MA 123 — Elem. Calculus Fall 2012
EXAM 1 9/19/2012

Name: Solution **Sec.:** _____

Do not remove this answer page — you will turn in the entire exam. You have two hours to do this exam. No books or notes may be used. You may use a graphing calculator during the exam, but NO calculator with a Computer Algebra System (CAS) or a QWERTY keyboard is permitted. Absolutely no cell phone use during the exam is allowed.

The exam consists of multiple choice questions. Record your answers on this page. For each multiple choice question, you will need to fill in the box corresponding to the correct answer. For example, if (b) is correct, you must write

[a] [b] [c] [d] [e]

Do not circle answers on this page, but please circle the letter of each correct response in the body of the exam. It is your responsibility to make it CLEAR which response has been chosen. You will not get credit unless the correct answer has been marked on both this page and in the body of the exam.

GOOD LUCK!

1. [a] [b] [c] [d] [e]
2. [a] [b] [c] [d] [e]
3. [a] [b] [c] [d] [e]
4. [a] [b] [c] [d] [e]
5. [a] [b] [c] [d] [e]
6. [a] [b] [c] [d] [e]
7. [a] [b] [c] [d] [e]
8. [a] [b] [c] [d] [e]
9. [a] [b] [c] [d] [e]
10. [a] [b] [c] [d] [e]
11. [a] [b] [c] [d] [e]
12. [a] [b] [c] [d] [e]
13. [a] [b] [c] [d] [e]
14. [a] [b] [c] [d] [e]
15. [a] [b] [c] [d] [e]
16. [a] [b] [c] [d] [e]
17. [a] [b] [c] [d] [e]
18. [a] [b] [c] [d] [e]
19. [a] [b] [c] [d] [e]
20. [a] [b] [c] [d] [e]

For grading use:

Number Correct	**(out of 20 problems)**

Total	**(out of 100 points)**

Multiple Choice Questions

Show all your work on the page where the question appears.
Clearly mark your answer both on the cover page on this exam
and in the corresponding questions that follow.

1. If $P(x) = x^2 + 2x + 4$ and we write $P(x)$ in the form

$$P(x) = A + B(x-1) + C(x-1)(x-2) + D(x-1)(x-2)(x-3),$$

what is the value of B? (HINT: you may need to find A before you can find B.)

Possibilities:

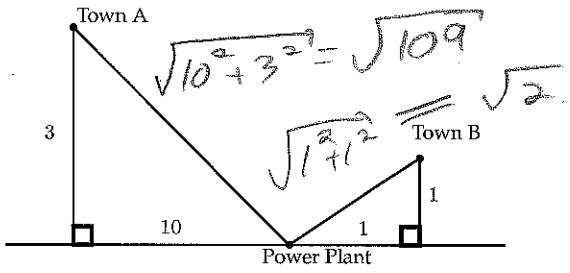
(a) 3
(b) 4
(c) 5
(d) 6
(e) 7

$P(1) = 1^2 + 2\cdot 1 + 4 = 7$
$= A + B\cdot 0 + C\cdot 0(-1) + D\cdot 0(-1)(-2)$
$= A$
So $A = 7$

$P(2) = 2^2 + 2\cdot 2 + 4 = 12$
$= 7 + B(2-1) + C(2-1)(2-2) + D(2-1)(2-2)(2-3)$

So $12 = 7 + B$
$B = 5$

2. A power plant is located on the bank of a river (the river indicated by the horizontal line). How many total miles of power line are needed to connect the power plant to both of the cities? (The figure is not necessarily drawn to scale.)



$\sqrt{10^2 + 3^2} = \sqrt{109}$
$\sqrt{1^2 + 1^2} = \sqrt{2}$

Possibilities:

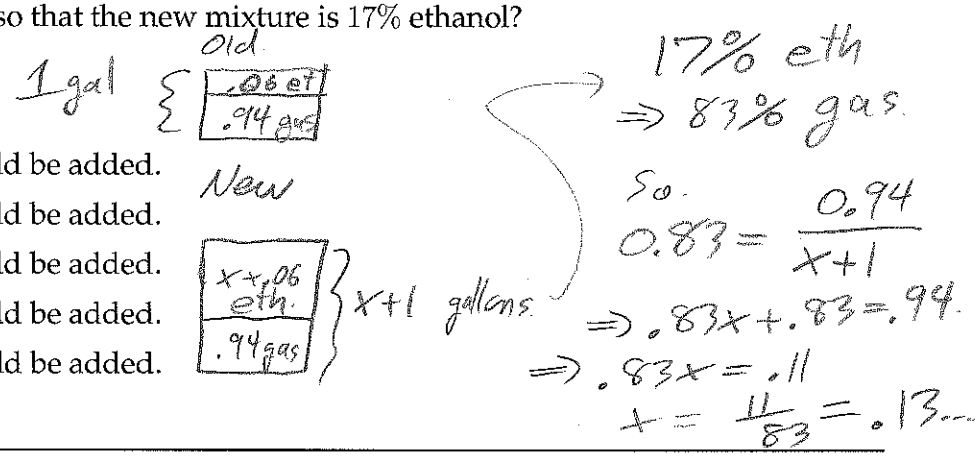
(a) $\sqrt{109} + \sqrt{2}$ miles.
(b) 15 miles.
(c) $\sqrt{111}$ miles.
(d) $\sqrt{3} + \sqrt{10} + \sqrt{1} + \sqrt{1}$ miles.
(e) 111 miles.

Total distance
$= \sqrt{109} + \sqrt{2}$

3. A fuel mixture consists of 94% gasoline and 6% ethanol. How many gallons of pure ethanol must be added to 1 gallon of the fuel so that the new mixture is 17% ethanol?

Possibilities:

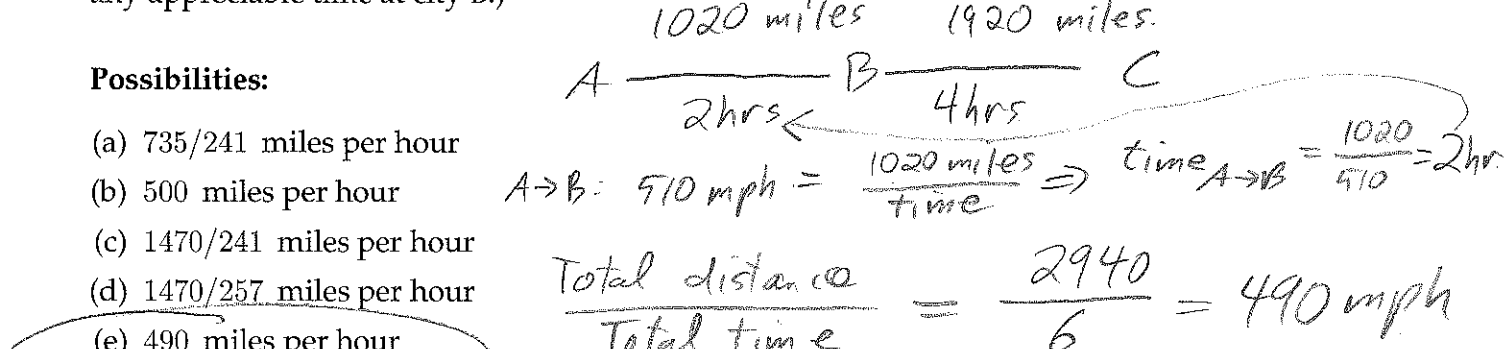
(a) 4.53 gallons of ethanol should be added.
(b) 1.13 gallons of ethanol should be added.
(c) 0.93 gallons of ethanol should be added.
(d) 0.13 gallons of ethanol should be added.
(e) 2.13 gallons of ethanol should be added.



4. A plane travels from city A to city B and then to city C. The distance from A to B is 1020 miles and the distance from B to C is 1920 miles. The average velocity from A to B was 510 miles per hour, and the time elapsed in travelling from from B to C was 4 hours. What was the average velocity from A to C (that is, for the entire trip) in miles per hour. (Assume the plane does NOT stop for any appreciable time at city B.)

Possibilities:

(a) 735/241 miles per hour
(b) 500 miles per hour
(c) 1470/241 miles per hour
(d) 1470/257 miles per hour
(e) 490 miles per hour



5. A particle moves along a straight line. The position of the particle is given by $s(t) = 3t^2 + 35$ where t is measured in seconds and $s(t)$ is measured in feet. Determine the average velocity of the particle from $t = 1$ to $t = 1 + h$.

Possibilities:

(a) $3h + 1$ feet per second
(b) $3h + 2$ feet per second
(c) $6h + 2$ feet per second
(d) $3h + 6$ feet per second
(e) $6h$ feet per second

$$\frac{s(1+h)-s(1)}{h} = \frac{[3(1+h)^2+35]-[3\cdot 1^2+35]}{h}$$

$$= \frac{3(1+2h+h^2)+35-3-35}{h}$$

$$= \frac{3+6h+3h^2+35-3-35}{h} = \frac{6h+3h^2}{h}$$

$$= \frac{(6+3h)h}{h} = 6+3h$$

6. Let $f(x) = \sqrt{x+5}$. Find the average rate of change of $f(x)$ from $x = 4$ to $x = 11$.

Possibilities:

(a) $-13/7$

(b) $-6/7$

(c) $1/7$

(d) $8/7$

(e) $15/7$

$$AROC = \frac{f(11)-f(4)}{11-4} = \frac{\sqrt{11+5}-\sqrt{4+5}}{7}$$

$$= \frac{\sqrt{16}-\sqrt{9}}{7} = \frac{4-3}{7} = \frac{1}{7}$$

---

7. Let $f(t) = t^2 + 4t + 7$. Find a value of t for which the average rate of change of $f(t)$ from 0 to t is equal to 15.

Possibilities:

(a) 11

(b) 12

(c) 13

(d) 14

(e) 15

$$AROC = \frac{f(t)-f(0)}{t-0} = \frac{[t^2+4t+7]-[0^2+4\cdot 0+7]}{t}$$

$$= \frac{t^2+4t}{t} = \frac{(t+4)t}{t} = t+4$$

Want $AROC = 15$

So $t+4 = 15$

$t = 11$

---

8. Determine the limit

$$\lim_{x\to 10}\sqrt{x-7} = \sqrt{10-7} = \sqrt{3}$$

Direct Substitution

Possibilities:

(a) $\sqrt{3}$

(b) $\sqrt{2}$

(c) $\sqrt{6}$

(d) $\sqrt{5}$

(e) The limit is infinite or the limit does not exist.

---

9. Determine the limit $\lim_{x\to 0}\left(\frac{6}{x}+\frac{8x-6}{x}\right)$

$= \lim_{x\to 0}\left[\frac{6+8x-6}{x}\right]$

$= \lim_{x\to 0}\frac{8x}{x} = 8$

Possibilities:

(a) 5

(b) 6

(c) 7

(d) 8

(e) Limit is infinite or limite does not exist

---

10. Compute $\lim_{t\to 2}\frac{t^2-t-2}{t^2-4}$ Direct sub gives 0/0, so must simplify!

Possibilities:

(a) 3/4

(b) 1

(c) 5/4

(d) 3/2

(e) The limit is infinite or the limit does not exist.

$\frac{t^2-t-2}{t^2-4} = \frac{(t-2)(t+1)}{(t-2)(t+2)} = \frac{t+1}{t+2}$

So $\lim_{t\to 2}\frac{t^2-t-2}{t^2-4} = \lim_{t\to 2}\frac{t+1}{t+2} = \frac{2+1}{2+2} = \frac{3}{4}$

---

11. Compute $\lim_{t\to 5^-}\frac{|5t|}{4t}$ $= \frac{|5\cdot 5|}{4\cdot 5} = \frac{|25|}{20} = \frac{25}{20} = \frac{5}{4}$

Possibilities:

(a) 5/4

(b) 3/2

(c) 7/4

(d) 2

(e) The limit is inifinite or the limit does not exist.

[Since denominator does not go to 0 as $t\to 5$, this limit can be computed by direct substitution]

---

12. Compute $\lim_{t \to 5^-} f(x)$, given that

Left side

$$f(x) = \begin{cases} x^2 - 7 & \text{for} \quad x < 5 \\ 5x + 4 & \text{for} \quad 5 \leq x \end{cases}$$

Possibilities:

(a) 18

(b) 29

(c) 4

(d) 5

(e) The limit is infinite or the limit does not exist.

$\lim_{x \to 5^-} f(x) = \lim_{x \to 5^-} x^2 - 7$

$= 5^2 - 7 = 25 - 7$

$= 18$

---

13. Determine the value of A which makes the function $F(x)$ continuous for all values of x, given that

$$F(x) = \begin{cases} Ax^2 + 8x + 2 & \text{for} \quad x \leq 1 \\ 3x - A & \text{for} \quad 1 < x \end{cases}$$

Possibilities:

(a) $-5/2$

(b) -4

(c) $-7/2$

(d) -3

(e) No such value of A exists.

$\lim_{x \to 1^-} Ax^2 + 8x + 2 = \lim_{x \to 1^+} 3x - A$

$A \cdot 1^2 + 8 \cdot 1 + 2 = 3 \cdot 1 - A$

$A + 10 = 3 - A$

$\Rightarrow 2A = -7 \Rightarrow A = \frac{-7}{2}$

---

14. Consider the function $f(x) = 2x^2 - 3$. The tangent line to $f(x)$ at $x = 3$ goes through the point $(4, y_0)$. Determine y_0.

Possibilities:

(a) 27

(b) 28

(c) 29

(d) 30

(e) 31

T. Line:

slope $= f'(3) = 2 \cdot 2 \cdot 3 = 12$

Point $= (3, 2 \cdot 3^2 - 3) = (3, 15)$

T. Line is $y - 15 = 12(x - 3)$

$\Rightarrow y = 12x - 36 + 15$

$y = 12x - 21$

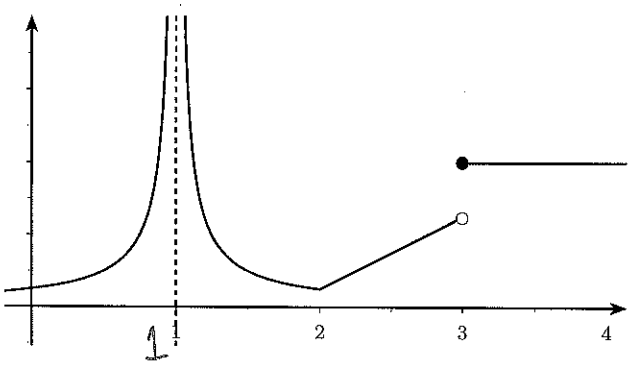
$(4, y_0)$ on T. line, so

$y_0 = 12 \cdot 4 - 21$

$= 27$

---

15. The graph of $y = f(x)$ is shown.



Which of the following statements are true?

(I) $f(x)$ is continuous but not differentiable at $x = 1$. False; Asymptote ⇒ Not Cont.

(II) $f(x)$ is differentiable but not continuous at $x = 2$. False; Corner cont but not diff.

(III) $f(x)$ is neither continuous nor differentiable at $x = 3$. TRUE ⇒ Jump ⇒ Not Cont & Not Diff

Possibilities:

(a) (II) and (III) are true

(b) (I) and (III) are true

(c) Only (II) is true

(d) Only (III) is true

(e) (I) and (II) are true

---

16. For the function $f(x) = 3x^2 - 3x + 10$, find the equation of the tangent line at $x = 2$.

Possibilities:

(a) $y = 9x - 3$

(b) $y = 9x - 2$

(c) $y = 9x - 1$

(d) $y = 9x$

(e) $y = 9x + 1$

$f(2) = 3 \cdot 2^2 - 3 \cdot 2 + 10 = 16$

slope $= f'(2) = 3 \cdot 2 \cdot 2 - 3 = 9$

T. Line

$y - 16 = 9(x - 2)$

$y - 16 = 9x - 18$

$y = 9x - 2$

17. Suppose $F(x) = 8x^2 - x - 5$. Find a value of b so that $F'(b) = 15$.

Possibilities:

(a) $1/2$

(b) 1

(c) $3/2$

(d) 2

(e) $5/2$

$F'(x) = 8 \cdot 2x - 1 = 16x - 1$

$F'(b) = 15$

$\Rightarrow \quad 16 \cdot b - 1 = 15$

$16b = 16$

$b = 1$

18. Which of the following is the correct expression for the slope of the tangent line to the graph of $g(x)$ at $x = 4$?

Possibilities:

(a) $\lim_{h \to 0} \dfrac{g(4) - g(4+h)}{h}$

(b) $\lim_{h \to 0} \dfrac{g(4+h) - g(4)}{h}$

(c) $\lim_{h \to 0} \dfrac{g(4-h) - g(4)}{h}$

(d) $\dfrac{g(4) - g(4+h)}{h}$

(e) $\dfrac{g(4+h) - g(4)}{h}$

19. The graph of $y = g(x)$ is shown (solid), as well as the tangent line to the graph (dotted) at $x = 1$. Determine $g'(1)$. (Graph is not drawn to scale.)

Possibilities:

(a) 5

(b) $-1/4$

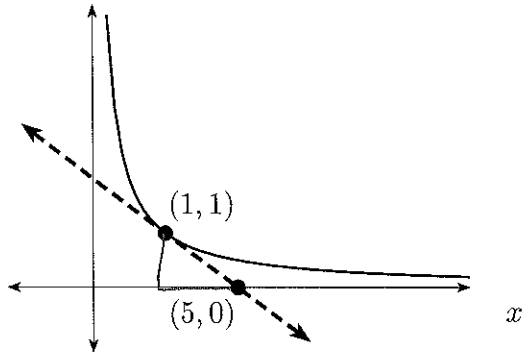
(c) 1

(d) $-1/2$

(e) $1/5$

$g'(1) =$ Slope T. Line at $x = 1$

Slope $= \dfrac{0-1}{5-1} = \dfrac{-1}{4}$



20. The tangent line to $f(x) = 4x^2 + bx + 4$ at $x = 10$ is parallel to the x-axis. Determine the value of b.

Possibilities:

(a) -83

(b) -82

(c) -81

(d) -80

(e) There is no such value of b.

Slope T. Line $= f'(10) = 2 \cdot 4 \cdot 10 + b = 80 + b$

T. Line parallel to x-axis

$\Rightarrow$ Slope T. Line is 0

So $80 + b = 0$

$\Rightarrow b = -80$