MA123 — Elem. Calculus Fall 2013
Exam 1 2013-09-26

Name: Solutions **Sec.:** _____

Do not remove this answer page — you will turn in the entire exam. You have two hours to do this exam. No books or notes may be used. You may use a graphing calculator during the exam, but NO calculator with a Computer Algebra System (CAS) or a QWERTY keyboard is permitted. Absolutely no cell phone use during the exam is allowed.

The exam consists of multiple choice questions. Record your answers on this page. For each multiple choice question, you will need to fill in the box corresponding to the correct answer. For example, if (a) is correct, you must write

[a] [b] [c] [d] [e]

Do not circle answers on this page, but please circle the letter of each correct response in the body of the exam. It is your responsibility to make it CLEAR which response has been chosen. You will not get credit unless the correct answer has been marked on both this page and in the body of the exam.

GOOD LUCK!

1.	a	**b**	c	d	e	11.	**a**	b	c	d	e
2.	**a**	b	c	d	e	12.	a	b	c	d	**e**
3.	a	**b**	c	d	e	13.	a	b	c	**d**	e
4.	**a**	b	c	d	e	14.	a	**b**	c	d	e
5.	a	b	c	**d**	e	15.	**a**	b	c	d	e
6.	a	b	**c**	d	e	16.	**a**	b	c	d	e
7.	a	b	c	**d**	e	17.	a	b	**c**	d	e
8.	a	b	c	**d**	e	18.	a	b	c	d	**e**
9.	**a**	b	c	d	e	19.	a	b	**c**	d	e
10.	a	b	**c**	d	e	20.	a	b	**c**	d	e

For grading use:

Number Correct	
	(out of 20 problems)

Total	
	(out of 100 points)

Please make sure to list the correct section number on the front page of your exam. In case you forgot your section number, consult the following table. Your section number is determined by your recitation time and location.

Section #	Instructor	Day	Time	Room
001-006	Jack Schmidt	MWF	08:00 am - 08:50 am	KAS 213
001	Jinping Zhuge	T	8:00 am - 9:15 am	FB B9
002	Yiyuan Wu	T	9:30 am - 10:45 am	NURS 501B
003	Devin Willmott	T	8:00 am - 9:15 am	CB 235
004	Tefjol Pllaha	T	8:00 am - 9:15 am	CB 237
005	Tefjol Pllaha	T	2:00 pm - 3:15 pm	CB 347
006	Tefjol Pllaha	T	3:30 pm - 4:45 pm	CB 347
007-012	Jack Schmidt	MWF	09:00 am - 09:50 am	KAS 213
007	Yiyuan Wu	R	8:00 am - 9:15 am	CB 217
008	Jinping Zhuge	R	9:30 am - 10:45 am	DH 323
009	Yiyuan Wu	R	11:00 am - 12:15 pm	EH 202
010	Jinping Zhuge	R	12:30 pm - 1:45 pm	DH 323
011	Dharma Maharjan	R	2:00 pm - 3:15 pm	CB 347
012	Dharma Maharjan	R	3:30 pm - 4:45 pm	CB 347
013-018	Paul Koester	MWF	1:00 pm - 1:50 pm	BS 116
013	Carolyn Troha	T	8:00 am - 9:15 am	CB 345
014	Carolyn Troha	T	9:30 am - 10:45 am	NURS 214
015	Morgan Schreffler	T	11:00 am - 12:15 pm	EH 202
016	Carolyn Troha	T	12:30 pm - 1:45 pm	MMRB 243
017	Morgan Schreffler	T	2:00 pm - 3:15 pm	BH 301
018	Morgan Schreffler	T	3:30 pm - 4:45 pm	CB 235
025-030	Paul Koester	MWF	2:00 pm - 2:50 pm	BS 107
025	Sarah Orchard	T	12:30 pm - 1:45 pm	TPC 212
026	Marie Meyer	R	8:00 am - 9:15 am	CB 240
027	Marie Meyer	T	2:00 pm - 3:15 pm	DH 331
028	Marie Meyer	R	2:00 pm - 3:15 pm	EH 304
029	Sarah Orchard	T	3:30 pm - 4:45 pm	OT OB7
030	Sarah Orchard	R	3:30 pm - 4:45 pm	OT 0B7
401	Brad Schwer	MTR	5:30 pm - 6:45 pm	CB 337

You may use the following formula for the derivative of a quadratic function.

$$\text{If} \quad p(x) = Ax^2 + Bx + C, \quad \text{then} \quad p'(x) = 2Ax + B.$$

Multiple Choice Questions

Show all your work on the page where the question appears.
Clearly mark your answer both on the cover page on this exam
and in the corresponding questions that follow.

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1. Find an equation for the line with slope (3/4) passing through the point $(x, y) = (8, 5)$.

Possibilities:

(a) $y = (3/4)x - 6$

(b) $y = (3/4)x - 1$

(c) $y = (5/8)x + (3/4)$

(d) $y = 8x + 5$

(e) $y = (3/4)x + 5$

Point-Slope form:

$y - 5 = \frac{3}{4}(x - 8)$

$y - 5 = \frac{3}{4}x - \frac{3}{4}\cdot 8 = \frac{3}{4}x - 6$

$y = \frac{3}{4}x - 6 + 5 = \frac{3}{4}x - 1$

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2. Solve the equation $x^3 + 3xy + 6y = 9$ for y in terms of x

Possibilities:

(a) $y = \dfrac{9 - x^3}{3x + 6}$

(b) $y = \dfrac{3x + 6}{9 - x^3}$

(c) $y = \dfrac{3x + 6}{x^3 - 9}$

(d) $y = \dfrac{x^3 - 9}{3x + 6}$

(e) $y = 9 - x^3 - 3x - 6$

$3xy + 6y = 9 - x^3$

$(3x + 6)y = 9 - x^3$

$y = \dfrac{9 - x^3}{3x + 6}$

---

3. Given $f(x) = \sqrt{4x - 6}$, write an expression for $(f(5 + h) - f(5)) \cdot (f(5 + h) + f(5))$

Possibilities:

(a) $6h$

(b) $4h$

(c) $28 + 4h$

(d) $\left(\sqrt{14 + h} - \sqrt{14}\right)^2$

(e) $\sqrt{14 + 4h} - \sqrt{14}$

$f(5) = \sqrt{5\cdot 4 - 6} = \sqrt{14}$

$f(5+h) = \sqrt{4(5+h) - 6} = \sqrt{14 + 4h}$

so $\left(\sqrt{14+4h} - \sqrt{14}\right)\left(\sqrt{14+4h} + \sqrt{14}\right)$

$= \sqrt{14+4h}^2 - \sqrt{14}\sqrt{14+4h} + \sqrt{14}\sqrt{14+4h} + \sqrt{14}^2$

$= 14 + 4h - 14 = 4h$

---

4. Evaluate $f(7)$ when $f(x)$ is given by the piecewise definition

$$f(x) = \begin{cases} 7x^2 + 2x + 2 & \text{if } x \leq -6 \\ 8 + 6x & \text{if } -6 < x \leq 7 \\ x^2 + 4x + 8 & \text{if } 7 < x \end{cases}$$

$f(7) = 8 + 6 \cdot 7 = 42 + 8 = 50$

Possibilities:

(a) 50

(b) 85

(c) 242

(d) 359

(e) 494

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5. A train travels from city A to city B, then travels from city B to city C. The train leaves city A at time 11:00am and arrives at city B at 12:30pm. The train leaves city B at 2:00pm and arrives at city C at 5:00pm. The average velocity of the train, while travelling from A to B, was 22 miles per hour. The average velocity of the train, while travelling from B to C, was 43 miles per hour. What was the average velocity of the train from city A to city C, including the wait at city B?

Possibilities:

(a) (65/2) miles per hour

(b) 28 miles per hour

(c) 65 miles per hour

(d) 27 miles per hour

(e) (21/2) miles per hour

$22 \cdot 1.5 = 33$ miles $\qquad 43 \cdot 3 = 129$ miles

A $\xrightarrow[1.5\text{ hr}]{33\text{ miles}}$ B $\xrightarrow[3\text{ hr}]{129\text{ miles}}$ C

$\frac{33 + 129}{6} = \frac{162}{6} = 27$

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6. Find the average rate of change of $f(x) = \sqrt{x+3}$ from $x = 6$ to $x = 33$.

Possibilities:

(a) 3

(b) $-7/22$

(c) 1/9

(d) $-1/9$

(e) $-1/11$

$\frac{f(33) - f(6)}{33 - 6} = \frac{\sqrt{36} - \sqrt{9}}{27} = \frac{6 - 3}{27} = \frac{3}{27}$

$= \frac{1}{9}$

---

7. Find the average rate of change of $f(x) = 12x^2 + 1$ from $x = 4$ to $x = 4 + h$.

Possibilities:

(a) $12h^2 + 96h$

(b) $-12h - 96$

(c) $-12h^2 - 96h$

(d) $12h + 96$

(e) h

$\frac{f(4+h) - f(4)}{h}$

$= \frac{193 + 96h + 12h^2 - 193}{h}$

$= \frac{96h + 12h^2}{h} = \frac{(96 + 12h)h}{h} = 96 + 12h$

$f(4) = 12 \cdot 4^2 + 1$

$= 193$

$f(4+h) = 12(4+h)^2 + 1$

$= 12(16 + 8h + h^2) + 1$

$= 192 + 96h + 12h^2 + 1$

$= 193 + 96h + 12h^2$

---

8. Let $f(x) = -9x^2 + 3x + 8$. Find the instantaneous rate of change of $f(x)$ at $x = -6$.

Possibilities:

(a) $-9h^2 + 111h$

(b) $-9h + 111$

(c) 0

(d) 111

(e) The instantaneous rate of change cannot be computed with the given information.

Inst ROC $= f'(x)$

$f'(x) = -9 \cdot 2x + 3 = -18x + 3$

$f'(-6) = (-18)(-6) + 3 = 111$

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9. Find an expression for the instantaneous rate of change of $f(x) = 3x^2 + 2x + 6$ at $x = a$.

Possibilities:

(a) $6a + 2$

(b) $12a + 4$

(c) 2

(d) $6a$

(e) $12a + 2$

$f'(x) = 3 \cdot 2x + 2$

$f'(a) = 6a + 2$

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10. Let $f(x) = 5x^2 + 2x + 5$. Find a value c between $x = 1$ and $x = 5$, so that the average rate of change of $f(x)$ from $x = 1$ to $x = 5$ is equal to the instantaneous rate of change of $f(x)$ at $x = c$.

Possibilities:

(a) 1

(b) 2

(c) 3

(d) 4

(e) 5

AROC $= \frac{f(5) - f(1)}{5 - 1} = \frac{(5 \cdot 5^2 + 2 \cdot 5 + 5) - (5 \cdot 1^2 + 2 \cdot 1 + 5)}{4}$

$= 32$

Inst ROC $= f'(c) = 5 \cdot 2c + 2 = 10c + 2$

So $10c + 2 = 32 \Rightarrow 10c = 30$

$c = 3$

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11. Determine the limit

$$\lim_{t\to 2}(t^3+t^2-2t+3)$$

Direct Sub

$= 2^3+2^2-2\cdot 2+3 = 11$

Possibilities:

(a) 11

(b) 12

(c) 13

(d) 14

(e) 15

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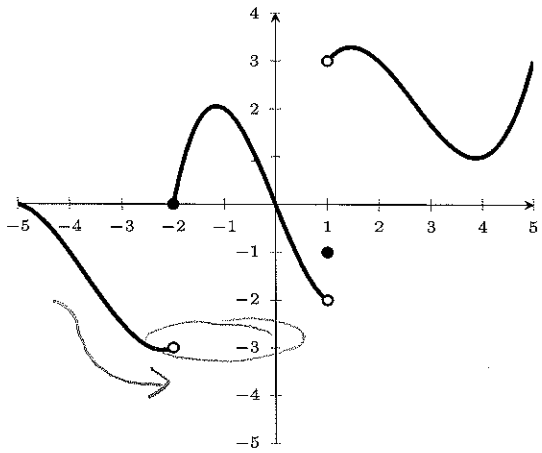
12. The graph of $y=f(x)$ is shown below. Compute $\lim_{x\to -2^-} f(x)$.



Possibilities:

(a) 3

(b) 0

(c) -1

(d) -2

(e) -3

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13. Find the limit $\lim_{t\to 0^+} \frac{50\sqrt{t}}{t}$

Algebraically

$\lim_{t\to 0^+} \frac{50}{\sqrt{t}}$ ← Num stays away from zero while denom $\to 0$

So limit DNE

Graphically

V. Asym @ $t=0$

So lim. DNE

Possibilities:

(a) $\frac{25}{\sqrt{t}}$

(b) 50

(c) 25

(d) This limit either tends to infinity or this limit fails to exist

(e) 0

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14. Find the limit

$$\lim_{x\to 0}\left(\frac{15}{x}+\frac{7x-15}{x}\right)$$

Simplify!

$= \lim_{x\to 0}\left(\frac{15+7x-15}{x}\right)$

$= \lim_{x\to 0}\frac{7x}{x} = \lim_{x\to 0} 7$

$= 7$

Possibilities:

(a) 1

(b) 7

(c) 0

(d) 15

(e) This limit does not exist.

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15. Find the limit

$$\lim_{x\to 1}\frac{x^2-3x+2}{x^2-4x+3}$$

top & bottom both go to 0, so need to simplify first

$= \lim_{x\to 1}\frac{(x-1)(x-2)}{(x-1)(x-3)} = \lim_{x\to 1}\frac{x-2}{x-3}$

$= \frac{1-2}{1-3} = \frac{-1}{-2} = \frac{1}{2}$

Possibilities:

(a) 1/2

(b) 1

(c) 3/2

(d) 2

(e) This limit does not exist

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16. Compute

$$\lim_{n\to\infty}\frac{7n^2+5n-7}{8n^2-5n-9}$$

same degree in num & denom, so lim = ratio of leading coeff

$= \frac{7}{8}$

If the limit tends to $\pm\infty$, select "Limit does not exist".

Possibilities:

(a) 7/8

(b) 0

(c) 7

(d) -1

(e) Limit does not exist

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17. Suppose $f(x) = Ax^3$ for $x < 2$ and $f(x) = 18 - Ax$ for $x \geq 2$. Find a value of A such that the function $f(x)$ is continuous at the point $x = 2$.

Possibilities:

(a) 7/5

(b) 8/5

(c) 9/5

(d) 2

(e) There is no such value of A.

$f(x) = \begin{cases} Ax^3 & x < 2 \\ 18 - Ax & x \geq 2 \end{cases}$

Need $\lim_{x \to 2^-} f(x) = \lim_{x \to 2^+} f(x)$

So $A \cdot 2^3 = 18 - 2 \cdot A$

$10A = 18$

$A = \frac{18}{10} = \frac{9}{5}$

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18. For the function $f(x) = 8x^2 + 8x + 8$, find the equation of the tangent line to graph of f at $x = -1$.

Possibilities:

(a) $y = 8$

(b) $y = 8x$

(c) $y = -8x + 8$

(d) $y = x^3 + 17$

(e) $y = -8x$

Slope: $f'(x) = 8 \cdot 2x + 8 = 16x + 8$

$f'(-1) = -16 + 8 = -8$

Point: $f(-1) = 8(-1)^2 + 8(-1) + 8 = 8$

So $y - 8 = -8(x - (-1))$

$y - 8 = -8x - 8 \Rightarrow y = -8x$

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19. The tangent line to graph of f at $x = -1$ has equation $y = -10x$. Find $f(-1)$ and $f'(-1)$.

Possibilities:

(a) $f(-1) = 0,\ f'(-1) = 0$

(b) $f(-1) = -10,\ f'(-1) = 0$

(c) $f(-1) = 10,\ f'(-1) = -10$

(d) $f(-1) = -10,\ f'(-1) = 10$

(e) $f(-1) = 0,\ f'(-1) = -10$

$f'(-1) = \text{Slope} = -10$

T. Line is

$y - f(-1) = -10(x - (-1))$

$y = -10x - 10 + f(-10) = -10x$

So $-10 + f(-10) = 0 \Rightarrow f(-10) = 10$

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20. Find all values of x where the derivative of the function $f(x) = |6x + 9|$ is not defined.

Possibilities:

(a) 6

(b) 0

(c) $-3/2$

(d) 9

(e) $-2/3$

Where $6x + 9 = 0$

$6x = -9$

$x = -\frac{9}{6} = -\frac{3}{2}$
