Exam 4

Name: ______________________ Section and/or TA: ______________

Do not remove this answer page — you will return the whole exam. You will be allowed two hours to complete this test. No books or notes may be used. You may use a graphing calculator during the exam, but NO calculator with a Computer Algebra System (CAS) or a QWERTY keyboard is permitted. Absolutely no cell phone use during the exam is allowed.

The exam consists of 12 multiple choice questions and 6 free response questions. Record your answers to the multiple choice questions on this page by filling in the circle corresponding to the correct answer.

Show all work to receive full credit on the free response problems. The wise student will show work for the multiple choice problems as well.

Multiple Choice Questions

1	A	B	C	D	E	**7**	A	B	C	D	E
2	A	B	C	D	E	**8**	A	B	C	D	E
3	A	B	C	D	E	**9**	A	B	C	D	E
4	A	B	C	D	E	**10**	A	B	C	D	E
5	A	B	C	D	E	**11**	A	B	C	D	E
6	A	B	C	D	E	**12**	A	B	C	D	E

SCORE

Multiple Choice	13	14	15	16	17	18	Total Score
36	12	11	12	10	11	10	100

THIS PAGE SHOULD BE BLANK

Multiple Choice Questions

1. The number of wolves in a national park is modeled by the function W that satisfies the logistic differential equation $\frac{dW}{dt} = 0.45W\left(1 - \frac{W}{400}\right)$, where t is the time in years and $W(0) = 28$. What is $\lim_{n\to\infty} W(t)$?

   A. 28

   B. 400

   C. 450

   D. 1260

   E. 1800

2. Let g be a differentiable function. Which of the following expression equals

$$\int \sin(x)g(x)dx\,?$$

   A. $g(x)\sin(x) + \int g'(x)\sin(x)dx.$

   B. $g(x)\sin(x) - \int g'(x)\cos(x)dx.$

   C. $-g'(x)\cos(x) + \int g(x)\cos(x)dx.$

   D. $-g(x)\cos(x) + \int g'(x)\cos(x)dx.$

   E. $g'(x)\sin(x) + \int g'(x)\cos(x)dx.$

3. What are all the values of x for which the series $\sum_{n=1}^{\infty} \frac{(x-2)^n}{n3^n}$ converges?

   A. $-3 \le x \le 3$

   B. $-3 < x < 3$

   C. $-1 < x \le 5$

   D. $-1 \le x \le 5$

   E. $-1 \le x < 5$

4. For $0 \le t \le 13$, an object travels along an elliptical path given by the parametric equations $x = 3\cos t$ and $y = 4\sin t$. At the point where $t = 13$, the object leaves the path and travels along the line tangent to the path at that point. What is the slope of the line on which the object travels?

   A. $-\frac{4}{3}$

   B. $-\frac{3}{4}$

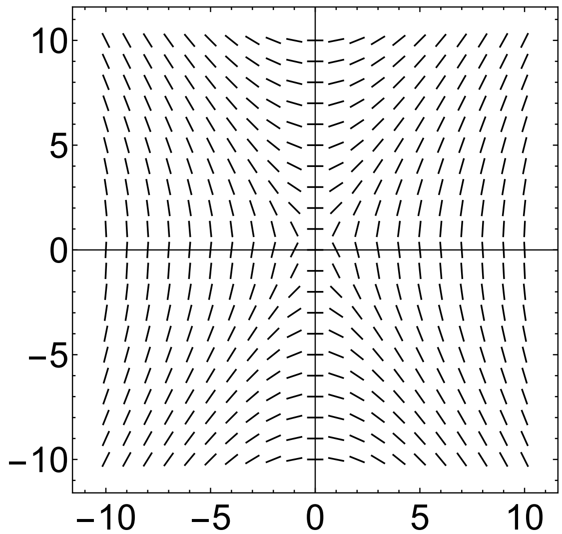
   C. $-\frac{4\tan 13}{3}$

   D. $-\frac{4}{3\tan 13}$

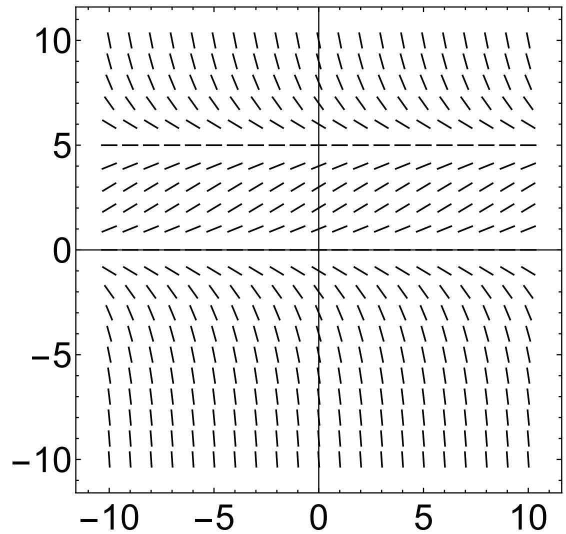
   E. $-\frac{3}{4\tan 13}$

5. Which of the following is the direction field for the differential equation $y' = x - 2y$?

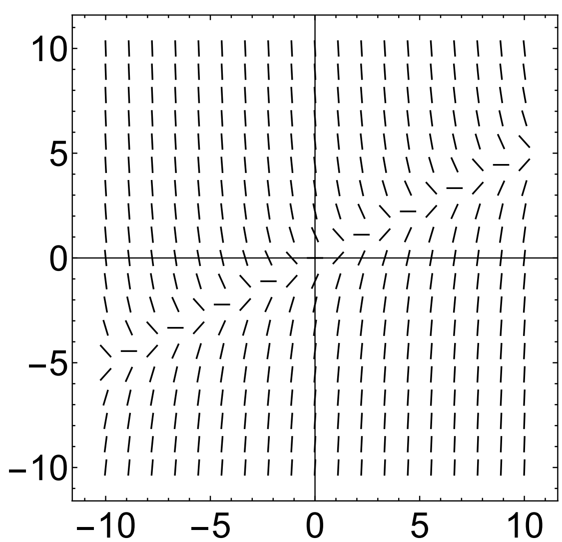
A.



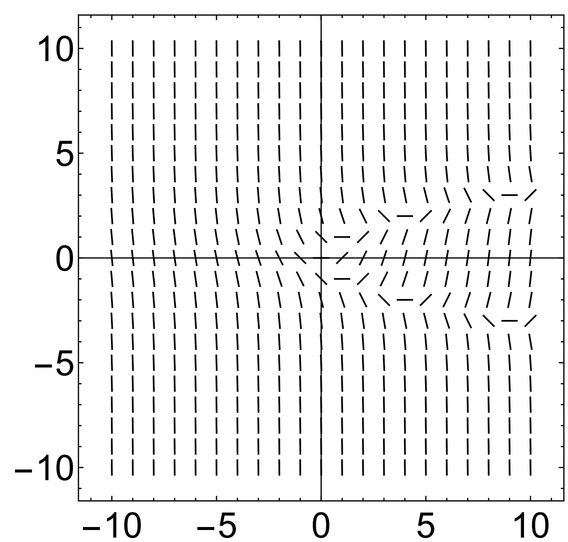
B.



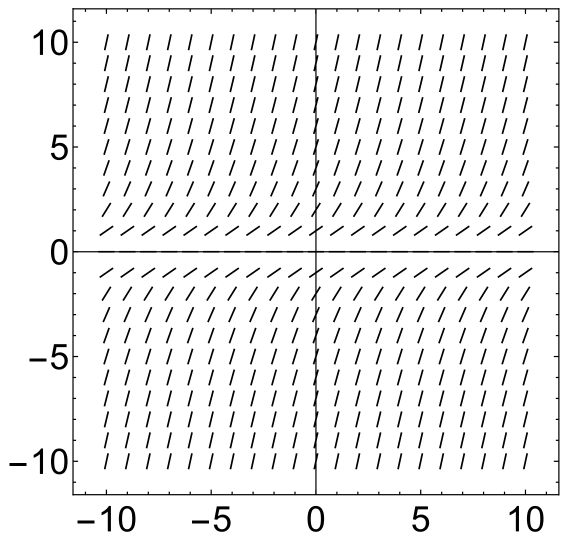
C.



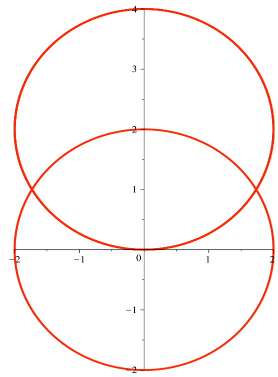
D.



E.



6. Which of the following integrals represents the area of the region enclosed by the graph of the polar curves $r = 4\sin(\theta)$ and $r = 2$?



A. $\frac{1}{2}\int_{\pi/6}^{5\pi/6} \left(16\sin^2(\theta) - 4\right) d\theta.$

B. $\frac{1}{2}\int_{\pi/6}^{5\pi/6} \left(4 - 16\sin^2(\theta)\right) d\theta.$

C. $\frac{1}{2}\int_{\pi/4}^{3\pi/4} \left(4\sin(\theta) - 2\right) d\theta.$

D. $\frac{1}{2}\int_{\pi/6}^{5\pi/6} 16\sin^2(\theta) d\theta.$

E. $\frac{1}{2}\int_{\pi/3}^{2\pi/3} \left(16\sin^2(\theta) - 4\right) d\theta.$

7. Which of the following is true for the sequence $\left\{\frac{100}{\sqrt{n^2 + 10n - 8}}\right\}$. There is only one correct answer.

A. The sequence is increasing and divergent.

B. The sequence is increasing and convergent.

C. The sequence is decreasing and divergent.

D. The sequence is decreasing and convergent.

E. The sequence is bounded and divergent.

8. The points on the polar curve $r = 1 - \cos\theta$ where the tangent line is horizontal are

A. $(0,0)$, $\left(\frac{1}{2}, \frac{\pi}{3}\right)$

B. $(0,0)$, $\left(\frac{3}{2}, \frac{2\pi}{3}\right)$, $\left(\frac{3}{2}, -\frac{2\pi}{3}\right)$

C. $(2,\pi)$, $\left(\frac{1}{2}, \frac{\pi}{3}\right)$, $\left(\frac{1}{2}, -\frac{\pi}{3}\right)$

D. $(2,\pi)$, $\left(\frac{3}{2}, \frac{2\pi}{3}\right)$, $\left(\frac{3}{2}, -\frac{2\pi}{3}\right)$

E. $\left(\frac{3}{2}, \frac{2\pi}{3}\right)$, $\left(\frac{3}{2}, -\frac{2\pi}{3}\right)$

9. The partial fraction expansion of $\frac{x^2+4}{x^2(x-4)}$ is of the form:

A. $\frac{Ax+B}{x^2} + \frac{C}{x-4}$

B. $\frac{A}{x} + \frac{B}{x^2} + \frac{C}{x-4}$

C. $\frac{A}{x^2} + \frac{B}{x-4}$

D. $\frac{A}{x} + \frac{B}{x} + \frac{C}{x-4}$

E. $\frac{Ax^2+Bx+C}{x^2} + \frac{D}{x-4}$

10. If $f(x) = x - \frac{1}{2}x^2 + \frac{1}{3}x^3 - \frac{1}{4}x^4 + ...$ converges for all x in $(-1, 1]$ then the value of $f'''(0)$ is:

A. 0

B. 1

C. 2

D. 6

E. does not exist

11. The Cartesian coordinates of the point with polar coordinates $(-2, \pi/3)$ are:

A. $(1, \sqrt{3})$

B. $(-1, -\sqrt{3})$

C. $(-\sqrt{3}, 1)$

D. $(1, -\sqrt{3})$

E. $(\sqrt{3}, -1)$

12. Which of the following are polar coordinates of the point with Cartesian coordinates $(-1, 1)$?

A. $(\sqrt{2}, \pi/4)$

B. $(\sqrt{2}, -\pi/4)$

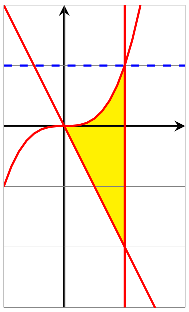
C. $(-\sqrt{2}, -\pi/4)$

D. $(\sqrt{2}/2, \pi/4)$

E. $(-\sqrt{2}, 3\pi/4)$

Free Response Questions

13. Let R be the shaded region bounded by the graph of $y = x^3$, $y = -2x$, and the vertical line $x = 1$, as shown in the figure below.



(a) Write, but do not evaluate the integral needed to find the volume of the solid generated when R is rotated about the line $y = 1$.

Solution:

$$V = \int_0^1 \pi \left[(1 - (-2x))^2 - (1 - x^3)^2 \right] \, dx = \frac{155\pi}{42} \approx 11.594.$$

(b) Write, but do not evaluate, the integral needed to find the volume of the solid generated when R is rotated about the y-axis.

Solution:

$$V = \int_0^1 2\pi x (x^3 + 2x) \, dx = \frac{26\pi}{15} \approx 5.4454$$

(c) Write, but do not evaluate, the integral that gives the length of the curve $y = x^3$ on $[0, 1]$.

Solution:

$$P = \int_0^1 \sqrt{1 + 9x^4} \, dx.$$

14. Find the most general solution to the following differential equation

$$(x^2+1)\frac{dy}{dx} = xy$$

Solution:

$$\begin{aligned}
(x^2+1)\frac{dy}{dx} &= xy \\
\frac{dy}{y} &= \frac{x}{x^2+1}\,dx \\
\int \frac{dy}{y} &= \int \frac{x}{x^2+1}\,dx \\
\ln|y| &= \frac{1}{2}\ln(x^2+1) + C \\
|y| &= A\sqrt{x^2+1} \\
y &= \pm A\sqrt{x^2+1}
\end{aligned}$$

15. (a) Show that every member of the family of functions $y = (\ln x + C)/x$ is a solution of the differential equation $x^2y' + xy = 1$.

Solution:

$$x^2y' + xy = x^2\left(\frac{1-(\ln x + C)}{x^2}\right) + \ln x + C = 1$$

(b) Find a solution of the differential equation that satisfies the initial condition $y(1) = 2$.

Solution: $y(1) = C = 2$, so the solution is

$$y = \frac{\ln x + 2}{x}.$$

16. (a) Find the average value f_{ave} of the function $f(x) = 3x^2 - 12x + 9$ over the interval $[1, 3]$.

Solution:

$$f_{\text{ave}} = \frac{1}{3-1}\int_1^3 3x^2 - 12x + 9\,dx = -2.$$

(b) Find all values c in $[1, 3]$ such that $f(c) = f_{\text{ave}}$.

Solution: Solve $f(c) = -2$.

$$3x^2 - 12x + 9 = -2$$
$$3x^2 - 12x + 11 = 0$$
$$x = \frac{12 \pm \sqrt{12}}{6} = 2 \pm \frac{\sqrt{3}}{3}$$

17. Use the ratio test to determine whether the series $\sum_{n=0}^{\infty} \frac{n^{10}}{10^n}$ converges or diverges.

Solution:

$$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim_{n\to\infty} \frac{(n+1)^{10}}{10^{n+1}}]frac10^n n^{10} = \lim_{n} \to \infty \frac{1}{10}\frac{(n+1)^{10}}{n^{10}} = \frac{1}{10} < 1$$

so the series converges.

18. The curve C is given parametrically by $x = t^2 + 4t - 7$ and $y = \frac{1}{2}t^2 + 2t + 9$ for $2 \le t \le 6$.

(a) Set up an integral for the length of the curve.

Solution:

$$\frac{dx}{dt} = 2t + 4 \qquad \frac{dy}{dt} = t + 2.$$

So, the arc length is

$$L = \int_2^6 \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt = \int_2^6 \sqrt{(2t+4)^2 + (t+2)^2}\, dt$$

(b) Solve the integral that you found in part (a) to find the exact length.

Solution:

$$\begin{aligned} L &= \int_2^6 \sqrt{(2t+4)^2 + (t+2)^2}\, dt \\ &= \int_2^6 \sqrt{5}(t+2)\, dt \\ &= \sqrt{5}\left[\frac{1}{2}t^2 + 2t\right|_2^6 \\ &= 24\sqrt{5} \end{aligned}$$