

Name: _____ Section: _____

Do not remove this answer page — you will return the whole exam. You will be allowed two hours to complete this test. No books or notes may be used. You may use a graphing calculator during the exam, but NO calculator with a Computer Algebra System (CAS) or a QWERTY keyboard is permitted. Absolutely no cell phone use during the exam is allowed.

The exam consists of 10 multiple choice questions and 5 free response questions. Record your answers to the multiple choice questions on this page by filling in the circle corresponding to the correct answer.

Show all work to receive full credit on the free response problems. You should also show your work on the multiple choice questions as it will make it easier for you to check your work. You should give exact answers, rather than a decimal approximation unless the problem asks for a decimal answer. Thus, if the answer is 2π , you should not give a decimal approximation such as 6.283 as your final answer.

Multiple Choice Questions

1 A B C D E**2** A B C D E**3** A B C D E**4** A B C D E**5** A B C D E**6** A B C D E**7** A B C D E**8** A B C D E**9** A B C D E**10** A B C D E

Multiple Choice	11	12	13	14	15	Total Score
50	10	10	10	10	10	100

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Multiple Choice Questions

1. (5 points) Select the integral which is equal to

$$\int x^2 f'(x) dx$$

- A. $x^2 f(x) - 2 \int x f(x) dx$
B. $x^2 f(x) + 2 \int x f(x) dx$
C. $\frac{x^3}{3} f(x) - 2 \int x f(x) dx$
D. $\frac{x^3}{3} f(x) - 2 \int x f'(x) dx$
E. $\frac{x^3}{3} f(x) + 2 \int x f'(x) dx$

2. (5 points) Four of the options below might appear in the partial fractions decomposition of

$$f(x) = \frac{x^3 + 1}{(x^2 - 4)(x^4 + 4x^2 + 4)(x - 2)}$$

Select the term that does *NOT* appear in the partial fractions decomposition of f .

- A. $\frac{A}{(x + 2)}$
B. $\frac{B}{(x - 2)}$
C. $\frac{C}{(x + 2)^2}$
D. $\frac{D_1 x + D_2}{x^2 + 2}$
E. $\frac{E_1 x + E_2}{(x^2 + 2)^2}$

3. (5 points) Find the sum of the series $\sum_{n=2}^{\infty} 4^{-n}$.

- A. $1/3$
- B. $4/3$
- C. $1/4$
- D. $1/8$
- E. $1/12$

4. (5 points) Choose the correct statement.

- A. The series $\sum_{n=1}^{\infty} \frac{1}{n}$ is conditionally convergent.
- B. The series $\sum_{n=1}^{\infty} \frac{1}{n}$ is absolutely convergent.
- C. The series $\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$ is absolutely convergent.
- D. The series $\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$ is conditionally convergent.
- E. The series $\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$ is divergent.

5. (5 points) Consider the curve C with parametric equations $x(t) = t^2$ and $y(t) = g(t)$ with $1 \leq t \leq 4$. The curve C is rotated about the x -axis. Write down an integral that gives the surface area of the resulting surface of revolution.

- A. $2\pi \int_1^4 t^2 \sqrt{4t^2 + (g'(t))^2} dt.$
- B. $2\pi \int_1^{16} g(t) \sqrt{4t^2 + (g'(t))^2} dt.$
- C. $2\pi \int_1^4 g(t) \sqrt{4t^2 + (g'(t))^2} dt.$
- D. $2\pi \int_1^4 g(t) \sqrt{t^4 + g(t)^2} dt.$
- E. $2\pi \int_1^{16} t^2 \sqrt{4t^2 + (g'(t))^2} dt.$

6. (5 points) Consider the curve with parametric equations $x(t) = t^2 - 2$ and $y(t) = t^3 - 3t$. Find the slope of the line that is tangent to the curve at the point $(2, -2)$.

- A. $3/2$
- B. $-3/2$
- C. -9
- D. $-9/2$
- E. $9/2$

7. (5 points) Find the directrix of the parabola with equation $x^2 + 4x = -4y$.

- A. $y = -2$
- B. $y = -1$
- C. $y = 0$
- D. $y = 1$
- E. $y = 2$

8. (5 points) For which values of r does the function $y(x) = e^{rx}$ satisfy the differential equation $y'' - 2y' - 3y = 0$.

- A. $r = 2$ or 3
- B. $r = -2$ or -3
- C. $r = -2$ or 3
- D. $r = 1$ or -3
- E. $r = -1$ or 3

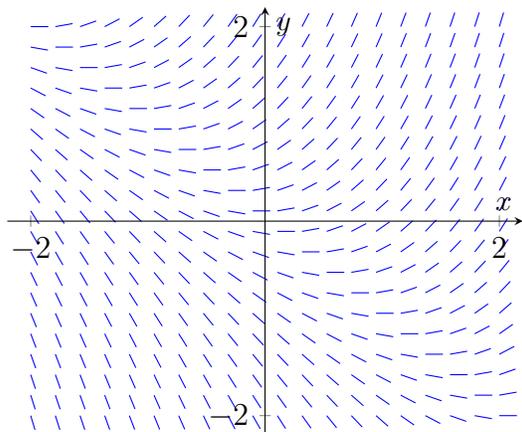
9. (5 points) We use Euler's method with step size 0.2 to approximate the solution to

$$y' = 5y \quad y(0) = 1.$$

Find the resulting value for $y(0.6)$ and give the exact value.

- A. 4.4
- B. 8
- C. 6
- D. 4
- E. 6.2

10. (5 points) Which of the following equations has the direction field shown?



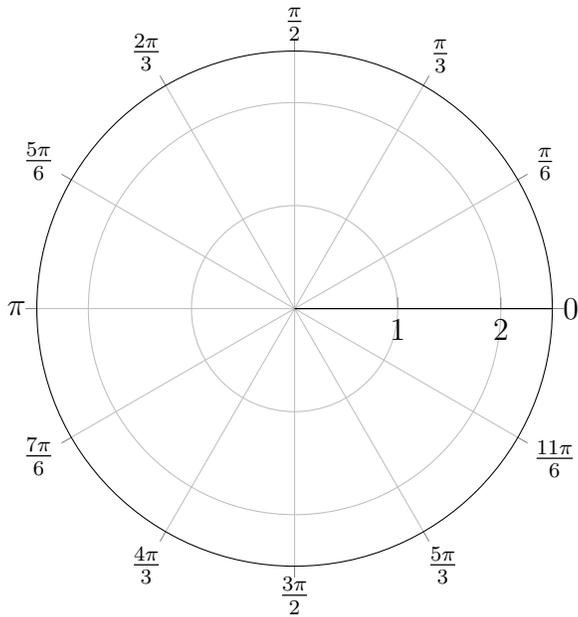
- A. $y' = y$
- B. $y' = x$
- C. $y' = -x - y$
- D. $y' = x + y$
- E. $y' = x - y$

Free Response Questions

11. Let R be the region enclosed by the line $y = 1$ and the parabola $y = (x - 2)^2$. We form a solid of revolution S by rotating the region R about the y -axis.
- (a) (6 points) Write an integral giving the volume of the solid S .
 - (b) (4 points) Evaluate the integral to find the volume of S .

12. (a) (3 points) Find a power series centered at 0 for the function $f(t) = \frac{1}{1+t^4}$. Give the interval of convergence of the series you found.
- (b) (3 points) Find a power series centered at 0 for the function $F(x) = \int_0^x \frac{1}{1+t^4} dt$.
- (c) (3 points) Substitute $x = 1/2$ into your answer in part b) and determine how many terms of the series are needed to approximate $F(1/2)$ with an error of at most 0.001. Hint: You may use the error estimate for the alternating series test.
- (d) (1 point) Use your answer to part c) to compute an approximate value for $F(1/2)$ and give the value correctly rounded to three decimal places.

13. Consider the curve defined by the polar equation $r = 2 \cos(3\theta)$.
- (a) (3 points) Sketch the curve on the polar axes provided.
- (b) (7 points) Write down an integral which gives the area enclosed by one loop of the curve. Evaluate the integral to find the area enclosed by one loop of the curve.

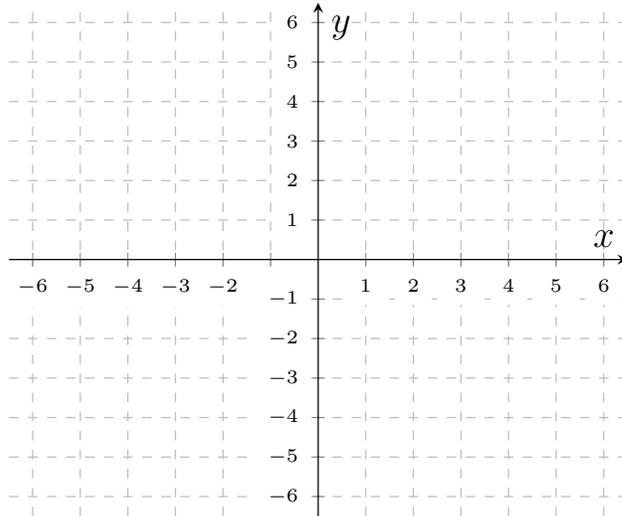


14. Consider the ellipse with equation $9x^2 + 18x + 4y^2 - 16y = 11$.

(a) (3 points) Complete the square to put in the form $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$.

(b) (4 points) Give the endpoints of the major and minor axes of the ellipse and the foci of the ellipse.

(c) (3 points) Use the information you found to make a sketch of the ellipse.



15. (10 points) Find the solution to the initial value problem

$$y' = 3yx^2, \quad y(0) = 2.$$