Exam 3

Name: _____ Section: _____

Do not remove this answer page — you will return the whole exam. You will be allowed two hours to complete this test. If you find you need scratch paper during the exam, please ask. You may not use any of your own notes, paper or anything else not provided. You may use a graphing calculator during the exam, but NO calculator with a Computer Algebra System (CAS). Absolutely no communication device use during the exam is allowed.

The exam consists of 10 multiple choice questions and 5 free response questions. Record your answers to the multiple choice questions on this page by filling in the circle corresponding to the correct answer.

Show all work to receive full credit on the free response problems. It will also help you check your answers to show work on multiple choice problems.

Multiple Choice Questions

1	A	B	C	D	E	**6**	A	B	C	D	E
2	A	B	C	D	E	**7**	A	B	C	D	E
3	A	B	C	D	E	**8**	A	B	C	D	E
4	A	B	C	D	E	**9**	A	B	C	D	E
5	A	B	C	D	E	**10**	A	B	C	D	E

Multiple Choice	11	12	13	14	15	Total Score
50	10	10	10	10	10	100

1. (5 points) Find the average value of $f(x) = 3\sqrt{x} + 1$ on the interval $[1, 16]$.

   A. 9

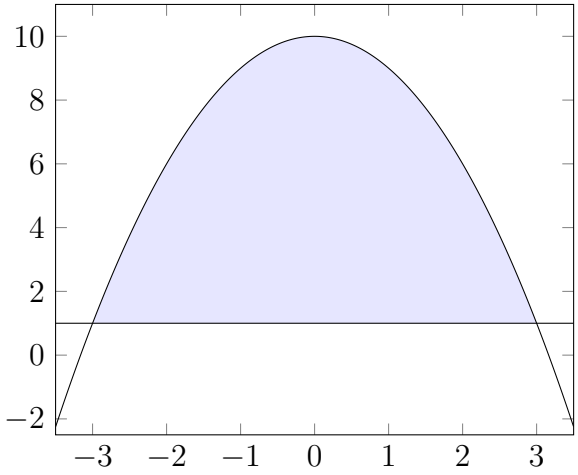
   B. 141

   C. $\frac{3}{5}$

   D. $\frac{17}{2}$

   E. $\frac{141}{15}$

2. (5 points) The region R bounded by $y = 10 - x^2$ and $y = 1$ is shown below.



   Consider the solid obtained by rotating R about the **horizontal line** $y = -2$. Which integral computes the volume of this solid using the **disk/washer** method?

   A. $\pi \int_{-3}^{3} (12 - x^2)^2 - 3^2 \, dx$

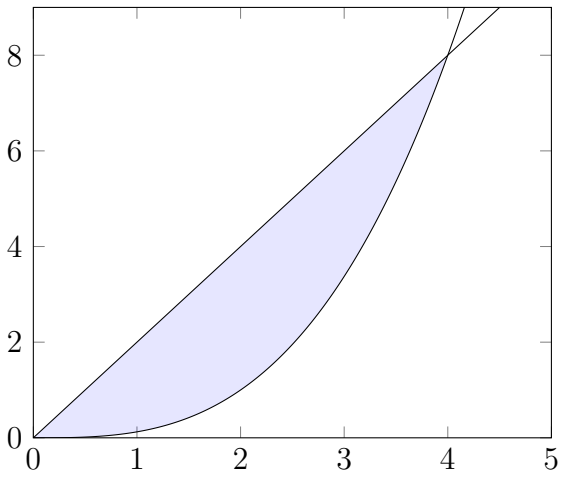
   B. $\pi \int_{-3}^{3} (10 - x^2)^2 - 1^2 \, dx$

   C. $\pi \int_{-3}^{3} (9 - x^2)^2 \, dx$

   D. $\pi \int_{-3}^{3} (10 - x^2)^2 - 3^2 \, dx$

   E. $\pi \int_{-3}^{3} (9 - x^2)^2 + (-2)^2 \, dx$

3. (5 points) The region R bounded by the curves $y = \dfrac{x^3}{8}$ and $y = 2x$ is shown below.



Consider the solid obtained by rotating R about the $\boldsymbol{y}$**−axis**. Which integral computes the volume of this solid using the **shell** method?

A. $\displaystyle\int_0^4 2\pi\left(2x - \frac{x^3}{8}\right)^2 dx$

B. $\displaystyle\int_0^4 2\pi x\left(2x - \frac{x^3}{8}\right) dx$

C. $\displaystyle\int_0^8 2\pi y\left(\sqrt[3]{8y} - \frac{y}{2}\right) dy$

D. $\displaystyle\int_0^8 2\pi\left((\sqrt[3]{8y})^2 - \left(\frac{y}{2}\right)^2\right) dy$

E. $\displaystyle\int_0^4 2\pi\left((2x)^2 - \left(\frac{x^3}{8}\right)^2\right) dx$

4. (5 points) Which integral computes the **arc length** of the curve $y = 3x^2 - 5x$ from the point $(1, -2)$ to the point $(3, 12)$?

A. $\displaystyle\int_{-2}^{12} \sqrt{1 + (6x-5)^2}\, dx$

B. $\displaystyle\int_{1}^{12} \sqrt{36x^2 - 60x + 24}\, dx$

C. $\displaystyle\int_{1}^{3} \sqrt{1 + (6x-5)^2}\, dx$

D. $\displaystyle\int_{1}^{3} \sqrt{1 + (3x^2-5x)^2}\, dx$

E. $\displaystyle\int_{-2}^{12} x\sqrt{1 + (6x-5)^2}\, dx$

5. (5 points) The curve $y = 4+3x^2$ from $x = 0$ to $x = 3$ is rotated about the $\boldsymbol{y}$**−axis**. (Be careful!) Which integral computes the area of the resulting surface?

   A. $\displaystyle\int_0^3 2\pi(4+3x^2)\sqrt{1+(6x)^2}\,dx$

   B. $\displaystyle\int_0^3 2\pi x\sqrt{1+(6x)^2}\,dx$

   C. $\displaystyle\int_0^3 2\pi x\sqrt{1+(4+3x^2)^2}\,dx$

   D. $\displaystyle\int_0^3 2\pi(4+3x^2)\sqrt{1+(4+3x^2)^2}\,dx$

   E. $\displaystyle\int_0^3 2\pi x(4+3x^2)\,dx$

6. (5 points) Two masses are located in the plane: 4g at $(3,-1)$ and 11g at $(5,7)$. Find the center of mass of this system.

   A. $(4,3)$

   B. $(4,4)$

   C. $(\frac{67}{15},\frac{73}{15})$

   D. $(\frac{8}{15},\frac{6}{15})$

   E. $(\frac{71}{37},\frac{37}{15})$

7. (5 points) Eliminate the parameter t to find a Cartesian equation for $x = t-4$ and $y = t^2+3$.

   A. $y = 2t$

   B. $y = x^2+x-1$

   C. $y = \sqrt{x-3}-4$

   D. $y = (x+4)^2+3$

   E. $y = (x-4)^2+3$

8. (5 points) Find dy/dx in terms of t if $x = te^t$ and $y = 8 + e^t$.

A. $\dfrac{dy}{dx} = 1$

B. $\dfrac{dy}{dx} = e^t$

C. $\dfrac{dy}{dx} = te^t + \dfrac{8}{e^t}$

D. $\dfrac{dy}{dx} = \dfrac{1}{t+1}$

E. $\dfrac{dy}{dx} = \dfrac{8+e^t}{te^t}$

9. (5 points) Which integral computes the **arc length** of the curve parametrized by $x(t) = \tan(t)$, $y(t) = \cos(t)$ for $0 \le t \le \pi/4$?

A. $\displaystyle\int_0^{\pi/4} \sqrt{\sec^2(t) - \sin^2(t)}\, dt$

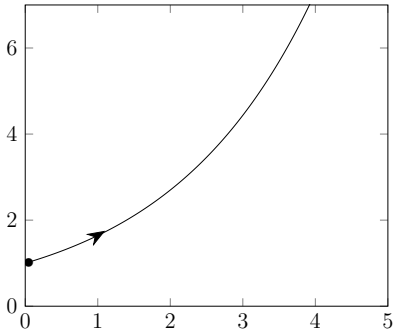
B. $\displaystyle\int_0^{\pi/4} \sqrt{\tan^2(t) + \cos^2(t)}\, dt$

C. $\displaystyle\int_0^{\pi/4} \sqrt{\sec^4(t) + \sin^2(t)}\, dt$

D. $\displaystyle\int_0^{\pi/4} \sqrt{1 + \sin^2(t)}\, dt$

E. $\displaystyle\int_0^{\pi/4} \sqrt{1 + \frac{\sec^4(t)}{\sin^2(t)}}\, dt$

10. (5 points) The graph below shows the plot of a parametric curve. Which parametrization is correct?



A. $x(t) = t + 5,\ y(t) = \sqrt{t+5},\ t > -5$

B. $x(t) = \dfrac{1}{t},\ y(t) = \dfrac{1}{t},\ t > 0$

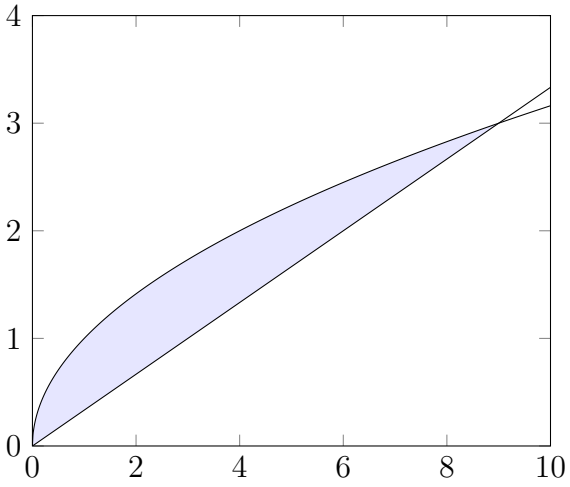
C. $x(t) = \cos(t),\ y(t) = 2\cos(t) + 1$

D. $x(t) = e^t,\ y(t) = e^{-t}$

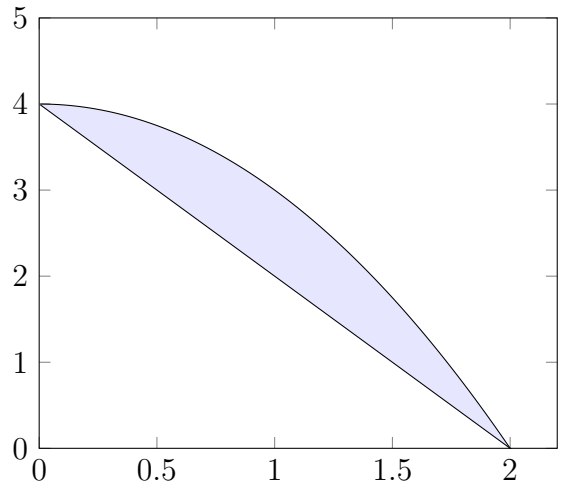
E. $x(t) = \ln(t),\ y(t) = \sqrt{t},\ t \ge 1$

Free Response Questions

11. (10 points) The base of a solid is the region enclosed by $y = \sqrt{x}$ and $y = x/3$, shown below. Cross-sections perpendicular to the x-axis are squares. Set up an integral which computes the volume of this solid, and then find the volume. Show all steps to compute the integral.



12. The region R bounded by $y = 4 - x^2$ and $y = 4 - 2x$ is shown below.



(a) (5 points) Set up but do not evaluate the integral that computes the volume when R is rotated about the $\boldsymbol{x}$**−axis**. Use the **disk/washer** method.

(b) (5 points) Set up but do not evaluate the integral that computes the volume when R is rotated about the **vertical line** $x = 3$. Use the **shell** method.

13. Let S be the region bounded by $y = \sqrt{x}$, $y = 3$ and the $y-$axis. Assume S has uniform density $\rho = 1$.

(a) (2 points) Find the total mass M for S.

(b) (3 points) Find the moment M_x for S.

(c) (3 points) Find the moment M_y for S.

(d) (2 points) Find the center of mass of S.

14. Let C be the curve defined by the graph of $f(x) = \sin(3x)$, $0 \leq x \leq \pi/3$.

(a) (5 points) Set up but do **not** evaluate an integral which computes the length of C.

(b) (5 points) Set up, but do **not** evaluate, an integral which computes the area of the surface obtained by revolving C about the $\boldsymbol{x}-$**axis**.

15. Let C be the curve parametrized by $x(t) = t^2 + 3$, $y(t) = t^5 - t$.

(a) (6 points) Find the slope of the tangent line to C at the point $(x, y) = (7, -30)$.

(b) (4 points) Find the second derivative, $\dfrac{d^2y}{dx^2}$, in terms of t. (You do not need to simplify your answer.)