MA 123 — Elem. Calculus Spring 2011
EXAM 2 3/9/2011

Name: Solutions **Sec.:** _____

Do not remove this answer page — you will turn in the entire exam. You have two hours to do this exam. No books or notes may be used. You may use a graphing calculator during the exam, but NO calculator with a Computer Algebra System (CAS) or a QWERTY keyboard is permitted. Absolutely no cell phone use during the exam is allowed.

The exam consists of multiple choice questions. Record your answers on this page. For each multiple choice question, you will need to fill in the box corresponding to the correct answer. For example, if (a) is correct, you must write

■ b c d e

Do not circle answers on this page, but please circle the letter of each correct response in the body of the exam. It is your responsibility to make it CLEAR which response has been chosen. You will not get credit unless the correct answer has been marked on both this page and in the body of the exam.

GOOD LUCK!

1. a b ■ d e
2. a b c ■ e
3. a b c ■ e
4. a b ■ d e
5. a b c d ■
6. a b c d ■
7. a b c ■ e
8. a b c ■ e
9. ■ b c d e
10. a b c ■ e
11. a b c ■ e
12. a ■ c d e
13. ■ b c d e
14. a ■ c d e
15. a b c ■ e
16. a b ■ d e
17. ■ b c d e
18. a b c ■ e
19. a b c d ■
20. ■ b c d e

For grading use:

Number Correct	
	(out of 20 problems)

Total	
	(out of 100 points)

Please make sure to list the correct section number on the front page of your exam. In case you forgot your section number, consult the following table. If you are enrolled in a lecture with recitation, then your time and location is based on your recitation, not your lecture.

Section #	Instructor	Lectures
001	Koester/Hamilton	T 8:00 - 9:15 am, CP 243
002	Koester/Hamilton	R 8:00 - 9:15 am, CP 243
003	Koester/Hamilton	T 9:30 - 10:45 am, MMRB 243
004	Koester/May	R 9:30 - 10:45 am, CB 342
005	Koester/May	T 11:00 - 12:15 pm, CP 220
006	Koester/May	R 11:30 - 12:15 pm, CP 220
007	Koester/Kyriopoulos	T 9:30 - 10:45 am, CP 367
008	Koester/Kyriopoulos	R 9:30 - 10:45 am, DH 323
009	Koester/Kyriopoulos	T 11:00 - 12:15 pm, FB 307A
010	Koester/Robinson	R 11:00 - 12:15 pm, CP 345
011	Koester/Robinson	T 12:30 - 1:45pm, CB 219
012	Koester/Robinson	R 12:30 - 1:45pm, CB 219
013	Shaw/Taylor	T 11:00 - 12:15 pm, CB 345
014	Shaw/Taylor	R 11:00 - 12:15 pm, MMRB 243
015	Shaw/Taylor	T 12:30 - 1:45 pm, Nurs 201
016	Shaw/Tarr	R 12:30 - 1:45 pm, Nurs 502A
017	Shaw/Tarr	T 2:00 - 3:15 pm, CB 233
018	Shaw/Tarr	R 2:00 - 3:15 pm, CB 245
019	Shaw/Ozbek	T 3:30 - 4:45pm, CP 208
020	Shaw/Ozbek	R 3:30 - 4:45 pm, CP 208
021	Shaw/Ozbek	T 2:00 - 3:15 pm, FB B2
022	Shaw/Zhi	R 2:00 - 3:15 pm, CP 233
023	Shaw/Zhi	T 9:30 - 10:45 am, CP 211
024	Shaw/Zhi	R 9:30 - 10:45 am, CB 341
025	Beth Kelly	MWF 12:00 - 12:50 pm, CP 153
026	John Maki	MWF 2:00 - 2:50 pm, KAS 213

Multiple Choice Questions

Show all your work on the page where the question appears.
Clearly mark your answer both on the cover page on this exam
and in the corresponding questions that follow.

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1. The equation of the tangent line to $y = f(x)$ at $x = 8$ is $y - 5 = 6(x - 8)$. Determine $f'(8)$.

Possibilities:

(a) 5
(b) -43
(c) 6
(d) 8
(e) 53

$f'(8)$ = Slope of tangent line at $x = 8$,

So $f'(8) = 6$

---

2. $f(x) = (x-2)^2$. Find the value of C, given that

$$\frac{f(x+h) - f(x)}{h} = Ax + Bh + C$$

Difference of Squares

Possibilities:

(a) -7
(b) -6
(c) -5
(d) -4
(e) -3

$$= \frac{(x+h-2)^2 - (x-2)^2}{h} = \frac{((x+h-2)+(x-2))((x+h-2)-(x-2))}{h}$$

$$= \frac{(2x+h-4)h}{h} = 2x + h - 4$$

C

---

3. Suppose that

$$\frac{f(x+h) - f(x)}{h} = \frac{7hx^2 - 10h^2}{h}$$

Find the derivative, $f'(3)$.

Possibilities:

(a) 0
(b) 7
(c) $63 - 10h$
(d) 63
(e) The derivative does not exist.

$$f'(3) = \lim_{h \to 0} \frac{f(3+h) - f(3)}{h}$$

$$= \lim_{h \to 0} \frac{7 \cdot h \cdot 3^2 - 10 \cdot h^2}{h} = \lim_{h \to 0} \frac{(63 - 10h)h}{h}$$

$$= \lim_{h \to 0} (63 - 10h) = 63.$$

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4. Find the equation of the tangent line to the graph of $f(x) = \sqrt{x} + 2$ at $x = 36$.

Possibilities:

(a) $y = (1/12)\,x + 3$

(b) $y = (1/12)\,x + 4$

(c) $y = (1/12)\,x + 5$

(d) $y = (1/12)\,x + 6$

(e) $y = (1/12)\,x + 7$

Slope $= f'(36) = \frac{1}{2} \cdot 36^{-1/2} = \frac{1}{12}$.

$y - (\sqrt{36}+2) = \frac{1}{12}(x-36)$

$y - 8 = \frac{1}{12}x - 3$

$y = \frac{1}{12}x + 5$

---

5. Find the derivative, $f'(x)$, of

$$f(x) = x^{(10/3)}$$

Possibilities:

(a) $(10/3)\,x^{13/3}$

(b) $(7/3)\,x^{10/3}$

(c) $(10/3)\,x^{3}$

(d) $(7/3)\,x^{7/3}$

(e) $(10/3)\,x^{7/3}$

Power Rule:

$\frac{10}{3} x^{\frac{10}{3}-1} = \frac{10}{3} x^{\frac{7}{3}}$

---

6. Find the derivative, $f'(7)$, where

$$f(x) = \sqrt{464 - x^3} = (464 - x^3)^{1/2}.$$

Possibilities:

(a) 0

(b) 21/22

(c) 147/22

(d) $-21/22$

(e) $-147/22$

Chain Rule:

Out $= (\ )^{1/2}$ In $= 464 - x^3$

Out$' = \frac{1}{2}(\ )^{-1/2}$ In$' = -3x^2$

$f'(7) = \frac{1}{2}(464 - 7^3)^{-1/2} \cdot (-3 \cdot 7^2)$

$= -\frac{147}{22}$.

---

7. Find the derivative, $f'(x)$, where

$$f(x) = \frac{6x+3}{x^2+5}$$

Possibilities:

(a) $\frac{3}{x}$

(b) $\frac{6x^2+6x-30}{x^2+5}$

(c) $\frac{6}{x}$

(d) $\frac{-6x^2-6x+30}{(x^2+5)^2}$

(e) $\frac{18x^2+6x+30}{(x^2+5)^2}$

Quotient Rule.

$$\frac{(6x+3)'(x^2+5) - (6x+3)(x^2+5)'}{(x^2+5)^2}$$

$$= \frac{6(x^2+5) - (6x+3)2x}{(x^2+5)^2}$$

$$= \frac{-6x^2 - 6x + 30}{(x^2+5)^2}$$

---

8. Find the derivative, $f'(6)$, where

$$f(x) = e^{-10x}$$

Possibilities:

(a) $-60e^{-61}$

(b) e^{-60}

(c) $-10e^{6}$

(d) $-10e^{-60}$

(e) $6e^{-60}$

Chain Rule:

$$(e^{-10x})' = -10e^{-10x}$$

At $x = 6$:

$$-10 \cdot e^{-60}$$

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9. Find the second derivative, $f''(x)$, where

$$f(x) = e^{18x}$$

Possibilities:

(a) $18^2 e^{18x}$

(b) e^{18x}

(c) e^{36}

(d) 0

(e) $2^{18}e^{18x}$

Chain Rule:

$$f'(x) = 18e^{18x}$$

$$f''(x) = 18^2 e^{18x}$$

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10. Find the derivative, $f'(x)$, where

$$f(x) = \ln\left(8x^2 + 6x + 1\right)$$

Possibilities:

(a) $\dfrac{1}{16x+6}$

(b) $16x + 6$

(c) $\dfrac{1}{8x^2+6x+1}$

(d) $\dfrac{16x+6}{8x^2+6x+1}$

(e) $\dfrac{8x^2+6x+1}{16x+6}$

Chain Rule:

$$\left(\ln\left(8x^2+6x+1\right)\right)' = \frac{\left(8x^2+6x+1\right)'}{8x^2+6x+1}$$

$$= \frac{16x+6}{8x^2+6x+1}$$

---

11. Suppose $f(36) = -7$, $f'(36) = 3$, and

$$g(x) = x^{(3/2)} + 2\,f(x).$$

Find $g'(36)$.

Possibilities:

(a) 12

(b) 13

(c) 14

(d) 15

(e) There is not enough information to find the requested derivative.

$$g'(x) = \frac{3}{2}x^{1/2} + 2\cdot f'(x)$$

$$g'(36) = \frac{3}{2}\sqrt{36} + 2\cdot f'(36)$$

$$= 9 + 2\cdot 3 = 15$$

---

12. Suppose $g(7) = 2$ and $g'(7) = -3$. Find $F'(7)$, given that

$$F(x) = (g(x))^4$$

Possibilities:

(a) -24

(b) -96

(c) 24

(d) -192

(e) 32

Chain Rule:

$$F'(x) = 4\,g(x)^3 \cdot g'(x)$$

$$F'(7) = 4\cdot g(7)^3 \cdot g'(7)$$

$$= 4\cdot 2^3\cdot(-3) = -96$$

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13. An object moves in a straight line so that after t seconds its distance from its starting point is $D(t) = t^4 + 7t^2 + 43t + 28$ feet. What is the acceleration of the object (measured in feet per second per second) at the end of 2 seconds?

Acceleration = Second derivative of position.

Possibilities:

(a) 62 feet per second^2

(b) 7 feet per second^2

(c) 48 feet per second^2

(d) 103 feet per second^2

(e) 158 feet per second^2

$D''(t) = 4t^3 + 14t + 43$

$D''(t) = 12t^2 + 14$

$D''(2) = 12 \cdot 2^2 + 14 = 62$

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14. Find the derivative, $f'(x)$, where

$$f(x) = x^2\, e^{5x}$$

Product Rule:

Possibilities:

(a) $10x\, e^{5x}$

(b) $2x\, e^{5x} + 5x^2\, e^{5x}$

(c) $2x\, e^{5x}$

(d) $5x^3\, e^{5x} - 1$

(e) $2x\, e^{5x} + 5x^2\, e^{4x}$

$(x^2)' e^{5x} + x^2 (e^{5x})'$

$= 2x e^{5x} + 5x^2 e^{5x}$

---

15. Find the second derivative, $f''(x)$, where $f(x) = (3x-1)^5$.

Chain Rule:

In $= 3x-1$, Out $= (\cdots)^5$

In$' = 3$ Out$' = 5(\cdots)^4$

Possibilities:

(a) $15\,(3x-1)^4$

(b) $60\,(3x-1)^3$

(c) $5\,(3x-1)^4$

(d) $180\,(3x-1)^3$

(e) $20\,(3x-1)^3$

$f'(x) = 5(3x-1)^4 \cdot 3 = 15(3x-1)^4$

$f''(x) = 15 \cdot 4(3x-1)^3 \cdot 3$ ← Chain Rule, Again.

$= 180(3x-1)^3$

16. Let $f(t) = t^3$. Find a value c in the interval $(6, 9)$ so that the average rate of change of $f(t)$ from $t = 6$ to $t = 9$ is equal to the instantaneous rate of change of $f(t)$ at $t = c$.

$AROC = \frac{9^3 - 6^3}{3} = 171$

Possibilities:

(a) 57

(b) $\sqrt{(171/2)}$

(c) $\sqrt{57}$

(d) 513

(e) 171/2

$InstROC = 3c^2$. Now set AROC = Inst ROC

$3c^2 = 171 \Rightarrow c^2 = 57$

$c = \pm\sqrt{57}$.

Only want c in $[6, 9]$, so $c = \sqrt{57}$.

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17. Find the maximum value of $f(x)$ on $[1, 8]$ where $f(x) = |x - 3| + 13$.

y-coordinate. Max at endpoint, $x = 1$ or 8, OR where $f'(x) = 0$ or $f'(x)$ DNE.

Possibilities:

(a) 18

(b) 15

(c) 13

(d) 8

(e) 3

$f'(x)$ DNE at $x = 3$:

$f(3) = 13$

$f(1) = 15$

$f(8) = 18$ Max

---

18. Find the value of x in the interval $[-3, 4]$ where $f(x) = 2x^3 - 3x^2 - 36x$ attains its minimum value.

x-coordinate. $f'(x) = 6x^2 - 6x - 36 = 6(x - 3)(x + 2)$

$f'(x) = 0 \Rightarrow x = 3$ or -2.

Possibilities:

(a) $x = 4$

(b) $x = 27$

(c) $x = -81$

(d) $x = 3$

(e) $x = -3$

check endpoints + critical pts.

$f(4) = -64$

$f(-3) = 27$

$f(-2) = 44$

$f(3) = -81$ min. occurs at $x = 3$.

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19. $2000 is invested today and earns interest at an annual rate of 7.5 %, compounded continuously. How much is this investment worth in 8 years?

$P(t) = P_0 e^{rt}$

$P_0 = 2000, \; r = 0.075, \; t = 8$

$P(8) = 2000\, e^{0.075 \cdot 8}$

$= 2000\, e^{0.60}$

Possibilities:

(a) $2000e^{-60}$

(b) $2000(1 + 0.08)^8$

(c) $2000e^{-0.6}$

(d) $2000e^{60}$

(e) $2000e^{0.600}$

---

20. The population of a certain country doubles every 35 years. If we express the population as $P(t) = P_0 e^{r \cdot t}$, then find r.

$P_0 =$ Initial,

$P(35) = 2P_0$ (Doubles in 35 years).

So $2P_0 = P_0 e^{35r}$

$2 = e^{35r}$

$\Rightarrow \ln(2) = 35r$

$r = \dfrac{\ln(2)}{35}$.

Possibilities:

(a) $\dfrac{\ln(2)}{35}$

(b) $\dfrac{35}{\ln(2)}$

(c) $\dfrac{2}{\ln(35)}$

(d) $\dfrac{\ln(35)}{2}$

(e) $35 \cdot \ln(2)$
