MA 123 — Elem. Calculus Spring 2011
EXAM 1 2/9/2011

Name: Solutions. **Sec.:** _____

Do not remove this answer page — you will turn in the entire exam. You have two hours to do this exam. No books or notes may be used. You may use a graphing calculator during the exam, but NO calculator with a Computer Algebra System (CAS) or a QWERTY keyboard is permitted. Absolutely no cell phone use during the exam is allowed.

The exam consists of multiple choice questions. Record your answers on this page. For each multiple choice question, you will need to fill in the box corresponding to the correct answer. For example, if (a) is correct, you must write

■ b c d e

Do not circle answers on this page, but please circle the letter of each correct response in the body of the exam. It is your responsibility to make it CLEAR which response has been chosen. You will not get credit unless the correct answer has been marked on both this page and in the body of the exam.

GOOD LUCK!

1. ■ b c d e
2. a b c d ■
3. a b c d ■
4. a b c ■ e
5. a b c ■ e
6. a b ■ d e
7. a b c ■ e
8. a b c ■ e
9. a b c ■ e
10. a b c ■ e
11. ■ b c d e
12. ■ b c d e
13. ■ b c d e
14. a b c d ■
15. ■ b c d e
16. a ■ c d e
17. ■ b c d e
18. a b c ■ e
19. a ■ c d e
20. a b ■ d e

For grading use:

Number Correct	
	(out of 20 problems)

Total	
	(out of 100 points)

Please make sure to list the correct section number on the front page of your exam. In case you forgot your section number, consult the following table. If you are enrolled in a lecture with recitation, then your time and location is based on your recitation, not your lecture.

Section #	Instructor	Lectures
001	Koester/Hamilton	T 8:00 - 9:15 am, CP 243
002	Koester/Hamilton	R 8:00 - 9:15 am, CP 243
003	Koester/Hamilton	T 9:30 - 10:45 am, MMRB 243
004	Koester/May	R 9:30 - 10:45 am, CB 342
005	Koester/May	T 11:00 - 12:15 pm, CP 220
006	Koester/May	R 11:30 - 12:15 pm, CP 220
007	Koester/Kyriopoulos	T 9:30 - 10:45 am, CP 367
008	Koester/Kyriopoulos	R 9:30 - 10:45 am, DH 323
009	Koester/Kyriopoulos	T 11:00 - 12:15 pm, FB 307A
010	Koester/Robinson	R 11:00 - 12:15 pm, CP 345
011	Koester/Robinson	T 12:30 - 1:45pm, CB 219
012	Koester/Robinson	R 12:30 - 1:45pm, CB 219
013	Shaw/Taylor	T 11:00 - 12:15 pm, CB 345
014	Shaw/Taylor	R 11:00 - 12:15 pm, MMRB 243
015	Shaw/Taylor	T 12:30 - 1:45 pm, Nurs 201
016	Shaw/Tarr	R 12:30 - 1:45 pm, Nurs 502A
017	Shaw/Tarr	T 2:00 - 3:15 pm, CB 233
018	Shaw/Tarr	R 2:00 - 3:15 pm, CB 245
019	Shaw/Ozbek	T 3:30 - 4:45pm, CP 208
020	Shaw/Ozbek	R 3:30 - 4:45 pm, CP 208
021	Shaw/Ozbek	T 2:00 - 3:15 pm, FB B2
022	Shaw/Zhi	R 2:00 - 3:15 pm, CP 233
023	Shaw/Zhi	T 9:30 - 10:45 am, CP 211
024	Shaw/Zhi	R 9:30 - 10:45 am, CB 341
025	Beth Kelly	MWF 12:00 - 12:50 pm, CP 153
026	John Maki	MWF 2:00 - 2:50 pm, KAS 213

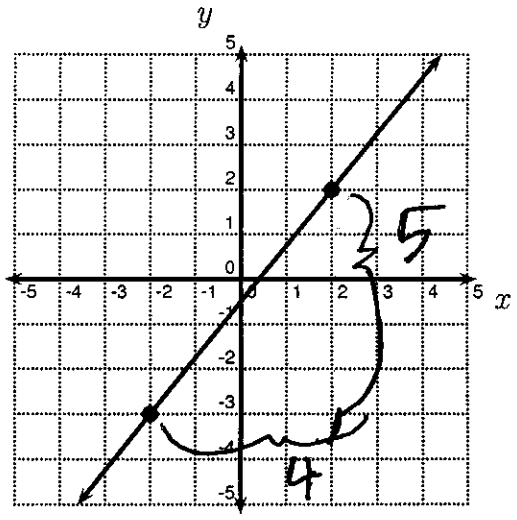
You may use the following formula for the derivative of a quadratic function.

If $p(x) = Ax^2 + Bx + C$, then $p'(x) = 2Ax + B$.

Multiple Choice Questions

Show all your work on the page where the question appears.
Clearly mark your answer both on the cover page on this exam
and in the corresponding questions that follow.

1. Find the slope of the line in the graph shown below.



Possibilities:

(a) $\frac{4}{5}$

(b) $\frac{5}{4}$

(c) $\frac{-5}{4}$

(d) $\frac{-4}{5}$

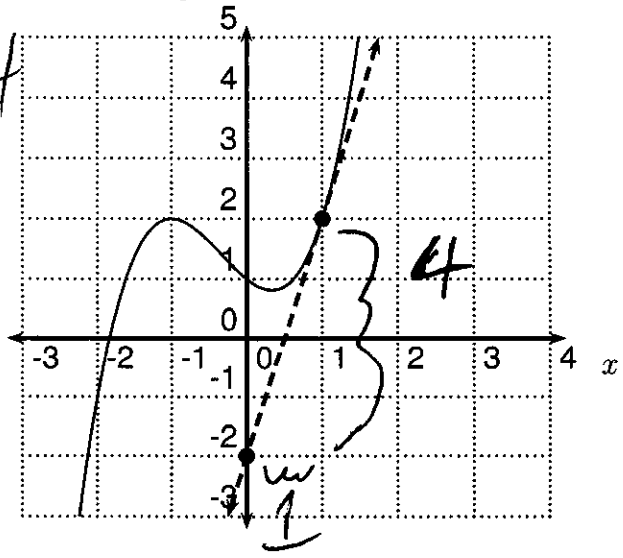
(e) -5

$\frac{5}{4}$

2. The graph of $y = g(x)$ is shown, as well as the tangent line to the graph at $x = 1$. Determine $g'(1)$.

$g'(1)$ = Slope of tangent Line at $x = 1$.

But slope is $\frac{4}{1} = 4$



Possibilities:

(a) -2

(b) $1/4$

(c) 1

(d) 2

(e) 4

3. Let $f(x) = 6x^2 + 4x - 5$. Find an equation for the line through the points $(2, f(2))$ and $(5, f(5))$.

$f(2) = 6 \cdot 2^2 + 4 \cdot 2 - 5 = 27$

$f(5) = 6 \cdot 5^2 + 4 \cdot 5 - 5 = 165$

Possibilities:

(a) $y - 2 = 46x + 27$

(b) $y + 27 = 46(x + 2)$

(c) $y = 46x - 119$

(d) $y - 2 = 46(x - 27)$

(e) $y - 27 = 46(x - 2)$

$\text{Slope} = \frac{165 - 27}{5 - 2} = \frac{138}{3} = 46.$

Use point-slope form with slope $= 46$, Point $= (2, 27)$:

$y - 27 = 46(x - 2)$

---

4. Find the average rate of change of $f(x) = \frac{2}{x}$ from $x = 5$ to $x = 9$.

$AROC = \frac{f(9) - f(5)}{9 - 5} = \frac{\frac{2}{9} - \frac{2}{5}}{4}$

Possibilities:

(a) 0

(b) 8/45

(c) 2/45

(d) $-2/45$

(e) $-8/45$

$= \frac{\frac{2 \cdot 5 - 2 \cdot 9}{9 \cdot 5}}{4} = \frac{\frac{-8}{45}}{4} = \frac{-8}{45 \cdot 4}$

$= \frac{-2}{45}$

---

5. Find the average rate of change of $f(x) = 2x + 5$ from $x = 2$ to $x = 2 + h$.

$AROC = \frac{f(2+h) - f(2)}{h} = \frac{(2(2+h) + 5) - (2 \cdot 2 + 5)}{h}$

Possibilities:

(a) h

(b) $2h$

(c) $-2h$

(d) 2

(e) -2

$= \frac{4 + 2h + 5 - 4 - 5}{h} = 2.$

Alternatively, AROC $= 2$ since $f(x)$ is a linear function with slope $= 2$.

---

6. Find the derivative, $f'(-1)$, where

$$f(x) = x^2 - 2x + 7$$

$f'(x) = 2x - 2$

$f'(-1) = 2(-1) - 2 = -4$

Possibilities:

(a) 10

(b) 4

(c) -4

(d) 3

(e) -3

---

7. Let $f(x) = 8x^2 + x - 9$. Find the instantaneous rate of change of $f(x)$ at $x = 5$.

Inst ROC $= f'(x)$.

$f'(x) = 2 \cdot 8x + 1 = 16x + 1$

$f'(5) = 16 \cdot 5 + 1 = 81$

Possibilities:

(a) $81h + 8h^2$

(b) 0

(c) $81 + 8h$

(d) 81

(e) The instantaneous rate of change cannot be computed with the given information.

---

8. Let $f(x) = 9x^2 - 4x - 6$. Find a value c in the interval $[5, 9]$ so that the average rate of change of $f(x)$ from $x = 5$ to $x = 9$ is equal to the instantaneous rate of change of $f(x)$ at $x = c$.

$f(5) = 9 \cdot 5^2 - 4 \cdot 5 - 6 = 199$

$f(9) = 9 \cdot 9^2 - 4 \cdot 9 - 6 = 687$

$AROC = \frac{f(9) - f(5)}{9 - 5} = \frac{687 - 199}{4} = 122$.

Inst ROC $= f'(c) = 2 \cdot 9c - 4 = 18c - 4$.

so AROC = Inst ROC

$\Rightarrow 122 = 18c - 4 \Rightarrow 18c = 126$

$c = 7$

Possibilities:

(a) 4

(b) 5

(c) 6

(d) 7

(e) 8

---

9. Let $f(x) = 3x^2 + 2x - 4$. Find an equation for the tangent line to the curve $y = f(x)$ at the point $x = 3$.

Point on Tangent Line is $(3, f(3))$

$f(3) = 3 \cdot 3^2 + 2 \cdot 3 - 4 = 29$, point is $(3, 29)$

Slope $= f'(3) = 2 \cdot 3 \cdot 3 + 2 = 20$.

Line is then

$y - 29 = 20(x - 3)$

Possibilities:

(a) $y = 20x - 89$

(b) $y + 29 = 20(x + 3)$

(c) $y - 3 = 20(x - 29)$

(d) $y - 29 = 20(x - 3)$

(e) $y - 3 = 20x + 29$

---

10. A particle is traveling along a straight line. Its position, measured in feet, after t seconds is given by $s(t) = 6t^2 + 202$. Find the instantaneous velocity of the particle at time $t = 1$.

Inst Velocity $= s'(t)$.

$s'(t) = 2 \cdot 6t = 12t$

$s'(1) = 12 \cdot 1 = 12$.

Possibilities:

(a) 202 feet per second

(b) 1 foot per second

(c) 208 feet per second

(d) 12 feet per second

(e) 6 feet per second

---

11. A car travels at 40 miles per hour from 8:00 am to 9:30 am. It then travels 100 miles from 9:30 to 11:30 am. What was the car's average velocity from 8:00 am to 11:30 am?

8:00 $\xrightarrow[3/2 \text{ hr}]{60 \text{ miles}}$ 9:30 $\xrightarrow[2 \text{ hr}]{100 \text{ miles}}$ 11:30

40 mph for $3/2$ hr $= 60$ miles.

Average Velocity $= \dfrac{\text{Distance}}{\text{Time}}$

$= \dfrac{100 + 60}{3/2 + 2} = \dfrac{160}{7/2} = \dfrac{320}{7}$.

Possibilities:

(a) 320/7 miles per hour

(b) 160 miles per hour

(c) 560 miles per hour

(d) 80 miles per hour

(e) 140 miles per hour

---

12. Compute

$$\lim_{t\to 4}\left(7-2t+\frac{t^2}{t-2}\right)$$

Denominator stays away from 0 as $t\to 4$, so limit is $7-2\cdot 4+\frac{4^2}{4-2}=7$

Possibilities:

(a) 7

(b) 8

(c) 9

(d) 10

(e) The limit does not exist.

---

13. Compute $\lim_{t\to 5}\frac{t^2-2t-15}{t-5}$

Numerator + Denominator both go to zero as $t\to 5$. Need to simplify!

$$\frac{t^2-2t-15}{t-5}=\frac{(t-5)(t+3)}{(t-5)}$$

So $\lim_{t\to 5}\frac{t^2-2t-15}{t-5}=\lim_{t\to 5}(t+3)=5+3=8$

Possibilities:

(a) 8

(b) 9

(c) 10

(d) 11

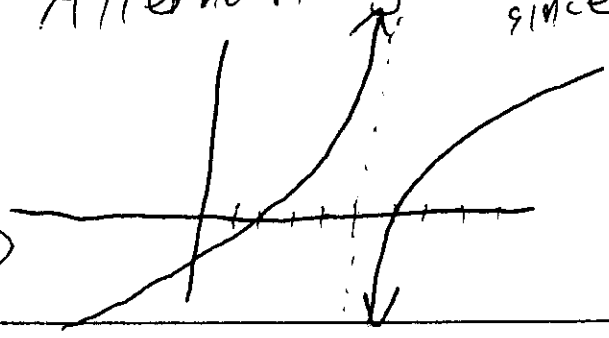
(e) The limit does not exist.

---

14. Compute $\lim_{t\to 5}\frac{t^2-5t-6}{t-5}$

Denominator, but not numerator, goes to zero as $t\to 5$, so limit does not exist.

Alternatively, limit does not exist since graph has vertical asymptote at $x=5$.



Possibilities:

(a) 5

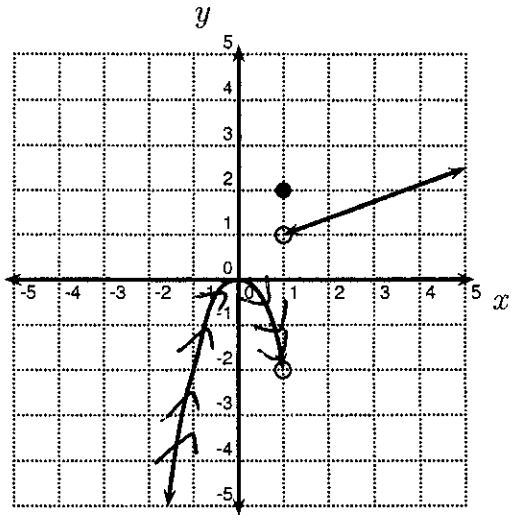
(b) 0

(c) 6

(d) −1

(e) The limit does not exist.

15. The graph of $y = f(x)$ is shown below. Compute $\lim_{x \to 1^-} f(x)$.



Possibilities:

(a) -2
(b) 0
(c) 1
(d) 2
(e) 4

16. Compute $\lim_{t \to 0^+} \frac{|6t|}{t}$

Possibilities:

(a) 0
(b) 6
(c) -6
(d) 5
(e) The limit does not exist.

method 1: $t > 0 \Rightarrow |6t| = 6t$, so

$\lim_{t \to 0^+} \frac{|6t|}{t} = \lim_{t \to 0^+} \frac{6t}{t} = 6.$

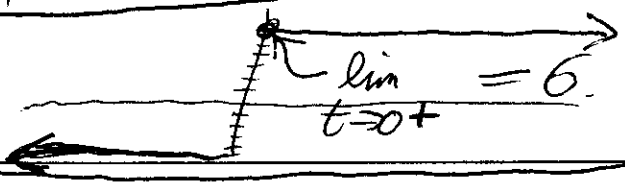
method 2: Plug in t values near zero, but $t > 0$,

$t = .1 \Rightarrow \frac{|6 \cdot .1|}{.1} = 6$

$t = .01 \quad \frac{|6 \cdot .01|}{.01} = 6$

} so limit should be 6

method 3



17. Find the value of A which makes $f(x)$ continuous everywhere, where

$$f(x) = \begin{cases} -4x + A, & \text{if } x \leq 2; \\ 6x^3 - 3, & \text{if } x > 2 \end{cases}$$

Possibilities:

(a) 53
(b) 45
(c) -3
(d) 8
(e) No such value of A exists

Need $\lim_{x \to 2^+} f(x) = \lim_{x \to 2^-} f(x)$

$\lim_{x \to 2^+} -4x + A = \lim_{x \to 2^-} 6x^3 - 3$

$-4 \cdot 2 + A = 6 \cdot 2^3 - 3$

$-8 + A = 45$, so $A = 53$

18. Suppose

$$-6x^2 - 8x + 10 = A + B(x-1) + C(x-1)^2.$$

Find A.

Plug in $x = 1$:

$$-6\cdot 1^2 - 8\cdot 1 + 10 = A + B(1-1) + C(1-1)^2$$

$$-4 = A$$

Possibilities:

(a) -7
(b) -6
(c) -5
(d) -4
(e) -3

---

19. Let $f(x) = 8x^2 - 4x - 3$. The tangent line to the curve $y = f(x)$ at $x = 2$ is given by $y = 28x + b$. Determine the value of b.

One method: $f(x)$ & tangent line at $x=2$ agree at $x = 2$, so

$f(2) = 28\cdot 2 + b$.

But $f(2) = 8\cdot 2^2 - 4\cdot 2 - 3 = 21$

so $21 = 56 + b$, so $b = -35$

Another method: $f(2) = 21$, $f'(2) = 2\cdot 8\cdot 2 - 4 = 28$

Tangent line is $y - 21 = 28(x-2)$

$\Rightarrow y = 28x - 28\cdot 2 + 21 = 28x - 35$

b!

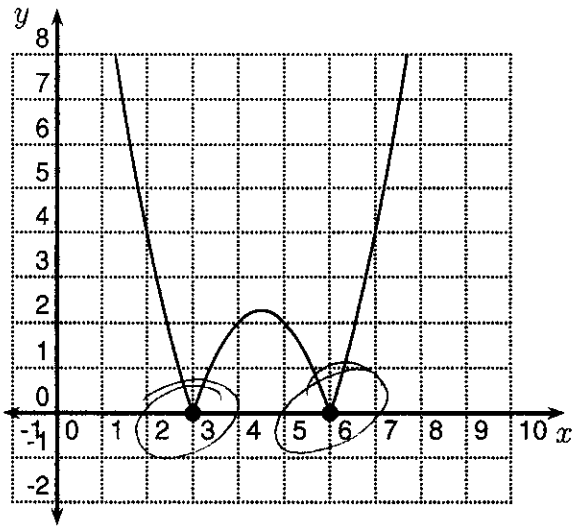
Possibilities:

(a) -36
(b) -35
(c) -34
(d) -33
(e) -32

---

20. Find all points where $g(x)$ is not differentiable, where $g(x)$ is defined by

$$g(x) = \begin{cases} x^2 - 9x + 18, & \text{if } x \le 3 \text{ or } x \ge 6; \\ -x^2 + 9x - 18, & \text{if } 3 < x < 6 \end{cases}$$



Possibilities:

(a) $x = 6$
(b) $x = 9/2$
(c) $x = 3$ and $x = 6$
(d) $g(x)$ is differentiable everywhere.
(e) $x = 3$
