MA123 — Elem. Calculus Fall 2013
Exam 2 2013-10-24

Name: Solutions **Sec.:** _____

Do not remove this answer page — you will turn in the entire exam. You have two hours to do this exam. No books or notes may be used. You may use a graphing calculator during the exam, but NO calculator with a Computer Algebra System (CAS) or a QWERTY keyboard is permitted. Absolutely no cell phone use during the exam is allowed.

The exam consists of multiple choice questions. Record your answers on this page. For each multiple choice question, you will need to fill in the box corresponding to the correct answer. For example, if (a) is correct, you must write

(a) b c d e

Do not circle answers on this page, but please circle the letter of each correct response in the body of the exam. It is your responsibility to make it CLEAR which response has been chosen. You will not get credit unless the correct answer has been marked on both this page and in the body of the exam.

GOOD LUCK!

1.	a	**b**	c	d	e	11.	**a**	b	c	d	e
2.	a	b	c	**d**	e	12.	a	**b**	c	d	e
3.	**a**	b	c	d	e	13.	a	b	c	d	**e**
4.	**a**	b	c	d	e	14.	a	b	c	d	**e**
5.	a	b	c	**d**	e	15.	a	**b**	c	d	e
6.	a	**b**	c	d	e	16.	a	b	**c**	d	e
7.	**a**	b	c	d	e	17.	a	b	**c**	d	e
8.	a	b	**c**	d	e	18.	a	b	**c**	d	e
9.	**a**	b	c	d	e	19.	**a**	b	c	d	e
10.	a	b	**c**	d	e	20.	a	b	**c**	d	e

For grading use:

Number Correct	(out of 20 problems)

Total	(out of 100 points)

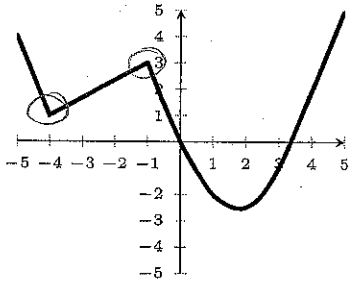
Please make sure to list the correct section number on the front page of your exam. In case you forgot your section number, consult the following table. Your section number is determined by your recitation time and location.

Section #	Instructor	Day	Time	Room
001-006	Jack Schmidt	MWF	08:00 am - 08:50 am	KAS 213
001	Jinping Zhuge	T	8:00 am - 9:15 am	FB B9
002	Yiyuan Wu	T	9:30 am - 10:45 am	NURS 501B
003	Devin Willmott	T	8:00 am - 9:15 am	CB 235
004	Tefjol Pllaha	T	8:00 am - 9:15 am	CB 237
005	Tefjol Pllaha	T	2:00 pm - 3:15 pm	CB 347
006	Tefjol Pllaha	T	3:30 pm - 4:45 pm	CB 347
007-012	Jack Schmidt	MWF	09:00 am - 09:50 am	KAS 213
007	Yiyuan Wu	R	8:00 am - 9:15 am	CB 217
008	Jinping Zhuge	R	9:30 am - 10:45 am	DH 323
009	Yiyuan Wu	R	11:00 am - 12:15 pm	EH 202
010	Jinping Zhuge	R	12:30 pm - 1:45 pm	DH 323
011	Dharma Maharjan	R	2:00 pm - 3:15 pm	CB 347
012	Dharma Maharjan	R	3:30 pm - 4:45 pm	CB 347
013-018	Paul Koester	MWF	1:00 pm - 1:50 pm	BS 116
013	Carolyn Troha	T	8:00 am - 9:15 am	CB 345
014	Carolyn Troha	T	9:30 am - 10:45 am	NURS 214
015	Morgan Schreffler	T	11:00 am - 12:15 pm	EH 202
016	Carolyn Troha	T	12:30 pm - 1:45 pm	MMRB 243
017	Morgan Schreffler	T	2:00 pm - 3:15 pm	BH 301
018	Morgan Schreffler	T	3:30 pm - 4:45 pm	CB 235
025-030	Paul Koester	MWF	2:00 pm - 2:50 pm	BS 107
025	Sarah Orchard	T	12:30 pm - 1:45 pm	TPC 212
026	Marie Meyer	R	8:00 am - 9:15 am	CB 240
027	Marie Meyer	T	2:00 pm - 3:15 pm	DH 331
028	Marie Meyer	R	2:00 pm - 3:15 pm	EH 304
029	Sarah Orchard	T	3:30 pm - 4:45 pm	OT OB7
030	Sarah Orchard	R	3:30 pm - 4:45 pm	OT 0B7
401	Brad Schwer	MTR	5:30 pm - 6:45 pm	CB 337

Multiple Choice Questions

Show all your work on the page where the question appears.
Clearly mark your answer both on the cover page on this exam and in the corresponding questions that follow.

1. The graph of $y = f(x)$ is shown below. The function is differentiable, except at:



Possibilities:

(a) 2 ~~only~~

(b) -4 and ~~-1~~

(c) -4 and 2

(d) -1 only

(e) -4 only

2. If $f(x) = \frac{1}{x+1}$ then choose the simplified form of $\frac{f(x+h)-f(x)}{h}$:

Possibilities:

(a) $\frac{1}{(x+h+1)(x+1)}$

(b) $-\frac{1}{(x+h+1)^2}$

(c) $\frac{2x+2+h}{(x+h+1)(x+1)(2x+h)}$

(d) $-\frac{1}{(x+h+1)(x+1)}$

(e) $\frac{hx^2+2hx+h-1}{(x+1)^2}$

$f(x+h) = \frac{1}{x+h+1}$

$\frac{f(x+h)-f(x)}{h} = \frac{\frac{1}{x+h+1} - \frac{1}{x+1}}{h} = \frac{(x+1)-(x+h+1)}{(x+h+1)(x+1)} \div h$

$= \frac{-h}{(x+h+1)(x+1)} \div h = \frac{-h}{(x+h+1)(x+1)} \cdot \frac{1}{h}$

$= \frac{-h}{h(x+h+1)(x+1)} = \frac{-1}{(x+h+1)(x+1)}$

3. If $f(x) = \sqrt{x+2}$ then choose the simplified form of $\frac{f(x+h)-f(x)}{h}$:

Possibilities:

(a) $\frac{1}{\sqrt{x+h+2}+\sqrt{x+2}}$

(b) $\frac{\sqrt{x+h+2}+\sqrt{x+2}}{h}$

(c) $\frac{\frac{1}{2}}{\sqrt{x+h+2}}$

(d) $\frac{h\sqrt{x+2}+\frac{1}{2}}{\sqrt{x+2}}$

(e) $\frac{1}{2}\sqrt{x+h+2} - \frac{1}{2}\sqrt{x+2}$

$= \frac{\sqrt{x+h+2} - \sqrt{x+2}}{h} = \frac{(\sqrt{x+h+2} - \sqrt{x+2})(\sqrt{x+h+2} + \sqrt{x+2})}{h(\sqrt{x+h+2} + \sqrt{x+2})}$

$= \frac{(x+h+2)-(x+2)}{h(\sqrt{x+h+2} + \sqrt{x+2})} = \frac{h}{h(\sqrt{x+h+2} + \sqrt{x+2})}$

$= \frac{1}{\sqrt{x+h+2} + \sqrt{x+2}}$

4. If $f(x) = 8x^3$ then choose the simplified form of $\frac{f(x+h)-f(x)}{h}$:

Possibilities:

(a) $8h^2 + 24xh + 24x^2$

(b) $-8h^2 - 24xh - 24x^2$

(c) $24x^2 + h$

(d) $8h^2 + 8xh + 8x^2$

(e) $24\,(x+h)^2$

$f(x+h) = 8(x+h)^3 = 8(x^3 + 3x^2h + 3xh^2 + h^3)$

so

$$\frac{f(x+h)-f(x)}{h} = \frac{8(x^3 + 3x^2h + 3xh^2 + h^3) - 8x^3}{h}$$

$$= \frac{24x^2h + 24xh^2 + 8h^3}{h} = \frac{(24x^2 + 24xh + 8h^2)h}{h}$$

$$= 24x^2 + 24xh + 8h^2$$

5. Find the derivative, $f'(x)$, of $f(x) = \frac{1}{x^8}$ $= x^{-8}$

Possibilities:

(a) $1/(8\,x^9)$

(b) $8x^7$

(c) $1/(8\,x^7)$

(d) $-8x^{-9}$

(e) $-8x^{-7}$

$f'(x) = -8x^{-9}$

6. Find the derivative, $f'(x)$, of $f(x) = x^{(8/3)}$

Possibilities:

(a) $(5/3)x^{8/3}$

(b) $(8/3)x^{5/3}$

(c) $(8/3)x^{7/3}$

(d) $(5/3)x^{5/3}$

(e) $(8/3)x^{11/3}$

$f'(x) = \frac{8}{3}x^{\frac{8}{3}-1}$

$= \frac{8}{3}x^{5/3}$

Product Rule

Chain Rule
$(x+2)' = 1$

7. Suppose $F(x) = g(x) \cdot h(x+2)$. If $g(0) = 5$, $g'(0) = 8$, $h(0) = 9$, $h'(0) = 3$, $h(2) = 6$, and $h'(2) = 7$, find $F'(0)$.

Possibilities:

(a) 83
(b) 67
(c) 38
(d) 94
(e) 118

$F'(x) = g'(x) \cdot h(x+2) + g(x) \cdot h'(x+2) \cdot 1$

$F'(0) = g'(0) \cdot h(0+2) + g(0) h'(0+2)$

$= g'(0) h(2) + g(0) h'(2)$

$= 8 \cdot 6 + 5 \cdot 7 = 83$

8. Suppose $F(x) = (g(x))^3 + 5$. If $g(2) = 11$, $g'(2) = 9$, and $g''(2) = 7$, then find $F'(2)$.

Possibilities: Chain

(a) 1336
(b) 368
(c) 3267
(d) 7
(e) 734

$F'(x) = 3 g(x)^2 \cdot g'(x)$

$F'(2) = 3 \cdot g(2)^2 g'(2)$

$= 3 \cdot 11^2 \cdot 9 = 3267$

9. If $f(x) = 3x^6 + 6x^5 + 9x^4 + 2x^2 + 4x + 1$ then choose the simplified form of $f''(x)$:

Possibilities:

(a) $90x^4 + 120x^3 + 108x^2 + 4$
(b) $90x^4 + 120x^3 + 198x^2 + 60x + 28$
(c) $18x^5 + 30x^4 + 36x^3 + 4x + 4$
(d) $18x^5 + 75x^4 + 156x^3 + 159x^2 + 88x + 24$
(e) $108x^6 + 150x^5 + 144x^4 + 8x^2$

$f'(x) = 3 \cdot 6 x^5 + 6 \cdot 5 x^4 + 9 \cdot 4 x^3 + 2 \cdot 2x + 4$

$= 18x^5 + 30x^4 + 36x^3 + 4x + 4$

$f''(x) = 18 \cdot 5 x^4 + 30 \cdot 4 x^3 + 36 \cdot 3 x^2 + 4$

$= 90x^4 + 120x^3 + 108x^2 + 4$

Chain

10. Find the derivative, $f'(x)$, if $f(x) = \sqrt{x^3+7x-1}$. $= (x^3+7x-1)^{1/2}$

Possibilities:

(a) $(1/2)(x^3+7x-1)(3x^2+7)$

(b) $(1/2)(x^3+7x-1)^{1/2}$

(c) $(1/2)(x^3+7x-1)^{-1/2}(3x^2+7)$

(d) $\dfrac{\sqrt{3x^2+7}}{\sqrt{x^3+7x-1}}$

(e) $\sqrt{3x^2+7}$

$f'(x) = \frac{1}{2}(x^3+7x-1)^{-1/2} \cdot (3x^2+7)$

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11. Find the derivative, $f'(x)$, if $f(x) = e^{9x^3+6x^2+7x+1}$.

Possibilities:

(a) $(27x^2+12x+7)e^{9x^3+6x^2+7x+1}$

(b) $\ln(9x^3+6x^2+7x+1)$

(c) $(27x^2+12x+7)e^x$

(d) $\dfrac{27x^2+12x+7}{9x^3+6x^2+7x+1}$

(e) $e^{27x^2+12x+7}$

$(e^{\text{Power}})' = \text{Power}' \cdot e^{\text{Power}}$

$f'(x) = (9x^3+6x^2+7x+1)' e^{9x^3+6x^2+7x+1}$

$= (27x^2+12x+7)e^{9x^3+6x^2+7x+11}$

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12. Find the derivative, $f'(x)$, if $f(x) = (5+6x)e^{8+3x}$.

Product

Possibilities:

(a) $(18)e^3$

(b) $(21+18x)e^{8+3x}$

(c) $(6)e^{8+3x}$

(d) $(6)e^3$

(e) $\dfrac{6}{8+3x}$

$(5+6x)'e^{8+3x} + (5+6x)(e^{8+3x})'$

$= 6e^{8+3x} + (5+6x)\cdot(8+3x)'\cdot e^{8+3x}$

$= 6e^{8+3x} + (5+6x)\cdot 3e^{8+3x}$

$= 6e^{8+3x} + 15e^{8+3x} + 18xe^{8+3x}$

$= 21e^{8+3x} + 18xe^{18+3x}$

$(\ln(\text{Inside}))' = \frac{\text{Inside}'}{\text{Inside}}$

13. Find the derivative, $f'(x)$, if $f(x) = \ln(-x^3 + 7x^2 + 8x + 3)$.

Possibilities:

(a) $e^{-3x^2+14x+8}$

(b) $(-3x^2 + 14x + 8)e^{-x^3+7x^2+8x+3}$

(c) $\dfrac{1}{-3x^2 + 14x + 8}$

(d) $\ln(-x^3 + 7x^2 + 8x + 3)$

(e) $\dfrac{-3x^2 + 14x + 8}{-x^3 + 7x^2 + 8x + 3}$

$f'(x) = \dfrac{(-x^3+7x^2+8x+3)'}{-x^3+7x^2+8x+3}$

$= \dfrac{-3x^2+14x+8}{-x^3+7x^2+8x+3}$

Product

14. Find the derivative, $f'(x)$, if $f(x) = (3 + 9x)\ln(2 + 4x)$.

Possibilities:

(a) $\dfrac{13}{2 + 4x}$

(b) $\dfrac{9}{2 + 4x}$

(c) $1/x$

(d) $9 + \dfrac{4}{2 + 4x}$

(e) $(9)\ln(2 + 4x) + \dfrac{12 + 36x}{2 + 4x}$

$(3+9x)'\ln(2+4x) + (3+9x)(\ln(2+4x))'$

$= 9\cdot\ln(2+4x) + (3+9x)\cdot\dfrac{(2+4x)'}{2+4x}$

$= 9\ln(2+4x) + (3+9x)\cdot\dfrac{4}{2+4x}$

$= 9\ln(2+4x) + \dfrac{12+36x}{2+4x}$

15. Suppose $F(x) = \ln(g(x))$. If $g(2) = 7$, $g'(2) = 3$, and $g''(2) = 11$, then find $F'(2)$.

Possibilities:

(a) $\ln(7)/3$

(b) $3/7$

(c) $\ln(11)$

(d) $7/\ln(3)$

(e) $7/3$

Chain

$F'(x) = \dfrac{g'(x)}{g(x)} \Rightarrow$

$F'(2) = \dfrac{g'(2)}{g(2)} = \dfrac{3}{7}$

16. If an amount of x dollars is invested at 2% interest compounded continuously, and at the end of 5 years the value of the investment is \$3000, find x.

Possibilities:

(a) \$3315.51

(b) \$300

(c) \$2714.51

(d) \$2000

(e) \$588.11

$P = P_0 e^{rt}$ $\quad r = 0.02$ $\quad t = 5$ $\quad P = 3000$, $P_0 = x$

$3000 = x e^{.02 \cdot 5} = x \cdot e^{0.1}$

So $x = \dfrac{3000}{e^{0.1}}$

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17. Find the maximum of $g(t) = (t-3)^2 + 4$ on the interval $[0, 5]$

Possibilities:

(a) 3

(b) 8

(c) 13

(d) 4

(e) 16

Check endpoints, where $g'(t) = 0$ & where $g'(t)$ dne

$g'(t) = 2(t-3) \Rightarrow$ Exists everywhere

$g'(t) = 0 \Rightarrow t = 3$

Test points: $t = 0, 3, 5$

$g(0) = (0-3)^2 + 4 = 13$ max

$g(3) = (3-3)^2 + 4 = 4$

$g(5) = (5-3)^2 + 4 = 8$

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18. Find the minimum of $g(t) = 2t^3 + 3t^2 - 12t + 1$ on the interval $[-1, 3]$

Possibilities:

(a) -16

(b) 14

(c) -6

(d) 46

(e) 21

$g'(t) = 6t^2 + 6t - 12 = 6(t^2 + t - 2)$

$= 6(t-1)(t+2)$

$g'(t) = 0 \Rightarrow t = 1$ OR $t = -2$ ← Not in interval

Test Points are $t = -1, \; t = 1, \; t = 3$

$g(-1) = 2(-1)^3 + 3(-1)^2 - 12(-1) + 1 = 14$

$g(1) = 2 + 3 - 12 + 1 = -6$ min

$g(3) = 2 \cdot 3^3 + 3 \cdot 3^2 - 12 \cdot 3 + 1 = 56$

19. The minimum value of $f(x) = 3x^{2/3} - x^2 + 3$ on the interval $[-2, 2]$ is:

Possibilities:

(a) 3

(b) 1

(c) 5

(d) 0

(e) $3\sqrt[3]{4} + 1$

$f'(x) = 3 \cdot \frac{2}{3} x^{-1/3} - 2x = 2x^{-1/3} - 2x$

$f'(x)$ DNE @ $x = 0$.

$f'(x) = 0 \Rightarrow 2x^{-1/3} = 2x \Rightarrow x^{-1/3} = x \Rightarrow 1 = x^{4/3}$

$\Rightarrow x = \pm 1$

Test points: $x = -2, -1, 0, 1, 2$.

$f(-2) = 3(-2)^{2/3} - (-2)^2 + 3 = 3\sqrt[3]{4} - 1 \approx 3.24$

$f(-1) = 3(-1)^{2/3} - (-1)^2 + 3 = 5$

$f(0) = 3 \cdot 0^{2/3} - 0^2 + 3 = 3$ Min

$f(1) = 3 \cdot 1^{2/3} - 1^2 + 3 = 5$

$f(2) = 3 \cdot 2^{2/3} - 2^2 + 3 = 3\sqrt[3]{4} - 1 \approx 3.24$.

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20. Suppose $g(s) = \sqrt{s}$. Find a value c in the interval $[0, 5]$ such that $g'(c)$ equals the average rate of change of $g(s)$ on the interval $[0, 5]$.

Possibilities:

(a) 5

(b) 5/2

(c) 5/4

(d) $\sqrt{5}$

(e) $1/(2\sqrt{5})$

$AROC = \frac{g(5) - g(0)}{5 - 0} = \frac{\sqrt{5} - \sqrt{0}}{5 - 0} = \frac{\sqrt{5}}{5}$

$g'(c) = \frac{1}{2} c^{-1/2} = \frac{1}{2\sqrt{c}}$

$\frac{1}{2\sqrt{c}} = \frac{\sqrt{5}}{5} \Rightarrow 5 = \sqrt{5} \cdot 2\sqrt{c}$

$\frac{5}{2\sqrt{5}} = \sqrt{c}$

$\left(\frac{5}{2\sqrt{5}}\right)^2 = c$

$\frac{25}{4 \cdot 5} = c \Rightarrow \frac{5}{4}$
