Exam 4

Name: ______________________________ Section: ______________

Do not remove this answer page — you will return the whole exam. You will be allowed two hours to complete this test. You are allowed to use notes on a single piece of 8.5" x 11" paper, front and back, including formulas and theorems. **You are required to turn this page in with your exam.** You may use a graphing calculator during the exam, but NO calculator with a Computer Algebra System (CAS). Absolutely no communication device use during the exam is allowed.

The exam consists of 10 multiple choice questions and 5 free response questions. Record your answers to the multiple choice questions on this page by filling in the circle corresponding to the correct answer.

Show all work to receive full credit on the free response problems. It will also help you check your answers to show work on multiple choice problems.

Multiple Choice Questions

1 (A) (B) (C) (D) (E) **6** (A) (B) (C) (D) (E)

2 (A) (B) (C) (D) (E) **7** (A) (B) (C) (D) (E)

3 (A) (B) (C) (D) (E) **8** (A) (B) (C) (D) (E)

4 (A) (B) (C) (D) (E) **9** (A) (B) (C) (D) (E)

5 (A) (B) (C) (D) (E) **10** (A) (B) (C) (D) (E)

Multiple Choice	11	12	13	14	15	Total Score
50	10	10	10	10	10	100

Multiple Choice Questions

1. (5 points) If $f(0) = 1, f(3) = 2, f'(0) = 8$ and $f'(3) = 5$ and $f''(x)$ is continuous, what is

$$\int_0^3 (x+1)f''(x)\, dx?$$

A. 11

B. 60

C. 18

D. 80

E. 16

2. (5 points) What is the best form of the partial fraction decomposition of

$$\frac{13}{(x+1)(x^2-9)}?$$

A. $\frac{A}{x+1} + \frac{Bx+C}{x^2-9}$

B. $\frac{A}{x+1} + \frac{B}{x-3} + \frac{C}{(x-3)^2}$

C. $\frac{A}{x+1} + \frac{B}{x^2-9}$

D. $\frac{A}{x+1} + \frac{B}{x+3} + \frac{C}{x-3}$

E. $\frac{Ax+B}{x+1} + \frac{Cx+D}{x^2-9}$

3. (5 points) Use the **midpoint rule** with $n = 4$ intervals to approximate $\int_1^9 \sqrt{1+x^5}\,dx$.

A. $\frac{2}{3}(\sqrt{1+(1)^5} + 4\sqrt{1+(3)^5} + 2\sqrt{1+(5)^5} + 4\sqrt{1+(7)^5} + \sqrt{1+(9)^5})$

B. $2(\sqrt{1+(2)^5} + \sqrt{1+(4)^5} + \sqrt{1+(6)^5} + \sqrt{1+(8)^5})$

C. $\sqrt{1+(1)^5} + 2\sqrt{1+(3)^5} + 2\sqrt{1+(5)^5} + 2\sqrt{1+(7)^5} + \sqrt{1+(9)^5}$

D. $2(\sqrt{1+(1)^5} + \sqrt{1+(3)^5} + \sqrt{1+(5)^5} + \sqrt{1+(7)^5})$

E. $2(\sqrt{1+(3)^5} + \sqrt{1+(5)^5} + \sqrt{1+(7)^5} + \sqrt{1+(9)^5})$

4. (5 points) Find the sum of the series $\sum_{n=1}^{\infty} \frac{2^n + 3^n}{5^n}$.

A. $\frac{25}{6}$

B. 1

C. $\frac{13}{6}$

D. $\frac{32}{15}$

E. This series diverges.

5. (5 points) Which of the following series converge?

   A. $\displaystyle\sum_{n=1}^{\infty} \frac{n^5}{n!}$

   B. $\displaystyle\sum_{n=2}^{\infty} \frac{\sqrt{n^6}}{n^4 - 1}$

   C. $\displaystyle\sum_{n=0}^{\infty} \cos(n\pi)$

   D. all of the given series converge

   E. none of the given series converge

6. (5 points) What is the radius of convergence for the power series

$$\sum_{n=1}^{\infty} \frac{(-1)^n (x-1)^n}{n^2 3^n}?$$

   A. 3

   B. ∞

   C. 0

   D. 1

   E. $\dfrac{1}{3}$

7. (5 points) The line $y = 4x + 1$ for $1 \leq x \leq 3$ is rotated about the x-axis. What is the area of the resulting surface?

A. 72π

B. $\sqrt{17}\pi$

C. $3\sqrt{17}\pi$

D. $18\sqrt{17}\pi$

E. $36\sqrt{17}\pi$

8. (5 points) The Cartesian coordinates of a point are $(-1, 1)$. Find polar coordinates (r, θ) of the point, where $r > 0$, and $0 \leq \theta < 2\pi$.

A. $r = \sqrt{2}, \theta = \dfrac{5\pi}{4}$

B. $r = \dfrac{1}{\sqrt{2}},\ \theta = \dfrac{\pi}{4}$

C. $r = -\sqrt{2},\ \theta = \dfrac{3\pi}{4}$

D. $r = \sqrt{2},\ \theta = \dfrac{3\pi}{4}$

E. $r = 1,\ \theta = -\dfrac{\pi}{4}$

9. (5 points) Which of the following integrals computes the **arc length** of the parametric curve $x(t) = 3 + \cos t$, $y(t) = \ln t$, $4 \leq t \leq 9$?

A. $\displaystyle\int_4^9 \sqrt{1 - \left(\frac{1}{t \sin t}\right)^2}\, dt$

B. $\displaystyle\int_4^9 \sqrt{\sin^2 t + \frac{1}{t^2}}\, dt$

C. $\displaystyle\int_4^9 \sqrt{(3 + \cos t)^2 + (\ln t)^2}\, dt$

D. $\displaystyle\int_4^9 \sin t + \frac{1}{t}\, dt$

E. $\displaystyle\int_4^9 2\pi \ln t \sqrt{1 + \sin^2 t}\, dt$

10. (5 points) Find the equation of the parabola which has vertex $(4, 1)$ and focus $(6, 1)$.

A. $x = 8(y - 1)^2 + 4$

B. $y = \frac{1}{8}(x - 4)^2 + 1$

C. $x = \frac{1}{8}(y - 1)^2 + 4$

D. $y = 8(x - 4)^2 + 1$

E. $x = \frac{1}{6}(y - 1)^2 - 4$

Free Response Questions

11. (a) (5 points) Evaluate $\displaystyle\int \frac{dx}{\sqrt{25-x^2}}$ using trigonometric substitution. Show all steps clearly.

Solution: Let $x = 5\sin\theta$, so $dx = 5\cos\theta\ d\theta$. The integral becomes

$$\int \frac{5\cos\theta}{\sqrt{25-25\sin^2\theta}}\ d\theta = \int \frac{5\cos\theta}{5\cos\theta}\ d\theta = \int d\theta = \theta + C$$

Since $\sin\theta = \frac{x}{5}$, $\theta = \arcsin(\frac{x}{5})$ and the integral is $\arcsin(\frac{x}{5}) + C$.

(b) (5 points) Evaluate $\displaystyle\int \sin^3(x)\ dx$. Show all steps clearly.

Solution:

$$\int \sin^3 x = \int \sin^2 x \sin x\ dx = \int (1-\cos^2 x)\sin x\ dx$$

Let $u = \cos x$, $du = -\sin x\ dx$, so that our integral becomes

$$-\int (1-u^2)\ du = -u + \frac{u^3}{3} + C = -\cos x + \frac{\cos^3 x}{3} + C$$

12. (a) (4 points) Write the Maclaurin series (i.e, the Taylor series centered at $x = 0$) for the function

$$f(x) = \frac{2x^5}{1 - x^6}.$$

Solution:

$$f(x) = 2x^5 \cdot \frac{1}{1 - x^6} = 2x^5 \cdot \sum_{n=0}^{\infty} (x^6)^n = \sum_{n=0}^{\infty} 2x^{6n+5}$$

(b) (6 points) Does the series $\sum_{n=6}^{\infty} \frac{(-1)^n}{n-4}$ converge absolutely, converge conditionally, or diverge? Show all steps to justify your answer, and clearly list which test(s) you use.

Solution: The series

$$\sum_{n=6}^{\infty} \left| \frac{(-1)^n}{n-4} \right| = \sum_{n=6}^{\infty} \frac{1}{n-4}$$

diverges by the comparison test since $\frac{1}{n-4} > \frac{1}{n}$ and $\sum_{n=6}^{\infty} \frac{1}{n-4}$ diverges. The series $\sum_{n=6}^{\infty} \frac{(-1)^n}{n-4}$ converges by the alternating series test since $\lim_{n \to \infty} \frac{1}{n-4} = 0$ and $\frac{1}{n-4}$ is decreasing. Therefore, $\sum_{n=6}^{\infty} \frac{(-1)^n}{n-4}$ converges conditionally.

13. Let S be the solid obtained by rotating the region in the first quadrant bounded by $y = x^3$ and $y = 9x$ about the $\boldsymbol{y}$**−axis**.

(a) (5 points) Set up the integral that computes the volume of S using the **disk/washer** method.

Solution: The curves intersect at $(0, 0)$ and $(3, 27)$. Therefore,

$$V = \int_0^{27} \pi \left((\sqrt[3]{y})^2 - (\tfrac{1}{9}y)^2 \right) \, dy$$

(b) (5 points) Set up the integral that computes the volume of S using the **cylindrical shells** method.

Solution:

$$V = \int_0^{3} 2\pi x(9x - x^3) \, dx$$

14. Consider the polar curve C defined by $r = \sqrt{6\sin\theta}$.

(a) (5 points) Set up an integral which computes the area bounded by C which lies in the sector $0 \le \theta \le \pi$, and then **evaluate** your integral to find the area.

Solution:

$$A = \int_0^\pi \frac{1}{2} r^2 \; d\theta = \int_0^\pi \frac{1}{2}(6\sin\theta) \; d\theta = -3\cos\theta \Big|_0^\pi = 6$$

(b) (5 points) Set up but do **not** evaluate an integral which computes the length of C for $0 \le \theta \le \pi$. You do not need to simplify.

Solution:

$$L = \int_0^\pi \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2} \; d\theta = \int_0^\pi \sqrt{6\sin\theta + \left(\tfrac{1}{2}(6\sin\theta)^{-\frac{1}{2}}(6\cos\theta)\right)^2} \; d\theta$$

15. (a) (5 points) Find the vertices and foci of the hyperbola defined by the equation

$$\frac{(x-3)^2}{1} - \frac{(y+2)^2}{9} = 1$$

Solution: The hyperbola is horizontal with center $(3, -2)$ and $a = 1$ so the vertices are $(4, -2)$ and $(2, -2)$. The foci are found using

$$c^2 = a^2 + b^2 = 1 + 9 = 10,$$

so $c = \pm\sqrt{10}$. Therefore the foci are $(3 + \sqrt{10}, -2)$, $(3 - \sqrt{10}, -2)$.

(b) (5 points) Find the vertices of the ellipse defined by the equation

$$16x^2 + y^2 - 6y = 7.$$

Solution: We proceed by completing the square.

$$16x^2 + y^2 - 6y + 9 - 9 = 7$$

$$16x^2 + (y-3)^2 = 16$$

$$x^2 + \frac{(y-3)^2}{16} = 1$$

The ellipse has center $(0, 3)$ with $a = 1$ and $b = 4$, so the vertices are $(0, -1)$, $(0, 7)$, $(1, 3)$ and $(-1, 3)$