Exam 4

Name: _____   Section: _____

Do not remove this answer page — you will return the whole exam. You will be allowed two hours to complete this test. If you find you need scratch paper during the exam, please ask. You may not use any of your own notes, paper or anything else not provided. You may use a graphing calculator during the exam, but NO calculator with a Computer Algebra System (CAS). Absolutely no communication device use during the exam is allowed.

The exam consists of 10 multiple choice questions and 5 free response questions. Record your answers to the multiple choice questions on this page by filling in the circle corresponding to the correct answer.

Show <u>all work</u> to receive full credit on the free response problems. It will also help you check your answers to show work on multiple choice problems.

Multiple Choice Questions

1 (A) (B) (C) (D) (E)   **6** (A) (B) (C) (D) (E)

2 (A) (B) (C) (D) (E)   **7** (A) (B) (C) (D) (E)

3 (A) (B) (C) (D) (E)   **8** (A) (B) (C) (D) (E)

4 (A) (B) (C) (D) (E)   **9** (A) (B) (C) (D) (E)

5 (A) (B) (C) (D) (E)   **10** (A) (B) (C) (D) (E)

Multiple Choice	11	12	13	14	15	Total Score
50	10	10	10	10	10	100

Multiple Choice Questions

1. (5 points) Find $\int (x+5)e^x \ dx$.

    A. $(x+4)e^x + C$

    B. $(x+5)e^x + C$

    C. $(x+6)e^x + C$

    D. $\left(\frac{1}{2}x^2 + 5x\right)e^x + C$

    E. $\left(\frac{1}{2}x^2 + 5x\right)\left(\frac{1}{2}e^{x^2}\right) + C$

2. (5 points) Find $\int \sin^2(5x) \ dx$.

    A. $\frac{1}{15}\cos^3(5x) + C$

    B. $\frac{1}{2}x - \frac{1}{20}\sin(10x) + C$

    C. $\frac{5}{2} - \frac{5}{2}\cos(2x) + C$

    D. $\frac{1}{3}\sin^2(5x) + C$

    E. $x - \frac{1}{3}\cos^3(5x) + C$

3. (5 points) Which of the following is equal to the integral $\displaystyle\int \frac{dx}{\sqrt{4-x^2}}$ after making the substitution $x = 2\sin\theta$?

A. $\displaystyle\int 2\cos\theta \, d\theta$

B. $\displaystyle\int \frac{1}{2}\sec\theta \, d\theta$

C. $\displaystyle\int \cot\theta \, d\theta$

D. $\displaystyle\int d\theta$

E. $\displaystyle\int \frac{\cos\theta}{1-\sin\theta} \, d\theta$

4. (5 points) For the graph of $y = f(x)$ shown, let I be the value of $\displaystyle\int_0^5 f(x)\,dx$, and let L_n, R_n, M_n and T_n be the approximations using left, right, midpoint, and trapezoidal integration. List these in order from smallest to largest.
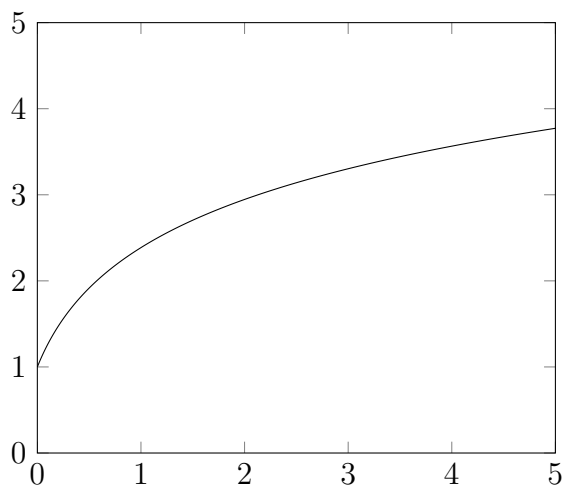
A. L_n, T_n, I, M_n, R_n

B. L_n, M_n, I, T_n, R_n

C. R_n, T_n, I, M_n, L_n

D. R_n, M_n, I, T_n, L_n

E. L_n, M_n, T_n, R_n, I

5. (5 points) Find the sum of the geometric series $\displaystyle\sum_{n=2}^{\infty} 4\left(\frac{1}{3}\right)^n$.

     A. $\dfrac{8}{9}$

     B. $\dfrac{16}{27}$

     C. $\dfrac{3}{2}$

     D. $\dfrac{2}{3}$

     E. $\dfrac{16}{9}$

6. (5 points) Find the interval of convergence of the power series $\displaystyle\sum_{n=1}^{\infty} \frac{(x+2)^n}{7^n \cdot n^2}$.

     A. $[-2, 2]$

     B. $[-7, 7]$

     C. $[-9, 5]$

     D. $[-\frac{15}{7}, -\frac{13}{7}]$

     E. $(-\infty, \infty)$

7. (5 points) Find the Taylor series centered at $x = 0$ (i.e., the Maclaurin series) for the function $f(x) = \dfrac{x^3}{1 - x^5}$.

     A. $\displaystyle\sum_{n=0}^{\infty} \frac{x^3}{x^{5n}}$

     B. $\displaystyle\sum_{n=0}^{\infty} x^{3n-5}$

     C. $\displaystyle\sum_{n=0}^{\infty} x^{5n+3}$

     D. $\displaystyle\sum_{n=0}^{\infty} (-1)^n x^{3n-5}$

     E. $\displaystyle\sum_{n=0}^{\infty} (-1)^n x^{5n+3}$

8. (5 points) Consider the curve C parametrized by $x(t) = 2t^2 + 5t$ and $y(t) = t^3 + 3$. Find the slope of the tangent line to C at the point $(x, y) = (7, 4)$.

    A. $\dfrac{3}{4}$

    B. $\dfrac{3}{5}$

    C. $\dfrac{3}{2}$

    D. $\dfrac{1}{3}$

    E. $\dfrac{4}{7}$

9. (5 points) Find the distance between the points with polar coordinates $(3, \frac{\pi}{2})$ and $(4, \pi)$.

    A. 5

    B. 1

    C. 7

    D. $1 + \frac{\pi}{2}$

    E. 1.862

10. (5 points) Find the **foci** of the ellipse with equation $9x^2 + y^2 - 10y + 16 = 0$.

    A. $(1, 5)$ and $(0, -8)$

    B. $(0, 5 + \sqrt{8})$ and $(0, 5 - \sqrt{8})$

    C. $(\sqrt{8}, 5)$ and $(-\sqrt{8}, 5)$

    D. $(\sqrt{6}, 5)$ and $(-\sqrt{6}, 5)$

    E. $(0, 5 + \sqrt{6})$ and $(0, 5 - \sqrt{6})$

Free Response Questions

11.   (a)   (5 points) Evaluate $\displaystyle\int \sin^4(x)\cos^3(x)\,dx$.

(b)   (5 points) Evaluate using partial fraction decomposition $\displaystyle\int \frac{6x+17}{(x+2)(x+3)}$.

12. Are the series below absolutely convergent, conditionally convergent, or divergent? Clearly state which test(s) are used, and show all steps.

(a) (6 points) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt[3]{n}}$

(b) (4 points) $\displaystyle\sum_{n=2}^{\infty} (-1)^n \frac{5n+1}{7n-2}$

13. The region R is in the first quadrant bounded by $y = 8 - x^3$, the x-axis and the y-axis. Let V be the solid obtained by rotating R about the vertical line $x = -3$.
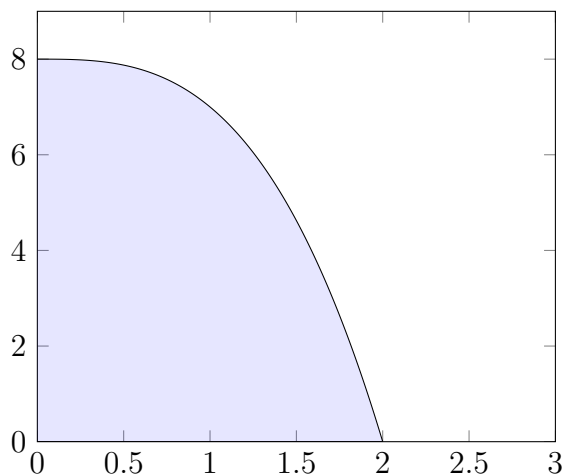
(a) (6 points) Set up but do not evaluate the integral that computes the volume of V using the **disk/washer** method.

(b) (4 points) Set up but do not evaluate the integral that computes the volume of V using the **shell** method.

14. (a) (6 points) Set up but do not evaluate an integral which computes the area of one leaf of the polar curve $r = \sin(2\theta)$. Show work to justify the bounds of integration.
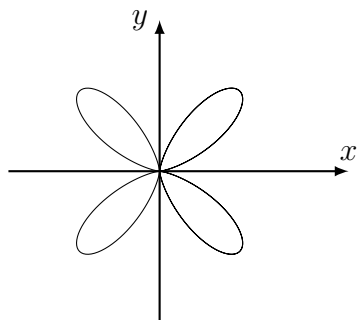
(b) (4 points) Set up but do not evaluate an integral which computes the length of the polar curve $r = \sin(2\theta)$ for $\frac{\pi}{6} \leq \theta \leq \frac{\pi}{3}$.

15. (a) (5 points) Find the center, vertices and foci of the hyperbola

$$\frac{(y+2)^2}{4} - \frac{(x-1)^2}{9} = 1.$$

Center: _____ Vertices: _____ Foci: _____

(b) (5 points) Find the vertex, focus and directrix of the parabola

$$y = \frac{1}{12}(x-5)^2 + 7.$$

Vertex: _____ Focus: _____ Directrix: _____