MA 123 — Elem. Calculus EXAM 1	Spring 2013 2/6/2013	Name: Solution	Sec.: ______

Do not remove this answer page — you will turn in the entire exam. You have two hours to do this exam. No books or notes may be used. You may use a graphing calculator during the exam, but NO calculator with a Computer Algebra System (CAS) or a QWERTY keyboard is permitted. Absolutely no cell phone use during the exam is allowed.

The exam consists of multiple choice questions. Record your answers on this page. For each multiple choice question, you will need to fill in the box corresponding to the correct answer. For example, if (b) is correct, you must write

a **b** c d e

Do not circle answers on this page, but please circle the letter of each correct response in the body of the exam. It is your responsibility to make it CLEAR which response has been chosen. You will not get credit unless the correct answer has been marked on both this page and in the body of the exam.

GOOD LUCK!

1.	a	**b**	c	d	e	**11.**	a	b	**c**	d	e
2.	**a**	b	c	d	e	**12.**	a	**b**	c	d	e
3.	a	b	**c**	d	e	**13.**	a	**b**	c	d	e
4.	a	b	c	d	**e**	**14.**	a	b	c	**d**	e
5.	**a**	b	c	d	e	**15.**	a	b	c	**d**	e
6.	a	b	c	d	**e**	**16.**	a	b	c	**d**	e
7.	a	b	c	**d**	e	**17.**	a	**b**	c	d	e
8.	**a**	b	c	d	e	**18.**	**a**	b	c	d	e
9.	a	b	c	**d**	e	**19.**	a	b	c	**d**	e
10.	a	b	c	**d**	e	**20.**	**a**	b	c	d	e

For grading use:

Number Correct	
	(out of 20 problems)

Total	
	(out of 100 points)

Please make sure to list the correct section number on the front page of your exam. In case you forgot your section number, consult the following table. Your section number is determined by your recitation time and location.

Section #	**Instructor**	**Day and Time**	**Room**
001	D. Akers	T, 8:00 am - 9:15 am	CB 213
002	W. Hough	R, 8:00 am - 9:15 am	CB 213
003	D. Akers	T, 12:30 pm - 1:45 pm	CB 342
004	W. Hough	R, 9:30 am - 10:45 am	CP 397
005	F. Smith	T, 11:00 am - 12:15 pm	TPC 212
006	W. Hough	R, 11:00 am - 12:15 pm	TPC 113
007	A. Happ	T, 2:00 pm - 3:15 pm	TPC 109
008	A. Hubbard	R, 2:00 pm - 3:15 pm	L 108
009	A. Happ	T, 11:00 am - 12:15 pm	TPC 113
010	A. Hubbard	R, 11:00 am - 12:15 pm	CB 340
011	A. Happ	T, 12:30 pm - 1:45 pm	TEB 231
012	A. Hubbard	R, 12:30 pm - 1:45 pm	EH 307
013	L. Solus	T, 11:00 am - 12:15 pm	CB 340
014	D. Akers	R, 11:00 am - 12:15 pm	TPC 101
015	L. Solus	T, 12:30 pm - 1:45 pm	OT 0B7
016	F. Smith	R, 12:30 pm - 1:45 pm	FB B4
017	L. Solus	T, 2:00 pm - 3:15 pm	FB B4
018	F. Smith	R, 2:00 pm - 3:15 pm	CB 245
019	X. Kong	T, 3:30 pm - 4:45 pm	BH 303
020	Q. Liang	R, 3:30 pm - 4:45 pm	EGJ 115
021	X. Kong	T, 12:30 pm - 1:45 pm	CB 205
022	X. Kong	R, 2:00 pm - 3:15 pm	CB 233
023	L. Davidson	T, 9:30 am - 10:45 am	OT 0B7
024	L. Davidson	R, 9:30 am - 10:45 am	OT 0B7
026	L. Davidson	R, 8:00 am - 9:15 am	CB 243
027	Q. Liang	T, 9:30 am - 10:45 am	DH 131

You may use the following formula for the derivative of a quadratic function.

If $p(x) = Ax^2 + Bx + C,$ then $p'(x) = 2Ax + B.$

Multiple Choice Questions

Show all your work on the page where the question appears.
Clearly mark your answer both on the cover page on this exam
and in the corresponding questions that follow.

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1. Suppose the graph of the equation

$$y = A + B(x-2) + C(x-2)(x-3)$$

contains the points $(2, 12)$ and $(3, 11)$ in the (x, y) plane. What is the value of B?

Possibilities:

(a) $B = -2$
(b) $B = -1$
(c) $B = 0$
(d) $B = 1$
(e) $B = 2$

When $x = 2$, then $y = 12$,
So $12 = A + B(2-2) + C(2-2)(2-3)$
$\Rightarrow 12 = A$

When $x = 3$, then $y = 11$
$11 = A + B(3-2) + C(3-2)(3-3)$
$\Rightarrow 11 = A + B$

$A = 12$
so
$12 + B = 11$
$B = -1$

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2. Suppose a fuel mixture is 15 % ethanol and 85 % gasoline. How much ethanol (in gallons) must you add to one gallon of the fuel so that the new fuel mixture is 19 % ethanol?

x = # gallons eth. added

Possibilities:

(a) 4/81 gallons of ethanol
(b) 5/81 gallons of ethanol
(c) 2/27 gallons of ethanol
(d) 7/81 gallons of ethanol
(e) 8/81 gallons of ethanol

| .15 gallon eth |
| .85 gallon gas |

1 gallon

| ? gallons eth |
| .85 gallon gas |

$1 + x$ gallons

19% eth $\Rightarrow$ 81% gas, so $.81 = \dfrac{.85}{1+x}$

$\Rightarrow .81 + .81x = .85 \Rightarrow .81 = .04x \Rightarrow x = \dfrac{4}{81}$

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3. Suppose $f(x) = x^2 + 5x + 15$. What is the average rate of change of $f(x)$ with respect to x as x changes from $x = -2$ to $x = 3$?

$$AROC = \frac{f(3) - f(-2)}{3 - (-2)} = \frac{(3^2 + 5\cdot 3 + 15) - ((-2)^2 + 5(-2) + 15)}{5}$$

Possibilities:

(a) 5
(b) 11/2
(c) 6
(d) 13/2
(e) 7

$$= \frac{39 - 9}{5} = 6$$

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4. Suppose $g(s) = s^3 + 13$. What is the average rate of change of $g(s)$ with respect to s as s changes from 2 to $2 + h$?

$$AROC = \frac{g(2+h) - g(2)}{h} = \frac{h^3 + 6h^2 + 12h + \cancel{21} - \cancel{21}}{h} = \frac{h(h^2+6h+12)}{h} = h^2 + 6h + 12$$

Possibilities:

(a) $3h^2$

(b) $h^2 + 3h + 3$

(c) $h^3 + 3h^2 + 3h$

(d) $h^3 + 6h^2 + 12h$

(e) $h^2 + 6h + 12$

$g(2) = 2^3 + 13 = 21$

$g(2+h) = (2+h)^3 + 13 = (2+h)(2+h)(2+h) + 13$

$= (4 + 4h + h^2)(2+h) + 13$

$= 8 + 8h + 2h^2 + 4h + 4h^2 + h^3 + 13$

$= h^3 + 6h^2 + 12h + 21.$

5. Suppose $f(x) = x^2 + bx + 10$ for some constant b. Suppose that the average rate of change of $f(x)$ from $x = 1$ to $x = 3$ is equal to 8. Determine the value of b.

$$AROC = \frac{f(3) - f(1)}{3 - 1} = 8$$

Possibilities:

(a) 4

(b) 5

(c) 6

(d) 7

(e) 8

$f(1) = 1^2 + b \cdot 1 + 10 = 11 + b$

$f(3) = 3^2 + 3b + 10 = 19 + 3b.$

So $\frac{(19+3b) - (11+b)}{2} = 8$

$\Rightarrow \frac{8+2b}{2} = 8$

$\Rightarrow 8 + 2b = 16.$

$2b = 8 \Rightarrow b = 4.$

6. If $h(t)$ represents the height of an object above ground level in feet at time t seconds and $h(t)$ is given by $h(t) = -16t^2 + 32t + 101$, find the height of the object at the time when the speed is zero.

Want $h(t)$ for some t.

Possibilities:

(a) 1 foot

(b) 32 feet

(c) 101 feet

(d) 116 feet

(e) 117 feet

Speed $= h'(t) = -16 \cdot 2t + 32 = -32t + 32$

So Speed $= 0 \Rightarrow 32t = 32 \Rightarrow t = 1.$

So height $= h(1) = -16 \cdot 1^2 + 32 + 101$

$= 117.$

7. Let $f(t) = 3t^2 + 2t - 84$. Find the value of t for which the tangent line to the graph of $f(t)$ at the point $(t, f(t))$ has slope equal to 6.

Slope tangent line $= f'(t) = 2 \cdot 3t + 2$
$= 6t + 2$

Possibilities:

(a) 1/6
(b) 1/3
(c) 1/2
(d) 2/3
(e) 5/6

Want slope $= 6$.
$6t + 2 = 6$
$6t = 4$
$t = 4/6 = 2/3$

8. Determine the limit

$$\lim_{t \to 2}(t^3 + t^2 - 2t - 3)$$

Direct Sub.

$= 2^3 + 2^2 - 2 \cdot 2 - 3$
$= 8 + 4 - 4 - 3 = 5$

Possibilities:

(a) 5
(b) 6
(c) 7
(d) 8
(e) 9

9. Find the limit

$$\lim_{x \to 0}\left(\frac{13}{x} + \frac{7x - 13}{x}\right)$$

$= \lim_{x \to 0}\left(\frac{\cancel{13} + 7x - \cancel{13}}{x}\right) = \lim_{x \to 0} \frac{7\cancel{x}}{\cancel{x}}$
$= 7$

Possibilities:

(a) 13
(b) 1
(c) 0
(d) 7
(e) This limit does not exist.

10. Find the limit

$$\lim_{x\to 1} \frac{x^2+x-2}{x^2-3x+2}$$

$$= \lim_{x\to 1} \frac{(x-1)(x+2)}{(x-1)(x-2)}$$

$$= \lim_{x\to 1} \frac{x+2}{x-2} = \frac{1+2}{1-2} = \frac{3}{-1} = -3$$

Possibilities:

(a) -6
(b) -5
(c) -4
(d) -3
(e) This limit does not exist

11. Suppose $f(x) = Ax^3$ for $x < 2$ and $f(x) = 14 - Ax$ for $x \geq 2$. Find a value of A such that the function $f(x)$ is continuous at $x = 2$.

Need $\lim_{x\to 2^-} f(x) = \lim_{x\to 2^+} f(x)$

So $\lim_{x\to 2^-} A\cdot x^3 = \lim_{x\to 2^+} 14 - Ax$

$A\cdot 2^3 = 14 - A\cdot 2$

$8A = 14 - 2A \Rightarrow 10A = 14$

$A = 14/10 = 7/5$

Possibilities:

(a) 1
(b) 6/5
(c) 7/5
(d) 8/5
(e) There is no such value of A.

12. Suppose $\lim_{x\to 0} f(x) = 4$. Find the limit

$$\lim_{x\to 0} \left(\frac{f(x)}{x+1} - (4+f(x))^2 \right)$$

Chapter 3 limit laws

$$= \frac{\lim_{x\to 0} f(x)}{\lim_{x\to 0}(x+1)} - \left(\lim_{x\to 0} 4 + f(x)\right)^2$$

$$= \frac{4}{1} - (4+4)^2 = 4 - 8^2$$
$$= 4 - 64$$
$$= -60$$

Possibilities:

(a) 1
(b) -60
(c) 0
(d) -28
(e) This limit does not exist.

13. Let $f(x) = x^2 + 3x + 7$. Find a value c between $x = 0$ and $x = 4$ so that the average rate of change of $f(x)$ from $x = 0$ to $x = 4$ is equal to the instantaneous rate of change of $f(x)$ at $x = c$.

$$\underset{0 \text{ to } 4}{\text{AROC}} = \frac{f(4) - f(0)}{4 - 0} = \frac{(4^2 + 3\cdot 4 + 7) - (0^2 + 3\cdot 0 + 7)}{4} = \frac{28}{4} = 7$$

Inst ROC @ $x = c$ $= 2\cdot c + 3$

Want Inst ROC = AROC

$7 = 2c + 3 \Rightarrow 4 = 2c \Rightarrow c = 2$

Possibilities:

(a) 1

(b) 2

(c) 3

(d) 4

(e) 5

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14. Find the limit

$$\lim_{t \to 0^+} \frac{42\sqrt{t}}{t} = \lim_{t \to 0^+} \frac{42}{\sqrt{t}}$$

Denominator $\to 0$

But Numerator $\not\to 0$

So limit DNE

Possibilities:

(a) 21

(b) 42

(c) 0

(d) This limit either tends to infinity or this limit fails to exist

(e) $\frac{21}{\sqrt{t}}$

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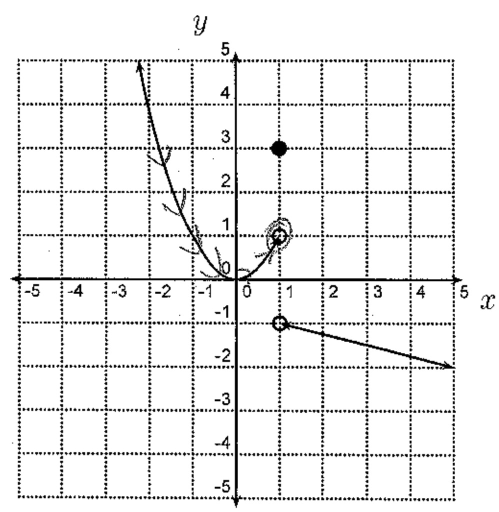
15. The graph of $y = f(x)$ is shown below. Compute $\lim_{x \to 1^-} f(x)$.



Possibilities:

(a) -4

(b) -2

(c) -1

(d) 1

(e) 3

16. Consider the function $f(x) = 5x^2 + 32$. Its tangent line at $x = 2$ goes through the point $(-2, y)$. Determine y.

$f(2) = 5 \cdot 2^2 + 32 = 52$

$f'(x) = 10x \Rightarrow f'(2) = 10 \cdot 2 = 20$

T. Line: $y - 52 = 20(x - 2)$

Put $x = -2$ in tangent line.

$y - 52 = 20(-2-2)$

$y = 52 - 80 = -28$

Possibilities:

(a) 52
(b) 20
(c) −52
(d) −28
(e) 28

17. You have to fly from city A to city B, then city B to city C. (You may assume that no time is spent on the ground between the two flights.) Suppose the plane flies with an average velocity of 600 mph from city A to city B, 550 mph from city B to city C, and the average velocity of the entire trip was 570 mph. The distance from city A to city B is 2400 miles and it took 6 hours to fly from B to C. Determine the total distance travelled from A to C.

Possibilities:

(a) 2970 miles
(b) 5700 miles
(c) 3600 miles
(d) 2950 miles
(e) 3300 miles

$$A \xrightarrow{2400 \text{ miles}} B \xrightarrow[6 \text{ hours}]{550 \text{ mph} \quad 3300 \text{ miles}} C$$

$$\begin{array}{r} 550 \\ \times\ 6 \\ \hline 3300 \end{array}$$

Total Distance $= 2400 + 3300 = 5700$

18. Suppose $g(x) = 6(x-2) + 19$. Compute

$$\lim_{h \to 0} \frac{g(2+h) - g(2)}{h}$$

Possibilities:

(a) 6
(b) 7
(c) 8
(d) 9
(e) 10

Quick way:

$g'(2) = \lim_{h \to 0} \frac{g(2+h) - g(2)}{h}$

But $g(x)$ is linear function with slope 6, so

$g'(2) = 6$

Less quick way.

$g(2) = 6(2-2) + 19 = 19$

$g(2+h) = 6(2+h-2) + 19 = 6h + 19$

So $\lim_{h \to 0} \frac{g(2+h) - g(2)}{h} = \lim_{h \to 0} \frac{6h + 19 - 19}{h}$

$= \lim_{h \to 0} \frac{6h}{h} = 6$

19. The graph of $y = g(x)$ is shown (solid), as well as the tangent line to the graph (dotted) at $x = 5$. Determine $g'(5)$. (Graph is not drawn to scale.)

Possibilities:

$g'(5) =$ Slope tangent line at $x = 5$

(a) 2

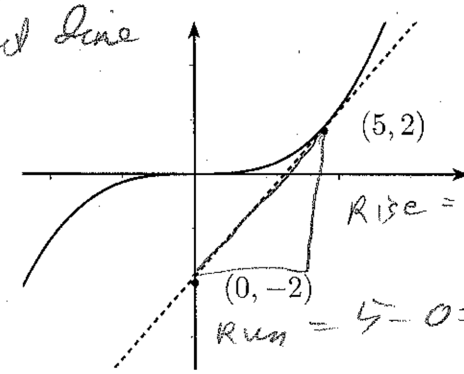
(b) 5

(c) 5/2

(d) 4/5

(e) 5/4

Slope $= \frac{\Delta y}{\Delta x} = \frac{4}{5}$



Rise $= 2-(-2) = 4$

Run $= 5 - 0 = 5$

20. Select the true statement

Theorem from end of Chapter 3 Lecture notes.

Possibilities:

(a) If $f(x)$ is differentiable at $x = a$, then $f(x)$ must be continuous at $x = a$.

(b) If $\lim_{x \to a} f(x)$ exists, then $f(x)$ must be differentiable at $x = a$.

(c) If $f(x)$ is continuous at $x = a$, then $f(x)$ must be differentiable at $x = a$. $y = |x|$ is continuous but not diff.

(d) If $\lim_{x \to a} f(x)$ exists, then $f(x)$ must be continuous at $x = a$.

(e) None of the above statements are true.