MA123 — Elem. Calculus Fall 2014
Exam 1 2014-09-25

Name: SOLUTIONS Sec.: _____

Do not remove this answer page — you will turn in the entire exam. You have two hours to do this exam. No books or notes may be used. You may use a graphing calculator during the exam, but NO calculator with a Computer Algebra System (CAS) or a QWERTY keyboard is permitted. Absolutely no cell phone use during the exam is allowed.

The exam consists of multiple choice questions. Record your answers on this page. For each multiple choice question, you will need to fill in the box corresponding to the correct answer. For example, if (a) is correct, you must write

(a) (b) (c) (d) (e)

Do not circle answers on this page, but please circle the letter of each correct response in the body of the exam. It is your responsibility to make it CLEAR which response has been chosen. You will not get credit unless the correct answer has been marked on both this page and in the body of the exam.

GOOD LUCK!

1.	**a**	b	c	d	e	11.	a	b	c	d	**e**
2.	a	**b**	c	d	e	12.	**a**	b	c	d	e
3.	a	b	**c**	d	e	13.	a	b	c	**d**	e
4.	a	b	c	d	**e**	14.	a	**b**	c	d	e
5.	a	b	**c**	d	e	15.	a	b	c	**d**	e
6.	a	b	c	d	**e**	16.	a	b	**c**	d	e
7.	a	b	c	**d**	e	17.	**a**	b	c	d	e
8.	a	**b**	c	d	e	18.	a	b	c	d	**e**
9.	a	b	**c**	d	e	19.	a	b	c	**d**	e
10.	**a**	b	c	d	e	20.	a	b	c	**d**	e

For grading use:

Number Correct	
	(out of 20 problems)

Total	
	(out of 100 points)

Please make sure to list the correct section number on the front page of your exam.
In case you forgot your section number, consult the following table.

Section	Instructor	Day	Time	Room
	Jack Schmidt	MWF	8:00 am - 8:50 am	FB 200
001	Nandita Sahajpal	Tu	8:00 am	FB B13
002	Nandita Sahajpal	Tu	9:30 am	NURS 501b
003	John Mosley	Tu	11:00 am	DH 353
004	John Mosley	Tu	12:30 pm	CB 337
005	John Mosley	Tu	2:00 pm	CB 233
006	John Mosley	Tu	3:30 pm	CB 341
	Jack Schmidt	MWF	9:00 am - 9:50 am	BS 107
007	Nandita Sahajpal	Th	8:00 am	TEB 207
008	Nandita Sahajpal	Th	9:30 am	TEB 231
009	Chad Linkous	Th	11:00 am	CP 111
010	Chad Linkous	Th	12:30 pm	CB 337
011	Bill Trok	Th	2:00 pm	CB 219
012	Bill Trok	Th	3:30 pm	CB 341
	erica Whitaker	MWF	1:00 pm - 1:50 pm	KAS 213
013	Dharma Maharjan	Tu	8:00 am	CP 397
014	Dharma Maharjan	Tu	9:30 am	NURS 511
015	Chad Linkous	Tu	11:00 am	FB B13
016	Chad Linkous	Tu	12:30 pm	CB 335
017	Bill Trok	Tu	2:00 pm	DH 301
018	Bill Trok	Tu	3:30 pm	CB 337
	erica Whitaker	MWF	3:00 pm - 3:50 pm	FB 200
019	Dharma Maharjan	Th	8:00 am	DH 203
020	Dharma Maharjan	Th	9:30 am	TEB 207
021	Kathy Effinger	Th	11:00 am	DH 353
022	Kathy Effinger	Th	12:30 pm	CB 335
023	Jonathan Thompson	Th	2:00 pm	FB B13
024	Jonathan Thompson	Th	3:30 pm	CB 303
401	Dustin Hedmark	MTR	5:30 pm - 6:45 pm	CB 343
402	Brad Fox	MTR	7:00 pm - 8:15 pm	CB 337

You may use the following formula for the derivative of a quadratic function.

If $p(x) = Ax^2 + Bx + C$, then $p'(x) = 2Ax + B$.

Multiple Choice Questions

Show all your work on the page where the question appears.
Clearly mark your answer both on the cover page on this exam
and in the corresponding questions that follow.

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1. Find an equation for the line with slope 2 passing through the point $(x, y) = (7, 6)$.

Possibilities:

(a) $y = 2x - 8$

(b) $y = 2x - 14$

(c) $y = (6/7)x + 2$

(d) $y = 2x + 6$

(e) $y = 7x + 6$

$y - y_1 = m(x - x_1)$

$y - 6 = 2(x - 7)$

$y - 6 = 2x - 14$

$+6 \quad +6$

$y = 2x - 8$

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2. Solve the equation $x^3 + 3xy + 6y = 8$ for y in terms of x

Possibilities:

(a) $y = \dfrac{x^3 - 8}{3x + 6}$

(b) $y = \dfrac{8 - x^3}{3x + 6}$

(c) $y = 8 - x^3 - 3x - 6$

(d) $y = \dfrac{3x + 6}{8 - x^3}$

(e) $y = \dfrac{3x + 6}{x^3 - 8}$

$x^3 + 3xy + 6y = 8$

$3xy + 6y = 8 - x^3$

$y(3x + 6) = 8 - x^3$

$y = \dfrac{8 - x^3}{3x + 6}$

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3. Find the point where the curve $y + 25 = (x - 7)^2$ intersects the y-axis.

Possibilities:

(a) $(-18, 0)$

(b) $(0, -24)$

(c) $(0, 24)$

(d) $(0, -18)$

(e) $(0, 32)$

y-axis is the line $x = 0$.

$y + 25 = (0 - 7)^2$ ← negative squared is positive

$y + 25 = 49$

$-25 \quad -25$

$y = 24$

(x, y)

$(0, 24)$

---

4. Evaluate $f(5)$ when $f(x)$ is given by the piecewise definition

$$f(x) = \begin{cases} x^2 - 5 & \text{if } x \leq 1 \\ 8x - 2 & \text{if } 1 < x \leq 3 \\ x^2 - 4x & \text{if } 3 < x \end{cases}$$

Since 5 is larger than 3, use this

$f(5) = 5^2 - 4(5) = 25 - 20 = 5$

Possibilities:

(a) -4

(b) 63

(c) 20

(d) 38

(e) 5

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5. A train travels from city A to city B, then travels from city B to city C. The train leaves city A at time 11:00am and arrives at city B at 12:30pm. The train leaves city B at 2:00pm and arrives at city C at 5:00pm. The average velocity of the train, while travelling from A to B, was 54 miles per hour. The average velocity of the train, while travelling from B to C, was 58 miles per hour. What was the average velocity of the train from city A to city C, including the wait at city B?

Possibilities:

(a) 56 miles per hour

(b) (87/2) miles per hour

(c) (85/2) miles per hour

(d) 2 miles per hour

(e) 112 miles per hour

Distance = Rate × Time

From A to B: $54 \times (1.5) = 81$ miles

From B to C: $58 \times (3) = 174$ miles

From A to C, Rate = $\frac{\text{Total dist}}{\text{Total time}} = \frac{174+81}{6} = \frac{255}{6}$

$= 42\frac{1}{2} = \boxed{\frac{85}{2}}$

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6. Find the average rate of change of $f(x) = \sqrt{x+3}$ from $x = 6$ to $x = 46$.

Possibilities:

(a) $-\frac{8}{23}$

(b) 4

(c) $-\frac{1}{10}$

(d) $\frac{43}{46}$

(e) $\frac{1}{10}$

$AROC = \frac{f(46) - f(6)}{46 - 6} = \frac{\sqrt{46+3} - \sqrt{6+3}}{40}$

$= \frac{\sqrt{49} - \sqrt{9}}{40} = \frac{7-3}{40} = \frac{4}{40} = \frac{1}{10}$

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7. Find the average rate of change of $f(x) = 7x^2 + 5$ from $x = 3$ to $x = 3 + h$.

Possibilities:

(a) h

(b) $-7h - 42$

(c) $-7h^2 - 42h$

(d) $7h + 42$

(e) $7h^2 + 42h$

$$\text{AROC} = \frac{f(3+h) - f(3)}{3+h-3} = \frac{7(3+h)^2 + 5 - (7\cdot 3^2 + 5)}{h}$$

$$= \frac{7(9 + 6h + h^2) + 5 - (7\cdot 9 + 5)}{h}$$

$$= \frac{63 + 42h + 7h^2 + 5 - 63 - 5}{h} = \frac{42h + 7h^2}{h} = 42 + 7h$$

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8. Find a value of x so that the instantaneous rate of change of $f(x) = 4x^2 + 8$ at x is equal to 48.

↳ this is $f'(x)$.

Possibilities:

(a) $x = 5$

(b) $x = 6$

(c) $x = 7$

(d) $x = 8$

(e) $x = 9$

Use the given formula: IF $f(x) = Ax^2 + Bx + C$,

$f'(x) = 2Ax + B$.

here, $A = 4$, $B = 0$, $C = 8$, so $f'(x) = 2(4)x + 0 = 8x$.

We want $8x = 48$, so $x = \frac{48}{8} = 6$

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9. Let $f(x) = 7x^2 + 4x + 5$. Find a value c between $x = 0$ and $x = 4$, so that the average rate of change of $f(x)$ from $x = 0$ to $x = 4$ is equal to the instantaneous rate of change of $f(x)$ at $x = c$.

Possibilities:

(a) 0

(b) 1

(c) 2

(d) 3

(e) 4

$$\text{AROC} = \frac{f(4) - f(0)}{4-0} = \frac{7\cdot 4^2 + 4(4) + 5 - (0+0+5)}{4}$$

$$= \frac{7(16) + 16 + 5 - 5}{4} = \frac{128}{4} = 32.$$

Inst. rate of ch = $f'(x) = 14x + 4$ (use formula above).

Set $14x + 4 = 32 \Rightarrow 14x = 28 \Rightarrow x = \frac{28}{14} = 2$

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10. If $\lim_{x\to 13} f(x) = 5$ and $\lim_{x\to 13} g(x) = 3$, then what is the value of $\lim_{x\to 13} \dfrac{(x+11)(f(x)+1)}{g(x)}$?

Possibilities:

(a) $\dfrac{(13+11)(5+1)}{3}$

(b) 0

(c) $\dfrac{(13)(5)}{3}$

(d) the limit is infinity or does not exist

(e) $\dfrac{5}{3}$

$= \dfrac{\lim_{x\to 13}(x+11)(f(x)+1)}{\lim_{x\to 13} g(x)} = \dfrac{\lim_{x\to 13}(x+11)\cdot \lim_{x\to 13}(f(x)+1))}{3}$

$= \dfrac{(13+11)\cdot(\lim_{x\to 13} f(x)+1)}{3} = \dfrac{(13+11)(5+1)}{3}$

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11. Find the limit

$\lim_{t\to 0^+} \dfrac{34\sqrt{t}}{t} = \lim_{t\to 0^+} \dfrac{34\sqrt{t}}{\sqrt{t}\cdot\sqrt{t}} = \lim_{t\to 0^+} \dfrac{34}{\sqrt{t}}$

test: $\frac{0}{0}$, "do more work"

one option ↑

test again: $\frac{34}{0}$; vertical asymptote

Possibilities:

(a) 34

(b) 0

(c) $\frac{17}{\sqrt{t}}$

(d) 17

(e) This limit either tends to infinity or this limit fails to exist

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12. Find the limit

$\lim_{x\to 0}\left(\dfrac{13}{x} + \dfrac{7x-13}{x}\right)$ ↙ add the fractions; already has common denominator

$= \lim_{x\to 0}\left(\dfrac{13+7x-13}{x}\right) = \lim_{x\to 0} \dfrac{7x}{x}$

$= \lim_{x\to 0} \dfrac{7}{1} = 7$

Possibilities:

(a) 7

(b) 0

(c) 1

(d) 13

(e) This limit does not exist.

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13. Compute

$$\lim_{n\to\infty} \frac{3n^2-8n+6}{7n^2+5n-5}$$

$= \lim_{n\to\infty} \frac{3n^2}{7n^2}$

$= \lim_{n\to\infty} \frac{3}{7} = \frac{3}{7}$

If the limit tends to $\pm\infty$, select "Limit does not exist".

Possibilities:

(a) 3

(b) 0

(c) -8

(d) 3/7

(e) Limit does not exist

(Since $n\to\infty$, and the expression is a single fraction, only the terms with the highest power in the numerator & denominator are significant.)

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14. For the function

$$f(x) = \begin{cases} |5+x| & \text{if } x < -2 \\ \sqrt{x^2+1} & \text{if } -2 \le x < 3 \\ 9x^2+x+2 & \text{if } 3 \le x \end{cases}$$

find $\lim_{x\to 4^+} f(x)$

Possibilities:

(a) 9

(b) 150

(c) 86

(d) $\sqrt{17}$

(e) $\sqrt{10}$

Values of x close to 4 and slightly larger than 4 are all larger than 3, so

$\lim_{x\to 4^+} f(x) = \lim_{x\to 4^+} (9x^2+x+2) = 9\cdot 4^2 + 4 + 2$

$= 9(16) + 6$

$= 144 + 6 = 150$

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15. The graph of $y = f(x)$ is shown below. Compute $\lim_{x\to -2^+} f(x)$.

Possibilities:

(a) 3

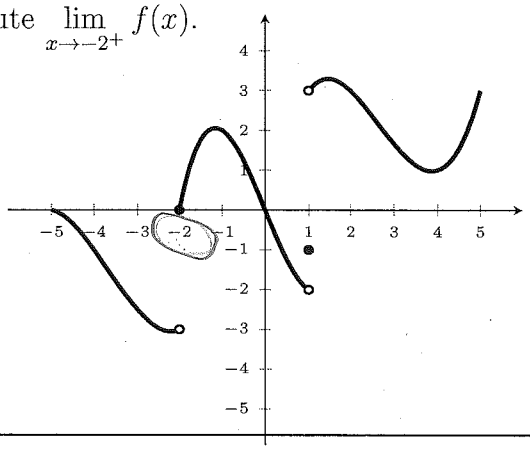
(b) -2

(c) -1

(d) 0

(e) -3

(Look for the y-values as we approach $x = -2$ from the right.)



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16. Suppose $f(x) = Ax^3$ for $x < 2$ and $f(x) = 14 - Ax$ for $x \geq 2$. Find a value of A such that the function $f(x)$ is continuous at the point $x = 2$.

↳ limit must exist

$$f(x) = \begin{cases} Ax^3 & x < 2 \\ 14 - Ax & x \geq 2 \end{cases}$$

Possibilities:

(a) 1

(b) $\frac{6}{5}$

(c) $\frac{7}{5}$

(d) $\frac{8}{5}$

(e) $\frac{9}{5}$

$\lim_{x \to 2^-} f(x) = \lim_{x \to 2^-} Ax^3 = A(2^3) = 8A$

$\lim_{x \to 2^+} f(x) = \lim_{x \to 2^+} (14 - Ax) = 14 - A(2) = 14 - 2A$

For $\lim_{x \to 2} f(x)$ to exist, we need $8A = 14 - 2A$

$\Rightarrow 10A = 14 \Rightarrow A = \frac{14}{10} = \frac{7}{5}$

17. Find the value of m which makes $f(x)$ differentiable everywhere, ~~where it~~

$$f(x) = \begin{cases} x^2, & \text{if } x \leq 2; \\ m(x-2)+4, & \text{if } x > 2 \end{cases}$$

Possibilities:

(a) 4

(b) 5

(c) 6

(d) 7

(e) 8

First Check continuity:

$\lim_{x \to 2^-} f(x) = \lim_{x \to 2^-} x^2 = 2^2 = 4$

$\lim_{x \to 2^+} f(x) = \lim_{x \to 2^+} m(x-2)+4 = m(0)+4 = 4$

(Since they match, continuous for all m)

Next check $f'(x) = \begin{cases} 2x & \text{if } x \leq 2 \\ m & \text{if } x > 2 \end{cases}$.

These must match at $x = 2$, so $2x = m \Rightarrow 2(2) = m$

(to avoid "sharp points") $\Rightarrow m = 4$

18. For the function $f(x) = 6x^2 + 5x + 9$, find the equation of the tangent line to graph of f at $x = -3$.

Possibilities:

(a) $y = 48x + 113$

(b) $y = x^3 + 17$

(c) $y = -31x + 48$

(d) $y = 48$

(e) $y = -31x - 45$

We need a point and a slope.

Point: $x = -3$. $y = f(x) = f(-3) = 6(-3)^2 + 5(-3) + 9$

$= 6(9) - 15 + 9 = 48$,

Slope $= m = f'(-3)$.

Use formula: $f'(x) = 12x + 5$, so $m = f'(-3) = 12(-3) + 5$

$= -36 + 5$

$= -31$.

$y - y_1 = m(x - x_1)$

$y - 48 = -31(x + 3) \Rightarrow y = -31x - 93 + 48$

$\Rightarrow \boxed{y = -31x - 45}$

19. Determine the value of $f'(1)$ from the graph of $f(x)$ given here:

Possibilities:

(a) $f'(1) = -1$

(b) $f'(1) = 0$

(c) $f'(1) = 1$

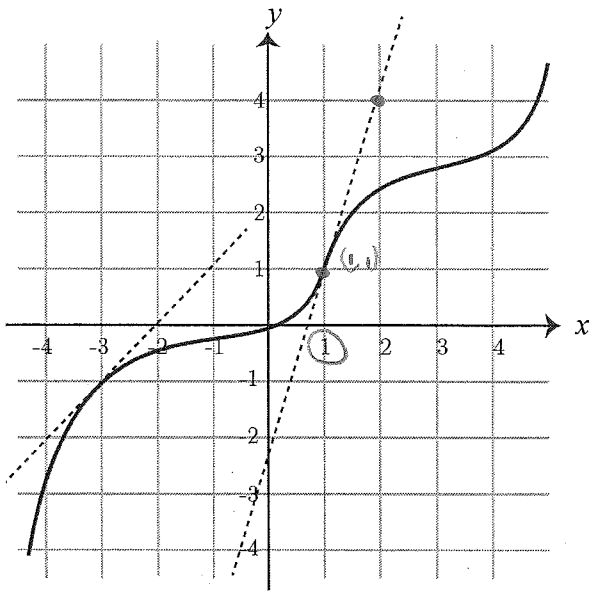
(d) $f'(1) = 3$

(e) $f'(1) = -3$

$f'(1)$ = slope of the tangent line at $x = 1$.

Line goes through $(1,1)$ and $(2,4)$,

$$f'(1) = m = \frac{4-1}{2-1} = \frac{3}{1} = 3$$



20. Determine the x values where the derivative is not defined (that is, the points where the function is not differentiable) on the function graphed here:

Possibilities:

(a) $x = -1$ and $x = 3$

(b) $x = -2$ and $x = 1$

(c) $x = -2$ and $x = 3$

(d) $x = -3$ and $x = 2$

(e) $x = -3$ and $x = 1$