MA123 — Elem. Calculus Fall 2014
Exam 2 2014-10-23

Name: SOLUTIONS **Sec.:** _____

Do not remove this answer page — you will turn in the entire exam. You have two hours to do this exam. No books or notes may be used. You may use an ACT-approved calculator during the exam, but NO calculator with a Computer Algebra System (CAS), networking, or camera is permitted. Absolutely no cell phone use during the exam is allowed.

The exam consists of multiple choice questions. Record your answers on this page. For each multiple choice question, you will need to fill in the box corresponding to the correct answer. For example, if (a) is correct, you must write

(a) (b) (c) (d) (e)

Do not circle answers on this page, but please circle the letter of each correct response in the body of the exam. It is your responsibility to make it CLEAR which response has been chosen. You will not get credit unless the correct answer has been marked on both this page and in the body of the exam.

GOOD LUCK!

1.	a	b	**c**	d	e	11.	a	b	c	**d**	e
2.	**a**	b	c	d	e	12.	a	b	c	**d**	e
3.	a	**b**	c	d	e	13.	a	b	**c**	d	e
4.	a	b	**c**	d	e	14.	**a**	b	c	d	e
5.	a	b	c	d	**e**	15.	a	b	c	d	**e**
6.	a	b	c	d	**e**	16.	**a**	b	c	d	e
7.	a	b	c	**d**	e	17.	a	b	c	**d**	e
8.	a	**b**	c	d	e	18.	a	**b**	c	d	e
9.	**a**	b	c	d	e	19.	a	b	c	**d**	e
10.	a	b	c	d	**e**	20.	a	b	**c**	d	e

For grading use:

Number Correct	
	(out of 20 problems)

Total	
	(out of 100 points)

Please make sure to list the correct section number on the front page of your exam.
In case you forgot your section number, consult the following table.

Section	Instructor	Day	Time	Room
	Jack Schmidt	MWF	8:00 am - 8:50 am	FB 200
001	Nandita Sahajpal	Tu	8:00 am	FB B13
002	Nandita Sahajpal	Tu	9:30 am	NURS 501b
003	John Mosley	Tu	11:00 am	DH 353
004	John Mosley	Tu	12:30 pm	CB 337
005	John Mosley	Tu	2:00 pm	CB 233
006	John Mosley	Tu	3:30 pm	CB 341
	Jack Schmidt	MWF	9:00 am - 9:50 am	BS 107
007	Nandita Sahajpal	Th	8:00 am	TEB 207
008	Nandita Sahajpal	Th	9:30 am	TEB 231
009	Chad Linkous	Th	11:00 am	CP 111
010	Chad Linkous	Th	12:30 pm	CB 337
011	Bill Trok	Th	2:00 pm	CB 219
012	Bill Trok	Th	3:30 pm	CB 341
	erica Whitaker	MWF	1:00 pm - 1:50 pm	KAS 213
013	Dharma Maharjan	Tu	8:00 am	CP 397
014	Dharma Maharjan	Tu	9:30 am	NURS 511
015	Chad Linkous	Tu	11:00 am	FB B13
016	Chad Linkous	Tu	12:30 pm	CB 335
017	Bill Trok	Tu	2:00 pm	DH 301
018	Bill Trok	Tu	3:30 pm	CB 337
	erica Whitaker	MWF	3:00 pm - 3:50 pm	FB 200
019	Dharma Maharjan	Th	8:00 am	DH 203
020	Dharma Maharjan	Th	9:30 am	TEB 207
021	Kathy Effinger	Th	11:00 am	DH 353
022	Kathy Effinger	Th	12:30 pm	CB 335
023	Jonathan Thompson	Th	2:00 pm	FB B13
024	Jonathan Thompson	Th	3:30 pm	CB 303
401	Dustin Hedmark	MTR	5:30 pm - 6:45 pm	CB 343
402	Brad Fox	MTR	7:00 pm - 8:15 pm	CB 337

Multiple Choice Questions

Show all your work on the page where the question appears.
Clearly mark your answer both on the cover page on this exam
and in the corresponding questions that follow.

1. Find the derivative, $f'(x)$, if $f(x) = \sqrt{4x^3 + 5x^2 + 6x + 2}$. $= (4x^3 + 5x^2 + 6x + 2)^{1/2}$

Possibilities:

(a) $(1/2)(4x^3 + 5x^2 + 6x + 2)(12x^2 + 10x + 6)$

(b) $\sqrt{12x^2 + 10x + 6}$

(c) $(1/2)(4x^3 + 5x^2 + 6x + 2)^{-1/2}(12x^2 + 10x + 6)$

(d) $(1/2)(4x^3 + 5x^2 + 6x + 2)^{1/2}$

(e) $\dfrac{\sqrt{12x^2 + 10x + 6}}{\sqrt{4x^3 + 5x^2 + 6x + 2}}$

$f'(x) = \underbrace{\frac{1}{2}(4x^3 + 5x^2 + 6x + 2)^{-1/2}}_{\text{power rule}} \cdot \underbrace{(12x^2 + 10x + 6)}_{\text{deriv. of inside}}$

2. Find the derivative, $f'(x)$, if $f(x) = e^{2x^3+6x^2+7x}$.

Possibilities:

(a) $(6x^2 + 12x + 7)e^{2x^3+6x^2+7x}$

(b) $\ln(2x^3 + 6x^2 + 7x)$

(c) $e^{6x^2+12x+7}$

(d) $(6x^2 + 12x + 7)e^x$

(e) $\dfrac{6x^2 + 12x + 7}{2x^3 + 6x^2 + 7x}$

$f'(x) = \underbrace{e^{2x^3+6x^2+7x}}_{\text{rule for } e} \cdot \underbrace{(6x^2 + 12x + 7)}_{\text{deriv. of exp.}}$

3. For the function $f(x) = 2x^3 + 4x^2 + 3x + 1$, find the equation of the tangent line to graph of f at $x = 3$.

Possibilities:

(a) $y = x^3 + 17$

(b) $y = 81x - 143$

(c) $y = 100x - 219$

(d) $y = 100$

(e) $y = 81x + 100$

y-value of point $= f(3) = 2 \cdot 3^3 + 4 \cdot 3^2 + 3 \cdot 3 + 1$

$= 2 \cdot 27 + 4 \cdot 9 + 9 + 1 = 100$

$f'(x) = 6x^2 + 8x + 3$

$m = f'(3) = 6 \cdot 3^2 + 8(3) + 3 = 6 \cdot 9 + 24 + 3 = 81$

$y - y_1 = m(x - x_1)$

$y - 100 = 81(x - 3) \Rightarrow y - 100 = 81x - 243$

$\Rightarrow y = 81x - 143$

4. Suppose $F(x) = \ln(g(x))$. If $g(2) = 7$, $g'(2) = 11$, and $g''(2) = 3$, then find $F'(2)$.

Possibilities:

(a) 7/11

(b) $\ln(7)/11$

(c) 11/7

(d) $\ln(3)$

(e) $7/\ln(11)$

$F'(x) = \underbrace{\frac{1}{g(x)}}_{\text{rule for ln}} \cdot \underbrace{g'(x)}_{\text{deriv. of } g}$

$F'(2) = \frac{1}{g(2)} \cdot g'(2) = \frac{1}{7}(11) = \frac{11}{7}$

---

5. Suppose $F(x) = g(x) \cdot h(x+2)$. If $g(0) = 6$, $g'(0) = 3$, $h(0) = 8$, $h'(0) = 7$, $h(2) = 4$, and $h'(2) = 9$, find $F'(0)$.

Possibilities:

(a) 130

(b) 120

(c) 74

(d) 37

(e) 66

PRODUCT RULE

$F'(x) = \underbrace{g(x)}_{\text{copy}} \cdot \underbrace{h'(x+2)}_{\text{der. of } h} \cdot \underbrace{1}_{\text{der. of inside}} + \underbrace{h(x+2)}_{\text{copy}} \cdot \underbrace{g'(x)}_{\text{der. of } g}$

$F'(0) = g(0) \cdot h'(0+2) + h(0+2) \cdot g'(0)$

$= 6 \cdot 9 + 4 \cdot 3$

$= 54 + 12 = 66$

---

6. Suppose $F(x) = (g(x))^5 + 7$. If $g(2) = 9$, $g'(2) = 13$, and $g''(2) = 3$, then find $F'(2)$.

Possibilities:

(a) $(5)(9^4) + 7$

(b) 3

(c) $9^5 + 7$

(d) $13^5 + 7$

(e) $(5)(9^4)(13)$

$F'(x) = \underbrace{5(g(x))^4}_{\text{power rule}} \cdot \underbrace{g'(x)}_{\text{inside}} + \underbrace{0}_{\text{der. of } 7}$

$F'(2) = 5(g(2))^4 \cdot g'(2) + 0$

$= 5 \cdot 9^4 \cdot 13$

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7. If $f(x) = \frac{8}{x+5}$ then choose the simplified form of $\frac{f(x+h)-f(x)}{h}$:

Possibilities:

(a) $\frac{16x+80+8h}{(x+h+5)(x+5)(2x+h)}$

(b) $\frac{hx^2+10hx+25h-8}{(x+5)^2}$

(c) $-\frac{8}{(x+h+5)^2}$

(d) $-\frac{8}{(x+h+5)(x+5)}$

(e) $\frac{8}{(x+h+5)(x+5)}$

$f(x+h) = \frac{8}{x+h+5}$, so

$$\frac{f(x+h)-f(x)}{h} = \frac{\frac{8}{x+h+5} - \frac{8}{x+5}}{h}$$

$$= \frac{\frac{8(x+5)}{(x+h+5)(x+5)} - \frac{8(x+h+5)}{(x+h+5)(x+5)}}{h/1}$$

cancel h's

$$= \frac{8x+40-(8x+8h+40)}{(x+h+5)(x+5)} \cdot \frac{1}{h} = \frac{-8h}{(x+h+5)(x+5)} \cdot \frac{1}{h}$$

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8. Find the derivative, $f'(x)$, if $f(x) = (6+9x)e^{2+9x}$.

Product Rule

Possibilities:

(a) $\frac{9}{2+9x}$

(b) $(63+81x)e^{2+9x}$

(c) $(81)e^9$

(d) $(9)e^9$

(e) $(9)e^{2+9x}$

$f'(x) = (6+9x) \cdot e^{2+9x} \cdot 9 + e^{2+9x} \cdot 9$

copy | rule for exp' | copy | der of $6+9x$

$= e^{2+9x}(9(6+9x)+9)$

$= e^{2+9x}(54+81x+9)$

$= e^{2+9x}(81x+63)$

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9. Find the derivative, $f'(x)$, of $f(x) = \frac{1}{x^8} = x^{-8}$

Possibilities:

(a) $-8x^{-9}$

(b) $-8x^{-7}$

(c) $8x^7$

(d) $1/(8x^7)$

(e) $1/(8x^9)$

$f'(x) = -8x^{-9}$

power rule.

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10. For the function $f(x) = \begin{cases} x^2 - 4 & x < 10 \\ \sqrt{x+7} & 10 \le x < 20 \\ x^3 - 6 & 20 \le x \end{cases}$, find the equation of the tangent line to the graph of f at $x = 19$.

$x = 19$ is between 10 and 20.

for these x, $f(x) = (x+7)^{1/2}$

So $f'(x) = \frac{1}{2}(x+7)^{-1/2}(1) = \frac{1}{2\sqrt{x+7}}$

and $m = f'(19) = \frac{1}{2\sqrt{19+7}} = \frac{1}{2\sqrt{26}} \cdot \frac{\sqrt{26}}{\sqrt{26}} = \frac{\sqrt{26}}{52}$

y-value is $f(19) = \sqrt{19+7} = \sqrt{26}$

$y - \sqrt{26} = \frac{\sqrt{26}}{52}(x - 19) \Rightarrow y = \frac{\sqrt{26}}{52}x - \frac{19}{52}\sqrt{26} + \sqrt{26} \cdot \frac{52}{52}$

$y = \frac{\sqrt{26}}{52}x + \frac{33}{52}\sqrt{26}$

Possibilities:

(a) $y = 357x - 2818$

(b) $y = 1083x - 13724$

(c) $y = \sqrt{26}x - \frac{415}{52}\sqrt{26}$

(d) $y = 38x - 365$

(e) $y = \frac{1}{52}\sqrt{26}x + \frac{33}{52}\sqrt{26}$

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11. Find the derivative, $f'(x)$, if $f(x) = (7 + 9x)\ln(6 + 7x)$.

Product Rule:

$f'(x) = \underbrace{(7+9x)}_{\text{copy}} \cdot \underbrace{\frac{1}{6+7x}}_{\ln \text{ rule}} \cdot \underbrace{7}_{\text{inside}} + \underbrace{\ln(6+7x)}_{\text{copy}} \cdot \underbrace{9}_{\text{der of } 7+9x}$

$= \frac{(7+9x)\cdot 7}{6+7x} + 9\ln(6+7x)$

Possibilities:

(a) $\frac{16}{6+7x}$

(b) $1/x$

(c) $\frac{9}{6+7x}$

(d) $(9)\ln(6+7x) + \frac{49+63x}{6+7x}$

(e) $9 + \frac{7}{6+7x}$

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12. For the function $f(x) = \ln(6x^2 + 8x + 9)$, find the equation of the tangent line to graph of f at $x = 0$.

$f'(x) = \underbrace{\frac{1}{6x^2+8x+9}}_{\ln \text{ rule}} \cdot \underbrace{12x+8}_{\text{inside}}$

$m = f'(0) = \frac{1}{0+0+9}(0+8) = \frac{8}{9}$

y-value $= f(0) = \ln(0+0+9) = \ln 9$

$y - \ln 9 = \frac{8}{9}(x - 0) \Rightarrow y - \ln 9 = \frac{8}{9}x \Rightarrow y = \frac{8}{9}x + \ln 9$

Possibilities:

(a) $y = 3x + \ln(9)$

(b) $y = \ln(9)\,x + \frac{8}{9}$

(c) $y = \ln(9)$

(d) $y = \frac{8}{9}x + \ln(9)$

(e) $y = x^3 + 17$

13. If $f(x) = 9x^4 + 7x$ then find the second derivative $f''(x)$:

Possibilities:

(a) $36x^3 + 7$

(b) $36x^3 + 54x^2 + 36x + 16$

(c) $108x^2$

(d) $144x^4$

(e) $108x^2 + 18$

$f'(x) = 36x^3 + 7$

$f''(x) = 36 \cdot 3x^2 + 0$

$= 108x^2$

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14. If $f(x) = (12x + 32)^{23}$ then $f''(x) =$?

Possibilities:

(a) $23(22)(12x + 32)^{21}(12)^2$

(b) $23^2(12)^{23}(12x + 32)$

(c) $23(12x + 32)^{22}$

(d) 0

(e) $23(22)12^{21}$

power rule, inside

$f'(x) = 23(12x + 32)^{22} \cdot 12$

$= 23 \cdot 12 (12x + 32)^{22}$

$f''(x) = \underbrace{23 \cdot 12}_{\text{coeff}} \cdot \underbrace{22(12x + 32)^{21}}_{\text{power rule}} \cdot \underbrace{12}_{\text{inside}}$

$= 23 \cdot 22 \cdot 12 \cdot 12 (12x + 32)^{21}$

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15. The function $f(x)$ is increasing on $(-\infty, 1] \cup [9, \infty)$ and decreasing on $[1, 9]$. The values $f(1) = 11$ and $f(9) = 3$ are known. Which of the following is possible?

Possibilities:

(a) $f(11) = 2$ ← $f(11)$ is > 3

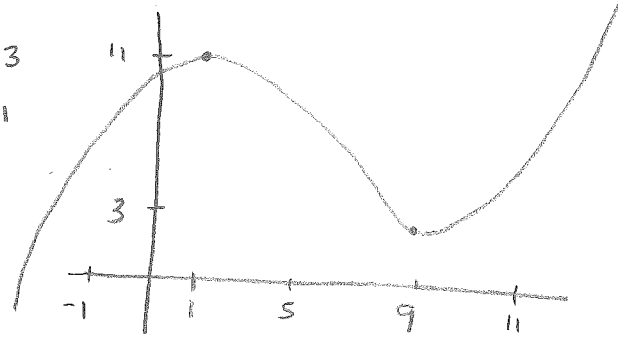
(b) $f(-1) = 12$ ← $f(-1)$ is < 11

(c) $f(5) = 12$ ← $f(5)$ is < 11

(d) $f'(5) = 7$ ← $f'(5)$ is negative

(e) $f'(5) = -7$

↑ only possibility



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16. If an amount of x dollars is invested at 2% interest compounded continuously, and at the end of 5 years the value of the investment is \$3000, find x.

Possibilities:

(a) \$2714.51

(b) \$3315.51

(c) \$300

(d) \$2000

(e) \$588.11

$P = P_0 e^{rt}$

$3000 = x e^{.02(5)}$

$x = \frac{3000}{e^{.02(5)}} = \frac{3000}{e^{.1}} \approx 2714.512$

[P = amount after t years
P_0 = initial investment
r = rate = 2% = .02
t = time in years]

17. The numbers of a bacteria in a culture doubles every 13 hours. How many hours will it take before 9 times the original number of bacteria is present?

Possibilities:

(a) $\frac{13}{9}$

(b) $\frac{117}{2}$

(c) $\frac{13}{2}$

(d) $13 \ln(9)/\ln(2)$

(e) $13 \ln(2)/\ln(9)$

$P = P_0 e^{rt}$

Suppose $P_0 = 1$. Then $2 = 1 e^{r(13)}$

$\Rightarrow e^{13r} = 2 \Rightarrow \ln e^{13r} = \ln 2$

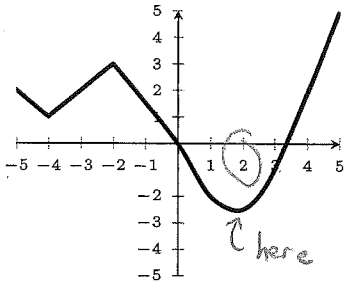
$\Rightarrow 13r = \ln 2 \Rightarrow r = \frac{\ln 2}{13}$.

Then $9 = 1 e^{\frac{\ln 2}{13} \cdot t} \Rightarrow e^{\frac{\ln 2}{13} t} = 9$

$\Rightarrow \ln e^{\frac{\ln 2}{13} t} = \ln 9 \Rightarrow \frac{\ln 2}{13} t = \ln 9$

$\Rightarrow t = \ln 9 \cdot \frac{13}{\ln 2} = \frac{13 \cdot \ln 9}{\ln 2}$

18. The graph of $y = f(x)$ is shown below. The minimum value of $f(x)$ on the interval $[-5, 3]$ occurs at which x?



Possibilities:

(a) -2

(b) 2

(c) -4

(d) 0

(e) -3

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19. Find the minimum of $g(t) = -(t+2)^2 + 7$ on the interval $[-3, 0]$

Possibilities:

(a) 7

(b) 6

(c) -2

(d) 3

(e) 0

$g'(t) = -2(t+2)^1(1) + 0 = -2t - 4$

$g'(t) = 0$ when $-2t - 4 = 0 \Rightarrow -2t = 4 \Rightarrow t = -2$, which is in $[-3, 0]$.

check endpoints and critical numbers.

$g(-3) = -(-3+2)^2 + 7 = -(-1)^2 + 7 = -1 + 7 = 6$

$g(0) = -(0+2)^2 + 7 = -4 + 7 = 3$ ← SMALLEST

$g(-2) = -(-2+2)^2 + 7 = -0^2 + 7 = 7$

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20. Find the minimum of $g(t) = -2t^3 - 3t^2 + 36t - 2$ on the interval $[1, 4]$

Possibilities:

(a) 42

(b) 29

(c) -34

(d) -83

(e) 0

$g'(t) = -6t^2 - 6t + 36$

$= -6(t^2 + t - 6) = -6(t+3)(t-2)$

$g'(t) = 0$ when $t = -3$ (not in $[1,4]$) and when $t = 2$,

check endpoints and critical numbers:

$g(1) = -2 \cdot 1^3 - 3 \cdot 1^2 + 36 \cdot 1 - 2 = -2 - 3 + 36 - 2 = 29$

$g(4) = -2 \cdot 4^3 - 3 \cdot 4^2 + 36 \cdot 4 - 2 = -128 - 48 + 144 - 2 = -34$

$g(2) = -2 \cdot 2^3 - 3 \cdot 2^2 + 36 \cdot 2 - 2 = -16 - 12 + 72 - 2 = 42$