

Do not remove this answer page — you will turn in the entire exam. You have two hours to do this exam. No books or notes may be used. You may use a graphing calculator during the exam, but NO calculator with a Computer Algebra System (CAS) or a QWERTY keyboard is permitted. Absolutely no cell phone use during the exam is allowed.

The exam consists of multiple choice questions. Record your answers on this page. For each multiple choice question, you will need to fill in the box corresponding to the correct answer. For example, if (b) is correct, you must write

☐ a ☒ b ☐ c ☐ d ☐ e

Do not circle answers on this page, but please circle the letter of each correct response in the body of the exam. It is your responsibility to make it CLEAR which response has been chosen. You will not get credit unless the correct answer has been marked on both this page and in the body of the exam.

GOOD LUCK!

1. ☐ a ☐ b ☒ c ☐ d ☐ e
2. ☐ a ☒ b ☐ c ☐ d ☐ e
3. ☐ a ☐ b ☐ c ☐ d ☒ e
4. ☐ a ☐ b ☐ c ☒ d ☐ e
5. ☐ a ☐ b ☐ c ☒ d ☐ e
6. ☐ a ☐ b ☒ c ☐ d ☐ e
7. ☐ a ☐ b ☒ c ☐ d ☐ e
8. ☐ a ☐ b ☒ c ☐ d ☐ e
9. ☐ a ☐ b ☐ c ☐ d ☒ e
10. ☐ a ☐ b ☒ c ☐ d ☐ e

11. ☐ a ☒ b ☐ c ☐ d ☐ e
12. ☐ a ☐ b ☐ c ☐ d ☒ e
13. ☒ a ☐ b ☐ c ☐ d ☐ e
14. ☐ a ☐ b ☐ c ☐ d ☒ e
15. ☐ a ☐ b ☒ c ☐ d ☐ e
16. ☐ a ☐ b ☐ c ☒ d ☐ e
17. ☒ a ☐ b ☐ c ☐ d ☐ e
18. ☐ a ☐ b ☐ c ☒ d ☐ e
19. ☐ a ☐ b ☒ c ☐ d ☐ e
20. ☐ a ☒ b ☐ c ☐ d ☐ e

For grading use:

Number Correct	
	(out of 20 problems)

Total	
	(out of 100 points)

Please make sure to list the correct section number on the front page of your exam. In case you forgot your section number, consult the following table. Your section number is determined by your recitation time and location.

Section #	Instructor	Day and Time	Room
001	J. Constable	T, 8:00 am-9:15 am	FB B2
002	J. Constable	T, 9:30 am-10:45 am	NURS 501C
003	J. Constable	T, 11:00 am-12:15 pm	TPC 212
004	L. Davidson	T, 12:30 pm-1:45 pm	CP 111
005	L. Davidson	T, 2:00 pm-3:15 pm	CB 219
006	L. Davidson	T, 3:30 pm-4:45 pm	CB 341
007	W. Hough	R, 8:00 am-9:15 am	FB B2
008	W. Hough	R, 9:30 am-10:45 am	MMRB 243
009	W. Hough	R, 11:00 am-12:15 pm	TPC 212
010	X. Kong	R, 12:30 pm-1:45 pm	Laferty 201
011	X. Kong	R, 2:00 pm-3:15 pm	FPAT 257
012	X. Kong	R, 3:30 pm-4:45 pm	CB 341
013	L. Solus	T, 8:00 am-9:15 am	CB 303
014	K. Effinger	T, 8:00 am-9:15 am	CB 233
015	K. Effinger	T, 11:00 am-12:15 pm	CB 347
016	Q. Liang	T, 12:30 pm-1:45 pm	NURS 501C
017	Q. Liang	T, 2:00 pm-3:15 pm	CB 247
018	Q. Liang	T, 3:30 pm-4:45 pm	CB 245
019	K. Effinger	R, 8:00 am-9:15 am	CB 303
020	L. Solus	R, 8:00 am-9:15 am	CB 233
021	L. Solus	R, 3:30 pm-4:45 pm	CB 214
022	A. Happ	R, 12:30 pm-1:45 pm	NURS 501C
023	A. Happ	R, 2:00 pm-3:15 pm	CB 338
024	A. Happ	R, 3:30 pm-4:45 pm	CB 245
025	F. Smith	T, 12:30 pm-1:45 pm	FPAT 263
026	D. Akers	R, 12:30 pm-1:45 pm	FPAT 263
027	F. Smith	T, 2:00 pm-3:15 pm	FPAT 259
028	D. Akers	R, 2:00 pm-3:15 pm	FPAT 259
029	F. Smith	T, 3:30 pm-4:45 pm	CB 205
030	D. Akers	R, 3:30 pm-4:45 pm	CB 205

Multiple Choice Questions

Show all your work on the page where the question appears.
Clearly mark your answer both on the cover page on this exam
and in the corresponding questions that follow.

1. If

$$f(x) = \frac{-1}{x+2}$$

then

$$\frac{f(x+h) - f(x)}{h} = \frac{1}{(x+2)(x+Ch+2)}$$

Determine the value of C.

Possibilities:

(a) -1

(b) 0

(c) 1

(d) 2

(e) 3

$$\begin{aligned} f(x+h) &= \frac{-1}{x+h+2} \\ \text{So } \frac{f(x+h) - f(x)}{h} &= \frac{1}{h} \left[\frac{-1}{x+h+2} - \frac{-1}{x+2} \right] \\ &= \frac{1}{h} \left[\frac{-(x+2) + (x+h+2)}{(x+2)(x+h+2)} \right] \\ &= \frac{1}{h(x+2)(x+h+2)} = \frac{1}{(x+2)(x+h+2)} \quad \text{C=1} \end{aligned}$$

2. Suppose

$$\frac{f(x+h) - f(x)}{h} = 7x^2 - 3h + 8$$

Determine the slope of the tangent line to $y = f(x)$ at $x = 2$.

Possibilities:

(a) 35

(b) 36

(c) 37

(d) 38

(e) 39

$$\begin{aligned} \text{Slope T-L} &= f'(2) = \lim_{h \rightarrow 0} \frac{f(2+h) - f(2)}{h} \\ &= \lim_{h \rightarrow 0} 7 \cdot 2^2 - 3h + 8 \\ &= 28 + 8 = 36 \end{aligned}$$

3. Determine the instantaneous rate of change of $f(x) = x^3 - x^2 - 3x + 6$ at $x = 2$.

Possibilities:

(a) 1

(b) 2

(c) 3

(d) 4

(e) 5

$$f'(x)$$

$$f'(x) = 3x^2 - 2x - 3$$

$$f'(2) = 3 \cdot 2^2 - 2 \cdot 2 - 3 = 5$$

4. The equation of the tangent line to the graph of $y = g(x)$ at $x = 3$ is given by $f(x) = 3(x - 3) + 5$. Determine $f'(3)$.

Determine $f'(3)$

This should have read: Determine $g'(3)$

Possibilities:

(a) 5

(b) 15

(c) 8

(d) 3

(e) $3x$

As stated, this just asks you to determine derivative of linear function,

$$[3(x - 3) + 5]' = 3$$

5. Find the derivative of

$$f(x) = \frac{x^2 - 2x - 15}{x + 3}$$

Note:
$$\frac{(x^2 - 2x - 15)}{x + 3} = \frac{(x + 3)(x - 5)}{x + 3}$$

Possibilities:

(a) -2

(b) -1

(c) 0

(d) 1

(e) 2

So
$$f'(x) = (x - 5)' = 1$$

6. Determine the derivative of

$$f(s) = (s^2 + 5s + 2)^6$$

Possibilities:

(a) $5(s^2 + 5s + 2)^6$

(b) $6(s^2 + 5s + 2)^5$

(c) $6(s^2 + 5s + 2)^5(2s + 5)$

(d) $6(2s + 5)^5(s^2 + 5s + 2)$

(e) $6(2s + 5)^5$

$$f'(s) = 6(s^2 + 5s + 2)^5 \cdot (2s + 5)$$

7. Suppose $f(4) = 4$, $g(4) = 2$, $f'(4) = 7$ and $g'(4) = 5$. Also, let $H(x) = f(x) \cdot g(x)$. Determine $H'(4)$.

Possibilities:

(a) 8

(b) 18

(c) 34

(d) 38

(e) 35

$$\begin{aligned} H'(4) &= f'(4)g(4) + f(4)g'(4) \\ &= 7 \cdot 2 + 4 \cdot 5 \\ &= 34 \end{aligned}$$

8. Suppose $h(x) = \sqrt{f(x)}$ and the equation of the tangent line to $f(x)$ at $x = 7$ is given by $y = 4 + 3(x - 7)$. Find $h'(7)$.

Possibilities:

(a) 3

(b) 2

(c) $3/4$

(d) $\sqrt{3}/6$

(e) 4

$$\begin{aligned} f(7) &= 4 \quad f'(7) = 3 \quad h'(x) = \frac{1}{2} f(x)^{-1/2} \cdot f'(x) = \frac{f'(x)}{2\sqrt{f(x)}} \\ \text{so } h'(7) &= \frac{f'(7)}{2\sqrt{f(7)}} = \frac{3}{2\sqrt{4}} = \frac{3}{4} \end{aligned}$$

9. Suppose $F(x) = (2x - 3)^5$. Find the third derivative, $F^{(3)}(2)$.

Possibilities:

- (a) 100
- (b) 1000
- (c) 60
- (d) 80
- (e) 480

$$\begin{aligned} F'(x) &= 5(2x-3)^4 \cdot 2 = 10(2x-3)^4 \\ F''(x) &= 10 \cdot 4(2x-3)^3 \cdot 2 = 80(2x-3)^3 \\ F'''(x) &= 80 \cdot 3(2x-3)^2 \cdot 2 = 480(2x-3)^2 \\ F'''(2) &= 480(2 \cdot 2 - 3)^2 = 480 \end{aligned}$$

10. Find $f'(x)$, where

$$f(x) = e^{2\sqrt{x}}$$

Possibilities:

- (a) $2x^{-1/2}e^{2\sqrt{x}}$
- (b) $2\sqrt{x}e^{2\sqrt{x}-1}$
- (c) $x^{-1/2}e^{2\sqrt{x}}$
- (d) $x^{1/2}e^{2\sqrt{x}}$
- (e) $e^{1x^{-1/2}}$

$$\begin{aligned} f'(x) &= (2\sqrt{x})' e^{2\sqrt{x}} \\ &= 2 \cdot \left(\frac{1}{2}x^{-1/2}\right) e^{2\sqrt{x}} \\ &= x^{-1/2} e^{2\sqrt{x}} \end{aligned}$$

11. Find $f'(x)$, where

$$f(x) = \ln(6x^2 + 7x + 7)$$

Possibilities:

- (a) $\frac{6x^2 + 7x + 7}{12x + 7}$
- (b) $\frac{12x + 7}{6x^2 + 7x + 7}$
- (c) $\frac{1}{12x + 7}$
- (d) e^{12x+7}
- (e) $\frac{1}{6x^2 + 7x + 7}$

$$\begin{aligned} f'(x) &= \frac{(6x^2 + 7x + 7)'}{6x^2 + 7x + 7} \\ &= \frac{12x + 7}{6x^2 + 7x + 7} \end{aligned}$$

12. Let

$$f(x) = (x - 1)^5$$

Compute

$$\lim_{h \rightarrow 0} \frac{f(5+h) - f(5)}{h}$$

This is just the definition of $f'(5)$!

Possibilities:

- (a) 1276
- (b) 1277
- (c) 1278
- (d) 1279
- (e) 1280

$$f'(5) = 5 \cdot (5-1)^4 = 5 \cdot 4^4$$

13. Find the second derivative, $f^{(2)}(x)$, where

$$f(x) = e^{5x^2}$$

Possibilities:

- (a) $10e^{5x^2} + 100x^2e^{5x^2}$
- (b) $100e^{5x^2}$
- (c) $4x^2e^{5x^2}$
- (d) $25e^{5x^2}$
- (e) $10e^{5x^2} + 10x^2e^{5x^2}$

$$\begin{aligned} f'(x) &= (5x^2)' e^{5x^2} = 10x e^{5x^2} \\ f''(x) &= (10x)' e^{5x^2} + (10x) \cdot (e^{5x^2})' \\ &= 10e^{5x^2} + 10x (10x e^{5x^2}) \\ &= 10e^{5x^2} + 100x^2 e^{5x^2} \end{aligned}$$

14. The population of a small country grows exponentially, with a growth rate of 4.0% per year. Suppose the population will be 3 million people in 45 years. What is the current population (measured in millions)?

Possibilities:

- (a) $3e^{1.800}$ million people
- (b) $3e^{180.0}$ million people
- (c) $3e^{-180.0}$ million people
- (d) $3e^{-.40}$ million people
- (e) $3e^{-1.800}$ million people

$$P(t) = P_0 e^{rt} = P_0 e^{0.04t}$$

$$\text{Know } P(45) = 3 \text{ million}$$

$$\text{so } P(45) = P_0 e^{.04 \cdot 45}$$

$$\Rightarrow P_0 e^{1.8} = 3 \text{ million}$$

$$\text{so } P_0 = 3e^{-1.8} \text{ mill.}$$

15. Find the derivative, $f'(x)$, where

$$f(x) = \frac{\ln(x+4)}{x+4}$$

Possibilities:

(a) $\frac{1}{x+4}$

(b) $\frac{\ln(x+4) - 1}{(x+4)^2}$

(c) $\frac{1 - \ln(x+4)}{(x+4)^2}$

(d) $\frac{e^{x+4}}{x+4}$

(e) $\frac{1}{(x+4)^2}$

$$\begin{aligned} f'(x) &= \frac{(x+4)(\ln(x+4))' - \ln(x+4) \cdot (x+4)'}{(x+4)^2} \\ &= \frac{(x+4) \frac{1}{x+4} - \ln(x+4)}{(x+4)^2} \\ &= \frac{1 - \ln(x+4)}{(x+4)^2} \end{aligned}$$

16. If the number of bacteria in a culture doubles every 10 hours, how many hours will it take before 25 times the original number is present? (HINT: The number of bacteria at time t follows an exponential model, $y(t) = P_0 e^{rt}$. You may need to find the value of r before you can solve this problem.)

Possibilities:

(a) 4

(b) $5/2$

(c) $\frac{\ln(2)}{10}$

(d) $\frac{10 \ln(25)}{\ln(2)}$

(e) $\frac{25 \ln(10)}{\ln(2)}$

Find r : $2P_0 = P_0 e^{10r} \Rightarrow \ln(2) = 10r$
 so $r = \frac{\ln(2)}{10}$

Now find t :

$$\begin{aligned} P(t) &= 25P_0 = P_0 e^{\frac{\ln(2)}{10} t} \\ \Rightarrow 25 &= e^{\frac{\ln(2)}{10} t} \Rightarrow \ln(25) = \frac{\ln(2)}{10} t \\ \Rightarrow t &= \frac{10}{\ln(2)} \ln(25) \end{aligned}$$

17. Let $f(t) = t^3$. Find a value c in the interval $(5, 10)$ so that the average rate of change of $f(t)$ from $t = 5$ to $t = 10$ is equal to the instantaneous rate of change of $f(t)$ at $t = c$.

Possibilities:

(a) $\sqrt{(175/3)}$

(b) $\sqrt{(175/2)}$

(c) 875

(d) $175/3$

(e) $175/2$

$$AROC = \frac{f(10) - f(5)}{10 - 5} = \frac{10^3 - 5^3}{5} = \frac{1000 - 125}{5} = 175$$

$$Inst. ROC = 3t^2$$

$$\begin{aligned} \text{Want } 3c^2 &= 175 \\ c^2 &= \frac{175}{3} \Rightarrow c = \sqrt{\frac{175}{3}} \end{aligned}$$

Endpoints or critical points

18. Find the value of x in the interval $[4, 6]$ where $f(x) = 2x^3 - 18x^2 + 30x + 10$ attains its minimum value.

Possibilities:

- (a) Minimum occurs at $x = 6$
- (b) Minimum occurs at $x = 4$
- (c) Minimum occurs at $x = -26$
- (d) Minimum occurs at $x = -5$
- (e) Minimum occurs at $x = -40$

$$f'(x) = 6x^2 - 36x + 30 = 6(x^2 - 6x + 5) = 6(x-1)(x-5)$$

c.p. are $x=1$ & 5 , But 1 not in $[4, 6]$

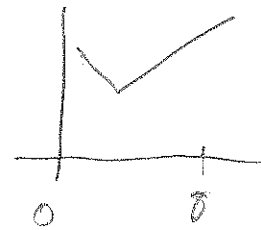
$$f(4) = -30$$
$$f(5) = -40 \text{ Min at } x=5$$
$$f(6) = -26$$

19. Find the maximum value of $f(x)$ on $[0, 8]$ where $f(x) = |x - 2| + 13$.

Possibilities: Max at an endpoint

- (a) 0
- (b) 15
- (c) 19
- (d) 13
- (e) 8

$$f(0) = |0 - 2| + 13 = 15$$
$$f(8) = |8 - 2| + 13 = 19$$



20. Which statement most accurately describes the Extreme Value Theorem?

Possibilities:

- (a) A continuous function on a closed and bounded interval may or may not attain a maximum value and may or may not attain a minimum value.
- (b) A continuous function on a closed and bounded interval must attain a maximum value and a minimum value.
- (c) A continuous function on an open and bounded interval must attain a maximum value and a minimum value.
- (d) A discontinuous function on a closed and bounded interval cannot attain a maximum value or a minimum value.
- (e) A continuous function on an unbounded interval cannot attain a maximum value or a minimum value.