Multiple Choice Questions
Show all your work on the page where the question appears.
Clearly mark your answer both on the cover page on this exam
and in the corresponding questions that follow.

Spring 2012 Exam 2

1. Suppose $f(x) = \dfrac{9}{x+6}$. Find the value of A, given that

$$\frac{f(x+h)-f(x)}{h} = \frac{A}{(x+6)(x+h+6)}.$$

Possibilities:

(a) -11
(b) -10
(c) -9
(d) -8
(e) -7

$$\frac{f(x+h)-f(x)}{h} = \frac{\left(\frac{9}{x+h+6}\right)-\left(\frac{9}{x+6}\right)}{h} = \frac{\frac{9(x+6)-9(x+h+6)}{(x+h+6)(x+6)}}{h}$$

$$= \frac{-9\cancel{h}}{(x+h+6)(x+6)\cancel{h}}$$

2. Suppose

$$\frac{f(x+h)-f(x)}{h} = 3x+3h+7.$$

Determine the slope of the tangent to the graph of $y = f(x)$ at $x = 3$.

Possibilities:

(a) 12
(b) 13
(c) 14
(d) 15
(e) 16

Slope T. L. $= f'(3)$

$$= \lim_{h\to 0} \frac{f(3+h)-f(3)}{h}$$

$$= \lim_{h\to 0} 3\cdot 3 + 3\cdot h + 7 = \lim_{h\to 0} 16 + \underbrace{3h}_{0}$$

$$= 16$$

3. Find $f'(x)$ where

$$f(x) = \frac{1}{\sqrt[3]{x}} = x^{-1/3}$$

$$f'(x) = \left(-\frac{1}{3}\right)x^{-\frac{1}{3}-1}$$

$$= \left(-\frac{1}{3}\right)x^{-\frac{4}{3}}$$

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Possibilities:

(a) $-\frac{1}{3}x^{-4/3}$

(b) $3x^2$

(c) $-3x^{-4}$

(d) $-\frac{1}{3}x^{-2/3}$

(e) $\frac{1}{(1/3)x^{-2/3}}$

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4. Suppose $F(x) = \frac{f(x)}{g(x)}$, $f(3) = 10$, $g(3) = 3$, $f'(3) = 3$, $g'(3) = 10$. Find $F'(3)$.

$$F'(3) = \frac{f'(3)g(3) - f(3)g'(3)}{g(3)^2}$$

$$= \frac{3 \cdot 3 - 10 \cdot 10}{3^2} = \frac{-91}{9}$$

Possibilities:

(a) $-91/9$

(b) $91/3$

(c) $91/9$

(d) $10/3$

(e) $-91/3$

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5. Determine $H'(2)$, provided that $H(t) = (t^2 + 3t - 1)(t^2 - 3)$.

$$H'(t) = (t^2+3t-1)' \cdot (t^2-3) + (t^2+3t-1)(t^2-3)'$$

$$= (2t+3)(t^2-3) + (t^2+3t-1)(2t)$$

$$H'(2) = (2\cdot 2+3)(2^2-3) + (2^2+3\cdot 2-1)(2\cdot 2)$$

Possibilities:

(a) 1

(b) -29

(c) 43

(d) 7

(e) 0

$$= 7 \cdot 1 + 9 \cdot 4 = 43$$

6. Find the derivative, $f'(3)$, where

$$f(x) = \sqrt{40 + x^2} = (40+x^2)^{1/2}$$

$$f'(x) = \tfrac{1}{\cancel{2}}(40+x^2)^{-1/2} \cdot \cancel{2}x$$

Der. of Inside

Possibilities:

(a) 1/7
(b) 2/7
(c) 3/7
(d) 4/7
(e) 5/7

$$= \frac{x}{\sqrt{40+x^2}}$$

So $f'(3) = \dfrac{3}{\sqrt{49}} = \dfrac{3}{7}$.

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7. Suppose $F(G(x)) = x^3$ and $G'(2) = 3$. Determine $F'(G(2))$.

Chain Rule.

$$(F(G(x)))' = F'(G(x)) \circ G'(x)$$

$$(x^3)' = 3x^2$$

So $3x^2 = F'(G(x)) \cdot G'(x)$

So $3 \cdot 2^2 = F'(G(2)) \cdot G'(2)$

$\Rightarrow 12 = F'(G(2)) \cdot 3$

$\Rightarrow F'(G(2)) = 4$

Possibilities:

(a) 3
(b) 12
(c) 4
(d) 1
(e) 6

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8. The tangent line to $y = f(x)$ at $x = 5$ is given by

$$y = -8(x-5) + 11.$$

Determine the $f(5) + f'(5)$. (Hint: use the tangent line to determine each of $f(5)$ and $f'(5)$. Then add.

$f'(5) = \text{slope} = -8$.

$f(5) \Rightarrow -8(5-5)+11 = 11$.

So $f(5) + f'(5)$

$= -8 + 11 = 3$

Possibilities:

(a) 3
(b) −29
(c) 8
(d) −5
(e) −11

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9. Find the fourth derivative, $f^{(4)}(x)$, where

$$f(x) = 2x^5 - 9x^2$$

Possibilities:

(a) $1250x^5$
(b) $28x - 18$
(c) $240x$
(d) $1250x$
(e) $28x$

$f'(x) = 10x^4 - 18x$
$f''(x) = 40x^3 - 18$
$f^{(3)}(x) = 120x^2$
$f^{(4)}(x) = 240x$

10. Find the derivative, $f'(t)$, where

$$f(t) = e^{t^2+4t+7}$$

$(2t+4)e^{t^2+4t+7}$

Possibilities:

(a) $(2t+4)e^{t^2+4t+7}$
(b) e^{2t+4}
(c) $\ln(t^2+4t+7)$
(d) e^{t^2+4t+7}
(e) $(t^2+4t+7)e^{t^2+4t+6}$

11. Find the derivative, $f'(x)$, where

$$f(x) = \ln(x^2+4x+3)$$

$f'(x) = \dfrac{2x+4}{x^2+4x+3}$

Possibilities:

(a) x^2+4x+3
(b) $\dfrac{1}{x^2+4x+3}$
(c) $\dfrac{2x+4}{x^2+4x+3}$
(d) $2x+4$
(e) $\dfrac{x^2+4x+3}{2x+4}$

12. Find the derivative, $f'(38)$, where

$$f(x) = x^2 + e^{-x}$$

$f'(x) = 2x - e^{-x}$

$f'(38) = 2\cdot 38 - e^{-38}$

$= 76 - e^{-38}$

Possibilities:

(a) $76 - 38e^{-37}$
(b) $76 + e^{-38}$
(c) $76 - e^{-38}$
(d) $76 + 38e^{-38}$
(e) $76 - 38e^{-39}$

13. Find the 12th derivative, $f^{(12)}(x)$, where

$$f(x) = e^{3x}$$

$f^{(12)}(x) = 3^{12} e^{3x}$

Possibilities:

(a) 0
(b) $3^{12}e^{3x}$
(c) $12e^{3x}$
(d) 12^3e^{3x}
(e) $3e^{3x-1}$

14. How much money must be invested now in order to have \$5000 in 6 years, assuming interest is compounded continuously at an annual rate of 7.5 % ?

$P(t) = P_0 e^{0.075t}$

$P(6) = 5000 = P_0 e^{0.075\cdot 6}$

So $P_0 = \dfrac{5000}{e^{.075\cdot 6}} = \dfrac{5000}{e^{.45}}$

$= 5000\, e^{-.45}$

Possibilities:

(a) $5000\, e^{0.450}$
(b) $5000\, e^{45.0}$
(c) $5000\, e^{-0.450}$
(d) $5000\, e^{-45.0}$
(e) $5000\,(1 + 0.08)^{-6}$

15. The population of a certain country doubles every 21 years. If we express the population as $P(t) = P_0 e^{r \cdot t}$, then find r.

Possibilities:

(a) $\frac{\ln(2)}{21}$

(b) $\frac{21}{\ln(2)}$

(c) $\frac{2}{\ln(21)}$

(d) $\frac{\ln(21)}{2}$

(e) $21 \cdot \ln(2)$

$P(21) = 2P_0 = P_0 e^{21r}$

$\Rightarrow 2 = e^{21r}$

$\ln(2) = 21r$

$r = \frac{\ln(2)}{21}$

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16. Find the maximum value of $f(x)$ on $[0, 4]$ where $f(x) = 2x^3 - 3x^2 - 12x$.

Possibilities:

(a) Maximum value = 32

(b) Maximum value = 4

(c) Maximum value = 0

(d) Maximum value = −20

(e) Maximum value = 7

$f'(x) = 0 \Rightarrow 6x^2 - 6x - 12 = 0$

$\Rightarrow x^2 - x - 2 = 0$

$\Rightarrow (x+1)(x-2) = 0$

Not in $[0, 4]$

$\Rightarrow \cancel{x = -1}$, or $x = 2$

$f(0) = 2 \cdot 0^3 - 3 \cdot 0^2 - 12 \cdot 0 = 0$

$f(2) = 2 \cdot 2^3 - 3 \cdot 2^2 - 12 \cdot 2 = -20.$

$f(4) = 2 \cdot 4^3 - 3 \cdot 4^2 - 12 \cdot 4 = 32$

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17. For which value of x does $f(x) = 1 + 2x + 3x^2 + 4x^3 + 5x^4$ attain it maximum value, on the interval $[2, 6]$?

Possibilities:

(a) Maximum value attained at $x = 3$

(b) Maximum value attained at $x = 2$

(c) Maximum value attained at $x = 6$

(d) Maximum value attained at $x = 4$

(e) $f(x)$ does not attain a maximimum value on $[2, 6]$

$f'(x) = 2 + 6x + 12x^2 + 20x^3$

For x in $[2, 6]$ $f'(x)$ is always positive, so max must be at an endpoint.

$f(2) = 129.$

$f(6) = 7965$

18. According to the Extreme Value Theorem, which of the functions are guaranteed to attain a maximum value on the given interval?

EVT: Continuous function on closed & bounded interval has extreme values.

(I) A continuous function on $(-\infty, \infty)$

(II) A continuous function on $[-1, 3]$ True

(III) A continuous function on $[0, 9]$ True

Possibilities:

(a) Only (II) is guaranteed to have a maximum and a minimum.

(b) (II) and (III) are guaranteed to have maxima and minima

(c) Only (I) is guaranteed to have a maximum and a minimum.

(d) Only (III) is guaranteed to have a maximum and a minimum.

(e) (I) and (III) are guaranteed to have maxima and minima

Function on $(-\infty,\infty)$ may or may not have extreme values.

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19. Let $f(x) = x^2 + 9$. Find a value c between $x = 4$ and $x = 8$ so that the average rate of change of $f(x)$ from $x = 4$ to $x = 8$ is equal to the instantaneous rate of change of $f(x)$ at $x = c$.

$$AROC = \frac{f(8) - f(4)}{8-4} = 12$$

$$\text{INST. ROC} = f'(x) = 2x$$

So $2c = 12 \Rightarrow c = 6$

Possibilities:

(a) 4

(b) 5

(c) 6

(d) 7

(e) 8

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20. Suppose $g(x) = 2x^3$ and the tangent line to $y = f(x)$ at $x = 2$ is given by $y = 5(x - 2) + 5$. Determine the slope of tangent line to $y = f(x) \cdot g(x)$ at $x = 2$

$$y' = f'(2)g(2) + f(2)g'(2)$$

$f'(2) = 5$ (slope), $f(2) = 5$

$g(2) = 2 \cdot 2^3 = 16$, $g'(2) = 2 \cdot 3 \cdot 2^2 = 24$

So $y' = 5 \cdot 16 + 5 \cdot 24 = 200$.

Possibilities:

(a) 200

(b) 80

(c) 29

(d) 21

(e) 120