))^2}\,dx \\ &= \int_1^{11} 2\pi\sqrt{x+1}\sqrt{1+\frac{1}{4(x+1)}}\,dx \\ &= \int_1^{11} 2\pi\sqrt{x+\frac{5}{4}}\,dx. \end{aligned}$$

(b) (5 points) Evaluate the integral to find the area of S. **Exact answer is required.**

Solution: We have

$$A = \int_1^{11} 2\pi\sqrt{x+\frac{5}{4}}\,dx = 2\pi\left[\frac{2}{3}\left(x+\frac{5}{4}\right)^{3/2}\right]_{x=1}^{x=11} = \frac{158\pi}{3}.$$

14. Consider the curve C with parametric equations

$$x(t) = e^t \cos(t), \quad y(t) = e^t \sin(t), \quad 0 \le t \le \pi$$

(a) (5 points) Set up an integral that represents the length L of the curve C.

Solution: We have

$$x'(t) = e^t(\cos(t) - \sin(t)), \quad y'(t) = e^t(\sin(t) + \cos(t))$$

and

$$L = \int_0^\pi \sqrt{e^{2t}\left[(\cos(t) - \sin(t))^2 + (\sin(t) + \cos(t))^2\right]}\, dt.$$

(b) (5 points) Evaluate the integral in part (a) to find the length of C.

Solution: Since

$$(\cos(t) - \sin(t))^2 + (\sin(t) + \cos(t))^2$$
$$= (\cos^2(x) - 2\cos(x)\sin(x) + \sin^2(x)) + (\sin^2(x) + 2\sin(x)\cos(x) + \cos^2(x))$$
$$= 2,$$

we have

$$L = \int_0^\pi \sqrt{2}\, e^t\, dt = \sqrt{2}(e^\pi - 1).$$

15. (10 points) Consider the curve C with parametric equations

$$x(t) = t^3 + 1, \quad y(t) = t^6 + t.$$

Check that C passes through the origin $(0, 0)$ by finding the value of t for which $(x(t), y(t)) = (0, 0)$. Then write the equation of the tangent to C at $(0, 0)$ in the slope-intercept form (i.e. $y = mx + b$).

Solution: If $(t^3 + 1, t^6 + t) = (0, 0)$, we must have $t^3 + 1 = 0$, i.e. $t = -1$. Since for $t = -1$ we also have $t^6 + t = 0$, we conclude that the origin $(0, 0)$ is on the curve C and it corresponds to $t = -1$.

The slope of C at $(0, 0)$ is

$$m = \frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{6t^5 + 1}{3t^2} = -\frac{5}{3}.$$

The y-intercept is 0 and therefore the equation of the tangent to C at $(0, 0)$ is $y = -\frac{5x}{3}$.