MA123 — Elem. Calculus Spring 2015
Exam 1 2015-02-12

Name: SOLUTIONS Sec.: _____

Do not remove this answer page — you will turn in the entire exam. You have two hours to do this exam. No books or notes may be used. You may use an ACT-approved calculator during the exam, but NO calculator with a Computer Algebra System (CAS), networking, or camera is permitted. Absolutely no cell phone use during the exam is allowed.

The exam consists of two short answer questions and eighteen multiple choice questions. Answer the short answer questions on the back of this page, and record your answers to the multiple choice questions on this page. For each multiple choice question, you will need to fill in the box corresponding to the correct answer. For example, if (a) is correct, you must write

(a) (b) (c) (d) (e)

Do not circle answers on this page, but please circle the letter of each correct response in the body of the exam. It is your responsibility to make it CLEAR which response has been chosen. You will not get credit unless the correct answer has been marked on both this page and in the body of the exam.

GOOD LUCK!

3. (a) (b) (c) (d) **(e)**
4. (a) (b) (c) (d) **(e)**
5. (a) (b) (c) **(d)** (e)
6. (a) **(b)** (c) (d) (e)
7. (a) **(b)** (c) (d) (e)
8. **(a)** (b) (c) (d) (e)
9. (a) (b) (c) **(d)** (e)
10. (a) **(b)** (c) (d) (e)
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13. **(a)** (b) (c) (d) (e)
14. (a) (b) (c) (d) **(e)**
15. (a) (b) (c) (d) **(e)**
16. (a) (b) **(c)** (d) (e)
17. **(a)** (b) (c) (d) (e)
18. (a) (b) (c) **(d)** (e)
19. (a) **(b)** (c) (d) (e)
20. (a) (b) (c) **(d)** (e)

For grading use:

Multiple Choice	Short Answer
(5 points each)	(out of 10 points)

Total	
	(out of 100 points)

Short Answer Questions

Write your answers on this page.

You must show proper, logical, sensible and legible work to be sure you will get full credit.

1. Find the **average rate of change** of $f(x)=5x^2+7$ from $x=2$ to $x=2+h$. Simplify your answer.

$$AROC = \frac{f(b)-f(a)}{b-a} = \frac{f(2+h)-f(2)}{2+h-2}$$

$$= \frac{5(2+h)^2+7-(5\cdot 2^2+7)}{h}$$

$$= \frac{5(4+4h+h^2)+7-27}{h} = \frac{20+20h+5h^2-20}{h}$$

$$= \frac{20h+5h^2}{h} = \frac{h(20+5h)}{h}$$

Final answer: $20+5h$

2. For the function $f(x)=x^2-5x+3$ find the **equation of the tangent line** to the graph of f at $x=6$.

Point: For $x=6$, $y=f(6)=6^2-5(6)+3=9$

Slope: $f'(x)=2x-5$

$m=f'(6)=2(6)-5=7$

point-slope form: $y-y_1=m(x-x_1)$

$y-9=7(x-6)$

$y-9=7x-42$

$y=7x-42+9$

Final answer: $\boxed{y=7x-33}$

Name: SOLUTIONS

Multiple Choice Questions

Show all your work on the page where the question appears.
Clearly mark your answer both on the cover page on this exam
and in the corresponding questions that follow.

3. Solve the equation $6x^2 + 102xy + 4y = 5$ for y in terms of x

Possibilities:

(a) $y = \dfrac{-102 \pm \sqrt{10308}}{12}$

(b) $y = \dfrac{5 - 6x^2 - 102x}{4}$

(c) $y = \dfrac{102x + 4}{6x^2 - 5}$

(d) $y = \dfrac{6x^2 - 5}{102x + 4}$

(e) $y = \dfrac{5 - 6x^2}{102x + 4}$

$6x^2 + 102xy + 4y = 5$

$102xy + 4y = 5 - 6x^2$

$y(102x + 4) = 5 - 6x^2$

$y = \dfrac{5 - 6x^2}{102x + 4}$

4. Evaluate $f(5)$ when $f(x)$ is given by the piecewise definition

$$f(x) = \begin{cases} x^2 - 2 & \text{if } x \leq 3 \\ 5x - 9 & \text{if } 3 < x \leq 7 \\ x^2 - 4x & \text{if } 7 < x \end{cases}$$

$x = 5$ is between 3 and 7

Possibilities:

(a) 5

(b) DNE

(c) 21

(d) 23

(e) 16

Thus $f(5) = 5(5) - 9$

$= 25 - 9$

$= 16$

5. A train travels from city A to city B, then travels from city B to city C. The train leaves city A a time 10:00am and arrives at city B at 12:30pm. The train leaves city B at 3:00pm and arrives at city C at 5:00pm. The average velocity of the train, while travelling from A to B, was 32 miles per hour. The average velocity of the train, while travelling from B to C, was 57 miles per hour. What was the average velocity of the train from city A to city C, including the wait at city B?

Possibilities:

(a) (201/7) miles per hour

(b) (89/2) miles per hour

(c) (25/2) miles per hour

(d) (194/7) miles per hour

(e) 89 miles per hour

From A to B:
velocity 32 mph, time 2.5 hours,
distance = $32(2.5) = 80$ miles

From B to C:
velocity 57 mph, time 2 hours,
distance = $57(2) = 114$ miles

Average velocity from A to C = $\frac{\text{Total distance}}{\text{total time}}$

↑ 10am to 5pm is 7 hours

$= \frac{80+114}{7 \text{ hours}} = \frac{194}{7}$ miles/hour

6. Find the average rate of change of $f(x) = \sqrt{x+3}$ from $x = 6$ to $x = 46$.

Possibilities:

(a) $-\frac{8}{23}$

(b) $\frac{1}{10}$

(c) 4

(d) $\frac{43}{46}$

(e) $-\frac{1}{10}$

$$\text{AROC} = \frac{f(b)-f(a)}{b-a}$$

$$= \frac{f(46)-f(6)}{46-6}$$

$$= \frac{\sqrt{46+3}-\sqrt{6+3}}{40}$$

$$= \frac{\sqrt{49}-\sqrt{9}}{40} = \frac{7-3}{40} = \frac{4}{40} = \frac{1}{10}$$

7. Find a value of x so that the instantaneous rate of change of $f(x) = 6x^2 + 5$ at x is equal to 24.

Possibilities:

(a) $x = 1$

(b) $x = 2$

(c) $x = 3$

(d) $x = 4$

(e) $x = 5$

"Instantaneous rate of change" is the derivative.

The formula on the last page says that if

$p(x) = Ax^2 + Bx + C$, then $p'(x) = 2Ax + B$.

For $f(x)$ we have $A = 6$, $B = 0$, $C = 5$.

Thus $f'(x) = 2(6)x + 0 = 12x$.

$12x = 24 \Rightarrow x = \frac{24}{12} = 2$.

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8. Let $f(x) = 6x^2 + 8x + 4$. Find a value c between $x = 1$ and $x = 5$, so that the average rate of change of $f(x)$ from $x = 1$ to $x = 5$ is equal to the instantaneous rate of change of $f(x)$ at $x = c$.

Possibilities:

(a) 3

(b) 4

(c) 5

(d) 6

(e) 7

AROC: $\frac{f(5) - f(1)}{5 - 1} = \frac{194 - 18}{4} = \frac{176}{4} = 44$.

For inst. rate of change, use deriv. formula above with $A = 6$, $B = 8$, $C = 4$; $f'(x) = 12x + 8$.

At $x = c$, $f'(c) = 12c + 8$.

We want c so these are equal:

$12c + 8 = 44$

$12c = 36$

$c = \frac{36}{12} = 3$. ← (this is between $x = 1$ and $x = 5$.)

9. If $\lim_{x\to 3} f(x) = 11$ and $\lim_{x\to 3} g(x) = 17$, then what is the value of $\lim_{x\to 3} \dfrac{(x+13)(f(x)+1)}{g(x)}$?

Possibilities:

(a) the limit is infinity or does not exist

(b) $\dfrac{11}{17}$

(c) $\dfrac{(3)(11)}{17}$

(d) $\dfrac{(3+13)(11+1)}{17}$

(e) 0

$= \dfrac{\lim_{x\to 3}(x+13) \cdot \lim_{x\to 3}(f(x)+1)}{\lim_{x\to 3} g(x)}$

$= \dfrac{(3+13)\cdot(11+1)}{17}$

(Since all limits exist, we are allowed to take the limit of each factor separately.)

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10. Find the limit

$$\lim_{t\to 0^+} \frac{46t^2}{t}$$

Test $t=0$: $\dfrac{46\cdot 0^2}{0} = \dfrac{0}{0}$,

do more work: cancel the t.

Possibilities:

(a) $\dfrac{23}{\sqrt{t}}$

(b) 0

(c) 23

(d) This limit either tends to infinity or this limit fails to exist

(e) 46

$= \lim_{t\to 0^+} (46t)$ ← test again: $46(0) = 0$.

no longer have zero in the denominator, so we have an answer:

Limit is 0.

11. Find the limit

$$\lim_{x\to 0}\left(\frac{12}{x}+\frac{6x-12}{x}\right)$$

separately these limits do not exist, so we cannot split and take them one at a time.

Possibilities:

(a) 12

(b) This limit does not exist.

(c) 6

(d) 0

(e) 1

$= \lim_{x\to 0} \frac{12+6x-12}{x}$ ← already have a common denominator; just add the fractions.

$= \lim_{x\to 0} \frac{6x}{x}$ ← combine terms. now cancel the x's

$= \lim_{x\to 0} 6 = \boxed{6}$

12. Find the limit

$$\lim_{n\to\infty}\frac{(n+3)^2}{7n+17}$$

Possibilities:

(a) $\frac{1}{7}$

(b) $\frac{1}{24}$

(c) The limit does not exist or approaches infinity

(d) $\frac{1}{17}$

(e) $\frac{9}{7}$

$= \lim_{n\to\infty} \frac{n^2+6n+9}{7n+17}$ "FOIL"

$= \lim_{n\to\infty} \frac{n^2}{7n}$ Since $n\to\infty$, only terms with highest power matter

$= \lim_{n\to\infty} \frac{n}{7}$ cancel one n

$= \lim_{n\to\infty} \frac{1}{7}n$ ← as $n\to\infty$, so does $\frac{1}{7}n$.

13. For the function

$$f(x) = \begin{cases} |3+9x| & \text{if } x < -2 \\ \sqrt{x^2+5} & \text{if } -2 \le x < 6 \\ 7x^2+3x+4 & \text{if } 6 \le x \end{cases}$$

find $\lim_{x \to 7^+} f(x)$

Since $x = 7$ is larger than 6, only the third line is relevant

Possibilities:

(a) 368

(b) 66

(c) $\sqrt{41}$

(d) 274

(e) $\sqrt{54}$

$$\lim_{x \to 7^+} f(x) = \lim_{x \to 7^+} (7x^2 + 3x + 4)$$

$$= 7 \cdot 7^2 + 3(7) + 4$$

$$= 343 + 21 + 4$$

$$= 368$$

14. The graph of $y = f(x)$ is shown below. Compute $\lim_{x \to -2^-} f(x)$.

Possibilities:

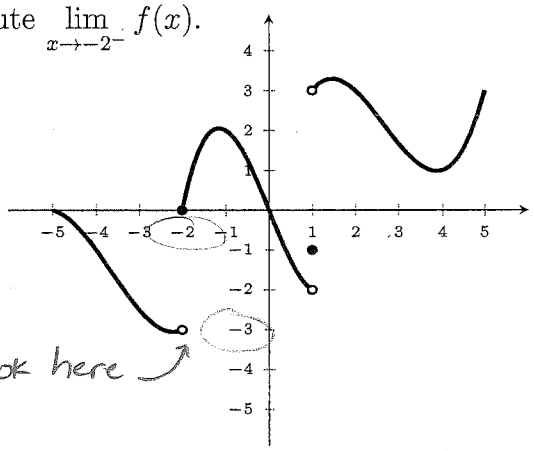
(a) 3

(b) 0

(c) -1

(d) -2

(e) -3



$\lim_{x \to -2^-} f(x)$ means we look for the y-value as we approach $x = -2$ from the left (from values smaller than $x = -2$).

15. Consider the function $f(x) = \begin{cases} x^2 - 2 & \text{if } x < 2 \\ 2x + B & \text{if } x \geq 2 \end{cases}$

Find a value of B so that the function is continuous at $x = 2$.

Possibilities:

(a) -6

(b) -5

(c) -4

(d) -3

(e) -2

To be continuous at $x=2$ we require $\lim_{x\to 2} f(x)$ to exist: it must match from the left and right.

$$\lim_{x\to 2^-} f(x) = \lim_{x\to 2^-} (x^2 - 2) = 2^2 - 2 = 2$$

must match

$$\lim_{x\to 2^+} f(x) = \lim_{x\to 2^+} (2x + B) = 2(2) + B = 4 + B$$

$$4 + B = 2 \quad \Rightarrow \quad B = 2 - 4 = \underline{-2}.$$

16. Find the value of m which makes $f(x)$ differentiable everywhere, where

$$f(x) = \begin{cases} x^2, & \text{if } x \leq 4; \\ m(x-4) + 16, & \text{if } x > 4 \end{cases}$$

Possibilities:

(a) 6

(b) 7

(c) 8

(d) 9

(e) 10

For $f(x)$ to be differentiable at $x=4$, both the function values and the derivative values must match from the left and right.

Function values: $\lim_{x\to 4^-} f(x) = \lim_{x\to 4^-} x^2 = 16$. ← These match no matter what we choose for m.

$$\lim_{x\to 4^+} f(x) = \lim_{x\to 4^+} m(x-4) + 16 = m(0) + 16 = 16$$

Deriv. values: For $x \leq 4$, $f'(x) = 2x$. (use quadratic formula.)

For $x > 4$, $f'(x) = m$, because f is a line and deriv. gives slope.

at $x=4$ these should match: $2x = m$

$2(4) = m$

$B = m$

17. Find the equation of the tangent line to the graph of the function $f(x) = \dfrac{1}{x^2+1} + 5$ at $x = 3$. You may use $f'(x) = -\dfrac{2x}{(x^2+1)^2}$

Possibilities:

(a) $y = -\dfrac{3}{50}x + \dfrac{132}{25}$

(b) $y = x^3 + 17$

(c) $y = -\dfrac{3}{50}x + \dfrac{51}{10}$

(d) $y = \dfrac{51}{10}x - \dfrac{384}{25}$

(e) $y = \dfrac{51}{10}$

We need a point and a slope.

Point: we have $x = 3$.

$$y = f(3) = \frac{1}{3^2+1} + 5 = \frac{1}{10} + 5 = \frac{51}{10}$$

Slope comes from the derivative.

$$f'(x) = -\frac{2x}{(x^2+1)^2} \text{ is given.}$$

$$m = f'(3) = -\frac{2(3)}{(3^2+1)^2} = -\frac{6}{10^2} = \frac{-6}{100} = \frac{-3}{50}$$

Line equation: point-slope form $y - y_1 = m(x - x_1)$

$$y - \frac{51}{10} = \frac{-3}{50}(x-3)$$

$$y = \frac{-3}{50}x + \frac{9}{50} + \frac{51}{10}$$

$$y = \frac{-3}{50}x + \frac{9}{50} + \frac{255}{50}$$

$$y = \frac{-3}{50}x + \frac{264}{50}$$

18. Consider the function $f(x) = 9x^2 + 8x + 2$. Its tangent line at $x = 3$ goes through the point $(9, y_1)$ where y_1 is:

Possibilities:

(a) 107

(b) -79

(c) 62

(d) 479

(e) 170

Find eqn of tang line.

point: we have $x = 3$,

$$y = f(3) = 9 \cdot 3^2 + 8(3) + 2 = 107$$

Slope comes from deriv; using formula for quadratics,

$$f'(x) = 18x + 8$$

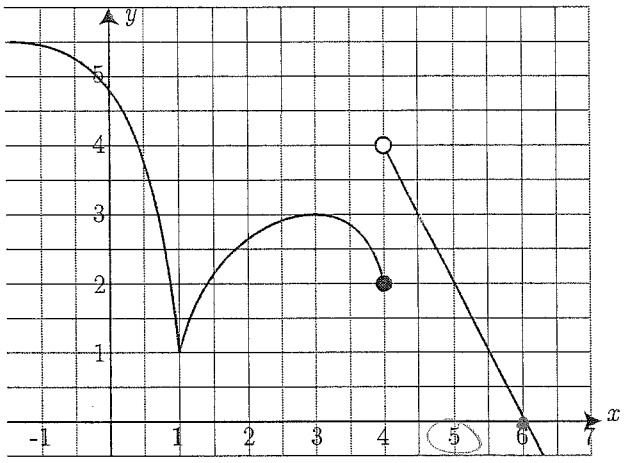
$$m = f'(3) = 18(3) + 8 = 62$$

$$y - 107 = 62(x - 3)$$

To answer their question, choose $x = 9$.

$$y - 107 = 62(9-3) \Rightarrow y = 107 + 372 = 479.$$

19. The graph of $y = f(x)$ is shown below. $f'(\frac{11}{2})$ is approximately :



Possibilities:

(a) 2

(b) -2

(c) $-\frac{1}{2}$

(d) The limit does not exist or tends to infinity

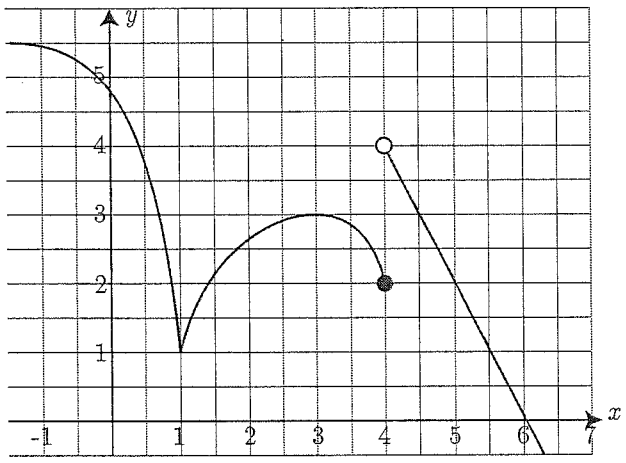
(e) $\frac{1}{2}$

At $x = \frac{11}{2} = 5.5$, the graph is a straight line. $f'(\frac{11}{2})$ is the slope of this line.

Find any two points on the line, e.g., (4, 4) and (6, 0);

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4 - 0}{4 - 6} = \frac{4}{-2} = -2$$

20. The graph of $y = f(x)$ is shown below. The function is differentiable, except at $x =$



Possibilities:

(a) x=1 only

(b) x=1, x=3, and x=4

(c) x=4 only

(d) x=1 and x=4

(e) x=1, x=3, x=4, and x=6

Look for

- any points of discontinuity, such as $x = 4$
- any "sharp points", such as $x = 1$
- vertical tangents (none pictured)

Please make sure to list the correct section number on the front page of your exam.
In case you forgot your section number, consult the following table.

Section	Instructor	Day	Time	Room
001-007	Jack Schmidt	MWF	2:00 pm - 2:50 pm	BS 107
001	Joseph Lindgren	Tu	8:00 am	CB 349
002	Joseph Lindgren	Tu	9:30 am	FB 213
003	Florian Kohl	Tu	11:00 am	FB 213
004	Carolyn Troha	Tu	12:30 pm	FB 213
005	Sarah Orchard	Tu	2:00 pm	FB 213
006	Sarah Orchard	Tu	3:30 pm	FB 213
007	Jing Wei	Tu	11:00 am	TPC 113
008-014	Jack Schmidt	MWF	12:00 pm - 12:50 pm	CB 106
008	Joseph Lindgren	Th	8:00 am	CB 349
009	Joseph Lindgren	Th	9:30 am	FB 213
010	Sarah Orchard	Th	11:00 am	FB 213
011	Sarah Orchard	Th	12:30 pm	FB 213
012	Yucong Sang	Th	2:00 pm	FB 213
013	Carolyn Troha	Th	3:30 pm	FB 213
014	Hao Wang	Th	11:00 am	TEB 231
015-021	erica Whitaker	MWF	1:00 pm - 1:50 pm	CB 118
015	Jing Wei	Tu	8:00 am	FB 213
016	Yucong Sang	Th	3:30 pm	CB 201
017	Carolyn Troha	Tu	11:00 am	CP 111
018	Yucong Sang	Tu	12:30 pm	DH 203
019	Jing Wei	Tu	2:00 pm	TPC 109
020	Hao Wang	Tu	3:30 pm	CB 240
021	Yucong Sang	Tu	3:30 pm	CB 246
022-028	erica Whitaker	MWF	2:00 pm - 2:50 pm	CB 106
022	Jing Wei	Th	8:00 am	FB 213
023	Hao Wang	Th	8:00 am	CB 303
024	Florian Kohl	Th	11:00 am	CB 234
025	Florian Kohl	Th	12:30 pm	CB 234
026	Hao Wang	Tu	8:00 am	CB 303
027	Florian Kohl	Th	3:30 pm	CB 244
028	Carolyn Troha	Tu	3:30 pm	CB 201

You may use the following formula for the derivative of a quadratic function.

If $p(x) = Ax^2 + Bx + C$, then $p'(x) = 2Ax + B$.