Answer all questions and show your work. Unsupported answers may receive *no credit*. You may not use a calculator on this quiz. Allow 15 minutes for the quiz.

Name: __________ Section: __________

Remember the trig identities: $\sin^2 + \cos^2 = 1$, $\sin^2 x = \frac{1}{2}(1 - \cos(2x))$ and $\cos^2 x = \frac{1}{2}(1 + \cos(2x))$

1. Find the following integrals.

   (a) (5 points) $\int (\sin^2(x) - \sin^3(x))dx$

Solution:

$$\begin{aligned}\int (\sin^2(x) - \sin^3(x))dx &= \int \left(\frac{1}{2}(1 - \cos(2x)) - \sin(x)(1 - \cos^2 x)\right) dx \\ &= \frac{t}{2} - \frac{\sin(2x)}{4} - \int \sin x dx + \int \sin x \cos^2 x dx \\ &= \frac{t}{2} - \frac{\sin(2x)}{4} + \cos x - \frac{\cos^3 x}{3}\end{aligned}$$

   (b) (5 points) $\int \frac{1}{x^2\sqrt{x^2 - 4}} dx.$

Solution: $\sin^2 t + \cos^2 t = 1$ so $\tan^2 t + 1 = \sec^2 t$ Take $x = 2\sec t$. Then $dx = 2\sec t \tan t dt$ and

$$\begin{aligned}\int \frac{1}{x^2\sqrt{x^2 - 4}} dx &= \int \frac{2\sec t \tan t dt}{4\sec^2 t\sqrt{4\sec^2 t - 4}} = \int \frac{2\sec t \tan t dt}{4\sec^2 t * 2\tan t} \\ &= \int \frac{dt}{4\sec t} = \int \frac{1}{4}\cos t dt = \frac{1}{4}\sin t + C\end{aligned}$$

Since $x = 2\sec t$, $\cos t = \frac{2}{x}$ and $\sin t = \frac{\sqrt{x^2-4}}{x}$. So

$$\int \frac{1}{x^2\sqrt{x^2 - 4}} dx = \frac{1}{4}\frac{\sqrt{x^2 - 4}}{x} + C$$