Exam 3

Name: ______________________________ Section: ______________

Do not remove this answer page — you will return the whole exam. You will be allowed two hours to complete this test. You are allowed to use notes on a single piece of 8.5"X11" paper, front and back, including formulas and theorems. **You are required to turn this page in with your exam.** You may use a graphing calculator during the exam, but NO calculator with a Computer Algebra System (CAS) or a QWERTY keyboard is permitted. Absolutely no cell phone use during the exam is allowed.

The exam consists of 10 multiple choice questions and 5 free response questions. Record your answers to the multiple choice questions on this page by filling in the circle corresponding to the correct answer.

Show all work to receive full credit on the free response problems. It will also help you check your answers to show work on multiple choice problems.

Multiple Choice Questions

1 (A) (B) (C) (D) (E) **6** (A) (B) (C) (D) (E)

2 (A) (B) (C) (D) (E) **7** (A) (B) (C) (D) (E)

3 (A) (B) (C) (D) (E) **8** (A) (B) (C) (D) (E)

4 (A) (B) (C) (D) (E) **9** (A) (B) (C) (D) (E)

5 (A) (B) (C) (D) (E) **10** (A) (B) (C) (D) (E)

Multiple Choice	11	12	13	14	15	Total Score
50	10	10	10	10	10	100

Trig identities

- $\sin^2(x) + \cos^2(x) = 1$,
- $\sin^2(x) = \frac{1}{2}(1 - \cos(2x))$ and $\cos^2(x) = \frac{1}{2}(1 + \cos(2x))$

Multiple Choice Questions

1. (5 points) What is the average value of the function $\sin(x)$ on the interval $[0, \pi]$?

   A. $-\dfrac{2}{\pi}$

   B. $\dfrac{\pi}{2}$

   C. $\dfrac{2}{\pi}$

   D. $-\dfrac{\pi}{2}$

   E. $\dfrac{4}{\pi}$

2. (5 points) Which of the following is a parametrization of the circle of radius 1 centered at $(1, 1)$?

   A. $x(t) = \cos(3t)$, $y(t) = 1 + \sin(3t)$

   B. $x(t) = 1 + \sin(5t)$**,** $y(t) = 1 + \cos(5t)$

   C. $x(t) = \sin(t)$, $y(t) = \cos(t)$

   D. $x(t) = 1 + 5\cos(t)$, $y(t) = 1 + 5\sin(t)$

   E. $x(t) = 2\sqrt{1 - t}$, $y(t) = 2\sqrt{1 + t}$

3. (5 points) Three masses are located in the plane: 1 gram at $(-2, 2)$, 1 gram at $(2, 2)$, and 6 grams at $(0, -6)$. Find the center of mass of this system.

   A. $(0, -4)$

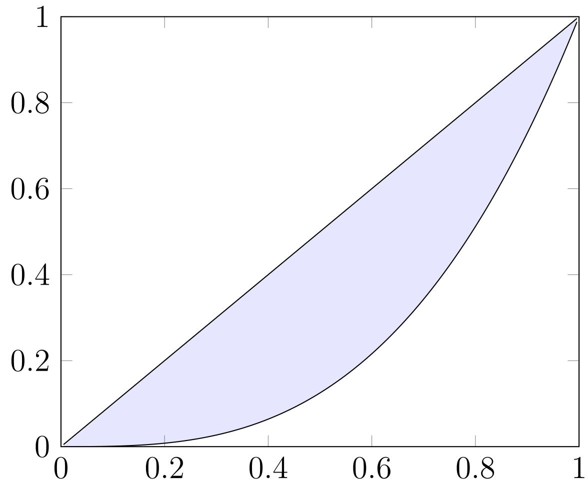
   B. $(\frac{1}{2}, -\frac{7}{2})$

   C. $(0, -\frac{49}{8})$

   D. $(0, -28)$

   E. $(-\frac{14}{3}, 0)$

4. (5 points) The region bounded by $y = x$ and $y = x^3$ is shown below.



Consider the solid obtained by rotating this region around the x-**axis**. Using the **disks/washers** method, which integral will compute the volume of this solid?

A. $\displaystyle\int_0^{\sqrt{3}} 2\pi\left(x - x^3\right)^2 dx.$

B. $\displaystyle\int_0^1 \pi\left(x^2 - x^6\right) dx.$

C. $\displaystyle\int_0^1 \pi\left(x^3 - x^6\right) dx.$

D. $\displaystyle\int_0^1 \pi\left(x - x^3\right)^2 dx.$

E. $\displaystyle\int_0^1 \pi\left(x - x^3\right) dx.$

5. (5 points) Which integral below computes the length of the arc parametrized by $(x, \sin(x^2))$ where $0 \leq x \leq \pi$?

A. $\displaystyle\int_0^{\pi} 1 + 4x^2[\cos(x^2)]^2 dx.$

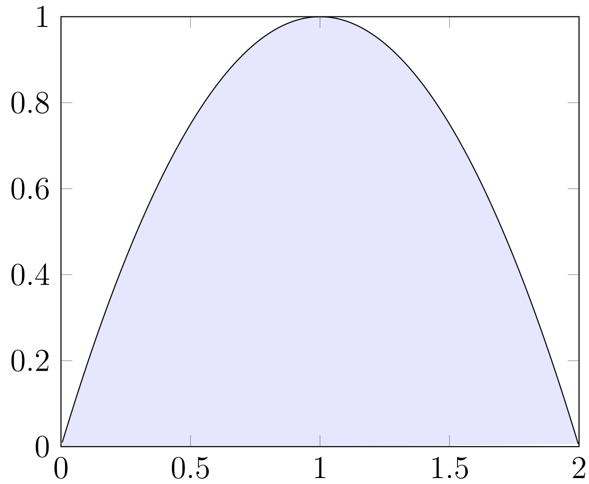
B. $\displaystyle\int_0^{\pi} \sqrt{x^2 + [\sin(x^2)]^2} dx.$

C. $\displaystyle\int_0^{\pi} \sqrt{1 + [\sin(x^2)]^2} dx.$

D. $\displaystyle\int_0^{\pi} 2\pi x^2\sqrt{1 + [\cos(x^2)]^2} dx.$

E. $\displaystyle\int_0^{\pi} \sqrt{1 + 4x^2[\cos(x^2)]^2} dx.$

6. (5 points) The region bounded by the curve $y = 2x - x^2$ and the x-axis is shown below.



Consider the solid obtained by rotating this region about the **y-axis**. Using the **shell** method, which integral will compute the volume of this solid?

A. $\int_0^2 2\pi x(2x - x^2)dx$.

B. $\int_0^2 \pi(2x - x^2)^2 dx$.

C. $\int_0^2 \pi(2x - x^2)dx$.

D. $\int_0^2 2\pi x^2(2x - x^2)^2 dx$.

E. $\int_0^2 2\pi(x^2 - 2x)dx$.

7. (5 points) The line $y = x + 1$ for $1 \leqslant x \leqslant 4$ is rotated about the x-axis. What is the **surface area** of the resulting surface?

A. 21π

B. $18\sqrt{2}\pi$

C. $20\sqrt{2}\pi$

D. $21\sqrt{2}\pi$

E. $2\sqrt{2}\pi$

8. (5 points) Find the slope of the tangent line to the curve parametrized by $x(t) = t - t^2, y(t) = t^3 + 1$ at the point $(-2, 9)$.

   A. 4
   B. $\frac{12}{5}$
   C. -4
   D. $\frac{11}{4}$
   E. -2

9. (5 points) Which of the following integrals computes the length of the curve parametrized by
$x(t) = (t+1)^2$, $y(t) = \frac{2}{3}(t+1)^3$, $-1 \leq t \leq 0$?

   A. $\int_{-1}^{0} 2\sqrt{1 + (t+1)^2}dt$
   B. $\int_{-1}^{0} 2t\sqrt{2 + 2t}dt$
   C. $\int_{-1}^{1} (t+1)\sqrt{1 + (t+1)^2}dt$
   D. $\int_{-1}^{1} (t+1)^2\sqrt{1 + (t+1)^2}dt$
   E. $\int_{-1}^{0} 2(t+1)\sqrt{2 + 2t + t^2}dt$

10. (5 points) The curve $y = e^x$ for $0 \leqslant x \leqslant 1$ is rotated about the y-axis, producing a surface. Which of the following integrals calculates its surface area?

   A. $\int_{0}^{1} 2\pi x\sqrt{1 + e^{2x}}\, dx$
   B. $\int_{1}^{e} 2\pi \ln(y-1)\sqrt{1 + \left(\frac{1}{y+1}\right)^2}\, dy$
   C. $\int_{1}^{e} 2\pi y\sqrt{1 + \left(\frac{1}{y}\right)^2}\, dy$
   D. $\int_{0}^{1} 2\pi e^x\sqrt{1 + e^{2x}}\, dx$
   E. $\int_{0}^{1} 2\pi e^x\sqrt{1 + x^2}\, dx$

Free Response Questions

11. Consider the circle C parametrized by the functions

$$x(t) = \sin(t^2),$$
$$y(t) = \cos(t^2).$$

(a) (5 points) Find the slope of the tangent line to the point on C given by $t = \frac{\pi}{4}$.

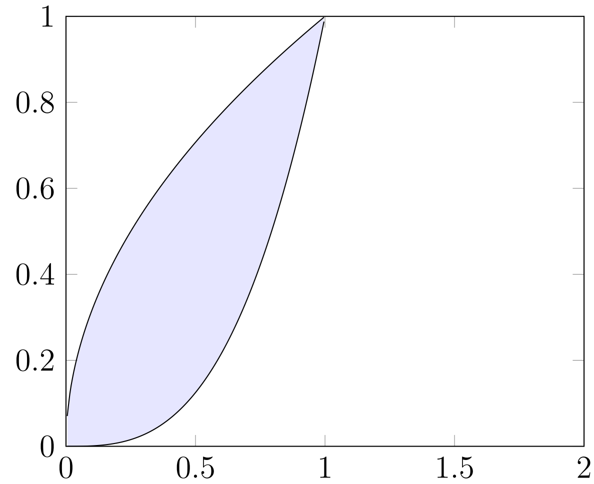
(b) (5 points) Find the arc length of the piece of C parametrized by $0 \le t \le \frac{\pi}{2}$.

12. Let S be the region in the plane bounded by the parabola $y = 1 - x^2$ and the x-axis for $0 \leq x \leq 1$. Assume that S has uniform density $\rho = 1$.

   (a) (8 points) Find the total mass M and the moments M_y and M_x for S.
   Clearly label each of your answers.

   (b) (2 points) Find the center of mass of S.

13. The region between the curves $y = \sqrt{x}$ and $y = x^3$ is shown below.



Let V be the solid obtained by rotating this region **around the vertical line** $x = 1$.

(a) (5 points) Set up but do not evaluate the integral that computes the volume of V using the disk/washer method.

(b) (5 points) Set up but do not evaluate the integral that computes the volume of V using the cylindrical shells method.

14. Let S be the surface obtained by revolving the arc L parametrized by $x(t) = t - 1$ $y(t) = \sqrt{t}$, $0 \leq t \leq 3$ around the x axis.

(a) (2 points) Set up but do not evaluate an integral which computes the arc length of L.

(b) (3 points) Set up but do not evaluate an integral which computes the surface area of S.

(c) (5 points) Find the surface area of S.

15. Let V be the shape bounded by the x-axis and the semicircle $y = \sqrt{4 - x^2}$, with cross sections given by squares perpendicular to the x-axis.

(a) (3 points) Find a function giving the area of the cross-section of V at x.

(b) (4 points) Set up an integral which computes the volume of V.

(c) (3 points) Find the volume of V.