Solutions to Selected Quiz Questions

Quiz 2, Question 1. Use the substitution $x = 6\tan(\theta)$ to evaluate the indefinite integral

$$\int \frac{79\,dx}{x^2\sqrt{x^2+36}}.$$

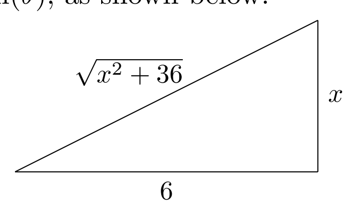
Solution: We take $x = 6\tan(\theta)$ with $-\pi/2 < \theta < \pi/2$. Then $\sqrt{x^2+36} = 6\sec(\theta)$ and $dx = 6\sec^2(\theta)\,d\theta$. Thus

$$\begin{aligned}\int \frac{79\,dx}{x^2\sqrt{x^2+36}} &= \int \frac{79\,[6\sec^2(\theta)\,d\theta]}{[36\tan^2(\theta)][6\sec(\theta)]}\\ &= \frac{79}{36}\int \frac{\sec(\theta)\,d\theta}{\tan^2(\theta)}\\ &= \frac{79}{36}\int \frac{\cos(\theta)\,d\theta}{\sin^2(\theta)}.\end{aligned}$$

Using the substitution $u = \sin(\theta)$, this reduces to

$$\begin{aligned}\frac{79}{36}\int \frac{du}{u} &= -\frac{79}{36}\cdot\frac{1}{u} + C\\ &= -\frac{79}{36}\cdot\frac{1}{\sin(\theta)} + C\\ &= -\frac{79\csc(\theta)}{36} + C.\end{aligned}$$

It now remains to express $\csc(\theta)$ in terms of $x = 6\tan(\theta)$. Consider a right triangle with legs x and 6, so that $x/6 = \tan(\theta)$, as shown below:



Here θ is the angle facing x. Then the hypotenuse is $\sqrt{x^2+36}$ and

$$\csc(\theta) = \frac{\sqrt{x^2+36}}{x}.$$

Finally,

$$\int \frac{79\,dx}{x^2\sqrt{x^2+36}} = -\frac{79\sqrt{x^2+36}}{36x} + C.$$