MA 114 — Calculus II Fall 2014
Sections 5 – 8 and 401, 402

Exam 4 Dec. 16, 2014

Name: ______________________________

Section: ______________________________

Last 4 digits of student ID #: ________

- No books or notes may be used.
- Turn off all your electronic devices and do not wear ear-plugs during the exam.
- You may use a calculator, but not one which has symbolic manipulation capabilities or a QWERTY keyboard.
- Additional blank sheets for scratch work are available upon request.
- **Multiple Choice Questions:** Record your answers on the right of this cover page by marking the box corresponding to the correct answer.
- **Free Response Questions:** Show all your work on the page of the problem. Clearly indicate your answer and the reasoning used to arrive at that answer.

Multiple Choice Answers

Question					
1	A	B	~~C~~	D	E
2	A	~~B~~	C	D	E
3	A	B	~~C~~	D	E
4	~~A~~	B	C	D	E

Exam Scores

Question	Score	Total
MC		20
5		17
6		15
7		15
8		18
9		15
Total		100

Unsupported answers for the free response questions may not receive credit!

Record the correct answer to the following problems on the front page of this exam.

1. What conclusion can we draw about the convergence of $\sum_{n=1}^{\infty} \frac{2n-1}{2n^2}$ if we use the limit comparison test with the series $\sum_{n=1}^{\infty} \frac{1}{n}$?

$\frac{2n-1}{2n^2}\cdot\frac{n}{1} = \frac{2n^2-n}{2n^2} \xrightarrow[n\to\infty]{} 1$

Since $\sum \frac{1}{n}$ diverges, so does $\sum \frac{2n-1}{2n^2}$.

A. The series converges absolutely.

B. The series converges conditionally.

C. The series diverges.

D. The series converges for $n > 1$ and diverges for $n < 1$.

E. The test is inconclusive.

2. Which of the following is the general solution of the differential equation

$$y' + \frac{3}{2+x}y = \frac{\sin(x)}{(2+x)^5}, \qquad x > 0.$$

A. $y = \int \frac{\sin(x)}{(2+x)^5}dx + \frac{C}{(2+x)^3}$.

B. $y = \frac{1}{(2+x)^3}\int \frac{\sin(x)}{(2+x)^2}dx + \frac{C}{(2+x)^3}$.

C. $y = (2+x)^3 \int \frac{\sin(x)}{(2+x)^2}dx + C(2+x)^3$.

D. $y = -\frac{\cos(x)}{(2+x)^4} + \frac{C}{(2+x)^3}$.

E. $y = (2+x)^3 \int \frac{\sin(x)}{(2+x)^5}dx + C(2+x)^3$.

- $\int \frac{3}{2+x}dx = 3\cdot\ln(2+x)$
- $\alpha = e^{3\ln(2+x)} = (2+x)^3$
- $\int (2+x)^3\cdot\frac{\sin(x)}{(2+x)^5}dx = \int \frac{\sin(x)}{(2+x)^2}dx$

So

$y = (2+x)^{-3}\left(\int \frac{\sin x}{(2+x)^2}dx + C\right)$

Record the correct answer to the following problems on the front page of this exam.

3. Which of the following integrals converge?

i. $\int_{-1}^{1} \frac{1}{x^2+1}\,dx$ $x^2+1>0$ for all x. So converges.

ii. $\int_{1}^{\infty} \frac{1}{e^x}\,dx = \lim_{R\to\infty} \int_1^R e^{-x}\,dx = \lim_{R\to\infty} (-e^{-R}+e^{-1}) = e^{-1}$

iii. $\int_{0}^{1} \frac{1}{x^{3/2}}\,dx = \lim_{R\to 0^+} \int_R^1 x^{-\frac{3}{2}}\,dx = \lim_{R\to 0^+} \left[(-2x^{-\frac{1}{2}})\Big|_R^1\right]$

$= \lim_{R\to 0^+} \left(-2 + \frac{2}{\sqrt{R}}\right) = \infty$

A. Only (i).

B. Only (iii).

~~C.~~ (i) and (ii).

D. (i) and (iii).

E. (ii) and (iii).

4. Which of the following is the Taylor polynomial of degree 3 for

$$f(x) = \sin(x)e^x + x$$

centered at $c = 0$?

$f'(x) = \cos(x)e^x + \sin(x)e^x + 1$

$= (\cos(x) + \sin(x))e^x + 1$

$f''(x) = (\cos(x) - \sin(x) + \cos(x) + \sin(x))e^x$

$= 2\cos(x)e^x$

$f(0) = 0, \quad f'(0) = 2, \quad f''(0) = 2$

$2x + x^2 + \ldots$

~~A.~~ $2x + x^2 + \frac{1}{3}x^3$.

B. $2 + x + x^2$.

C. $2x + 2x^2 + x^3$.

D. $1 + 3x + 4x^2 - x^3$.

E. $x + \frac{1}{2}x^2 - \frac{1}{3}x^3$

5. Given the curve with parametrization $c(t) = (t^2 - 4, t^3 + 6)$.

(a) Show that the point $(-3, 7)$ is on the parametric curve.

$-3 = t^2 - 4 \Rightarrow t^2 = 1 \Rightarrow t = \pm 1.$

$t = 1: \; t^3 + 6 = 7$ ✓

$t = -1: \; t^3 + 6 = 5$ ✗

Hence $c(1) = (-3, 7)$

(b) Find the equation for the tangent line to the curve at the point $(-3, 7)$.

$\text{slope} = \frac{y'(1)}{x'(1)} = \frac{3}{2}.$ Equation is

$y - 7 = \frac{3}{2}(x + 3)$

$y'(t) = 3t^2$

$x'(t) = 2t$

$\boxed{y = \frac{3}{2}x + \frac{23}{2}}$

(c) Find the arclength for the curve from $0 \le t \le 2$.

$$s = \int_0^2 \sqrt{x'(t)^2 + y'(t)^2}\, dt = \int_0^2 \sqrt{4t^2 + 9t^4}\, dt$$

$$= \int_0^2 t\sqrt{4 + 9t^2}\, dt \; \underset{\substack{u = 4 + 9t^2 \\ du = 18t\,dt}}{=} \; \frac{1}{18}\int_4^{40} u^{\frac{1}{2}}\, du$$

$$= \frac{1}{18} \cdot \frac{2}{3} u^{\frac{3}{2}} \Big|_4^{40} = \underline{\frac{1}{27}\left(\sqrt{40}^{\,3} - 8\right)} \approx 9.07$$

$$= \underline{\frac{80}{27}\sqrt{10} - \frac{8}{27}}$$

Free Response Questions: Show your work!

6. (a) Use calculus to compute the integral $\int \tan^3(x)\sec^4(x)\,dx$.

$$= \int \tan^3(x)(1+\tan^2(x))\sec^2(x)\,dx$$

$$= \int u^3(1+u^2)\,du = \frac{1}{4}u^4 + \frac{1}{6}u^6 + C$$

$u = \tan(x)$
$du = \sec^2(x)\,dx$

$$= \frac{1}{4}\tan^4(x) + \frac{1}{6}\tan^6(x) + C$$

(b) Use calculus to compute the integral $\int \frac{x^3}{x^2-4}\,dx$.

$$\begin{array}{r} x \\ (x^2-4)\,\overline{)\,x^3} \\ \underline{x^3-4x} \\ 4x \end{array}$$

, so $x^3 = x(x^2-4) + 4x$.

$$\int \frac{x^3}{x^2-4}\,dx = \int \frac{x(x^2-4)+4x}{x^2-4}\,dx = \int x\,dx + \int \frac{4x}{x^2-4}\,dx$$

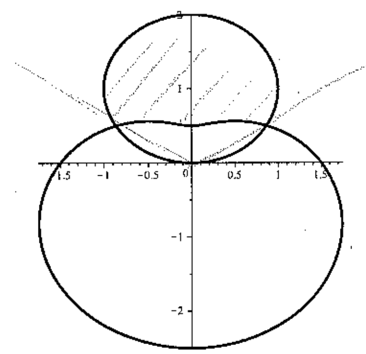
$$= \frac{1}{2}x^2 + 2\int \frac{du}{u} = \frac{1}{2}x^2 + 2\ln|u| + C$$

$$= \frac{1}{2}x^2 + 2\ln|x^2-4| + C$$

7. Below is a graph of

the limaçon curve $r = \frac{3}{2} - \sin(\theta)$ and the circle $r = 2\sin(\theta)$ for θ in $[0, 2\pi]$.

(a) Identify the polar coordinates of the two points of intersection.



$\frac{3}{2} - \sin\theta = 2\sin\theta$

$\Rightarrow \frac{3}{2} = 3\sin\theta$

$\Rightarrow \sin\theta = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{6}, \frac{5\pi}{6}$

then $r = 1$.

Points are $\boxed{\left(1, \frac{\pi}{6}\right), \left(1, \frac{5\pi}{6}\right)}$ in polar coordinates.

(b) Find the rectangular coordinates of the two intersection points.

$x = r\cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$, $y = r\sin\left(\frac{\pi}{6}\right) = \frac{1}{2}$.

the first point has rect. coordinates $\boxed{\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)}$

$x = r\cos\left(\frac{5\pi}{6}\right) = -\frac{\sqrt{3}}{2}$, $y = r\sin\left(\frac{5\pi}{6}\right) = \frac{1}{2}$.

the second point has rect. coord. $\boxed{\left(-\frac{\sqrt{3}}{2}, \frac{1}{2}\right)}$.

(c) Set up the integral for computing the area inside the circle but outside of the limaçon. **Do not evaluate the integral!**

$$\frac{1}{2}\int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} \left(4\sin^2(\theta) - \left(\frac{3}{2} - \sin\theta\right)^2\right) d\theta$$

Free Response Questions: Show your work!

8. Determine the radius and interval of convergence for the power series

$$\sum_{n=2}^{\infty} \frac{(2x)^n}{n^2+n}.$$

State the tests you use and verify that all assumptions are satisfied.

Ratio Test:

$$\left|\frac{a_{n+1}}{a_n}\right| = \left|\frac{(2x)^{n+1}\cdot(n^2+n)}{((n+1)^2+(n+1))\cdot(2x)^n}\right| = \left|2x\cdot\frac{n^2+n}{n^2+3n+2}\right|$$

$$\xrightarrow[n\to\infty]{} |2x|$$

For convergence we need $|2x|<1$, thus $|x|<\frac{1}{2}$. Thus $\boxed{\text{radius of convergence is } R=\frac{1}{2}.}$

The interval of convergence is $(-\frac{1}{2},\frac{1}{2})$ plus potentially endpoints:

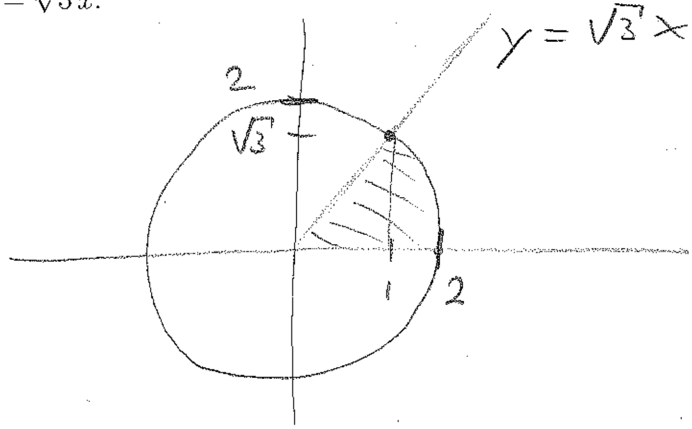
$x=-\frac{1}{2}$: $\sum_{n=2}^{\infty}\frac{(-1)^n}{n^2+n}$ converges by Leibniz test b/c $\frac{1}{n^2+n}$ is positive, decreasing and has limit 0.

$x=\frac{1}{2}$: $\sum_{n=2}^{\infty}\frac{1}{n^2+n}$ converges by comparison test b/c $\frac{1}{n^2+n}<\frac{1}{n^2}$ and $\sum\frac{1}{n^2}$ converges (p-series w/ $p=2$).

Thus $\boxed{\text{interval of convergence is } [-\frac{1}{2},\frac{1}{2}]}$.

Free Response Questions: Show your work!

9. (a) Sketch the region inside the circle $x^2 + y^2 = 4$, above the x-axis, and below the line $y = \sqrt{3}\,x$.



(b) Find the volume of the solid obtained by rotating the above region about the y-axis.

$$2\pi\int_0^1 x\sqrt{3}\,x\,dx + 2\pi\int_1^2 x\sqrt{4-x^2}\,dx$$

$$= 2\pi\left(\sqrt{3}\,\frac{1}{3}x^3\Big|_0^1 - \frac{1}{2}\int_3^0 u^{\frac{1}{2}}\,du\right)$$

$u = 4 - x^2$
$du = -2x\,dx$

$$= 2\pi\left(\frac{\sqrt{3}}{3} - \frac{1}{2}\left(\frac{2}{3}u^{\frac{3}{2}}\Big|_3^0\right)\right)$$

$$= 2\pi\left(\frac{\sqrt{3}}{3} + \frac{1}{3}\sqrt{3}^3\right) = 2\pi\left(\frac{\sqrt{3}}{3} + \sqrt{3}\right)$$

$$= \frac{8\pi}{3}\sqrt{3} \approx 14.51$$

Alternatively with washer method along y-axis:

$$\pi\int_0^{\sqrt{3}} \sqrt{4-y^2}^2 - \left(\frac{1}{\sqrt{3}}y\right)^2 dy = \pi\int_0^{\sqrt{3}}\left(4 - y^2 - \frac{1}{3}y^2\right)dy = \pi\int_0^{\sqrt{3}}\left(4 - \frac{4}{3}y^2\right)dy$$

$$= \pi\left(4y - \frac{4}{9}y^3\right)\Big|_0^{\sqrt{3}} = \frac{8\pi}{3}\sqrt{3}$$