| MA 123 — Elem. Calculus<br>EXAM 2 | Spring 2013<br>3/6/2013 | **Name:** Solutions. | **Sec.:** |
|---|---|---|---|

Do not remove this answer page — you will turn in the entire exam. You have two hours to do this exam. No books or notes may be used. You may use a graphing calculator during the exam, but NO calculator with a Computer Algebra System (CAS) or a QWERTY keyboard is permitted. Absolutely no cell phone use during the exam is allowed.

The exam consists of multiple choice questions. Record your answers on this page. For each multiple choice question, you will need to fill in the box corresponding to the correct answer. For example, if (b) is correct, you must write

[a] [**b**] [c] [d] [e]

Do not circle answers on this page, but please circle the letter of each correct response in the body of the exam. It is your responsibility to make it CLEAR which response has been chosen. You will not get credit unless the correct answer has been marked on both this page and in the body of the exam.

**GOOD LUCK!**

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| **1.** | a | b | c | **d** | e | **11.** | a | b | c | **d** | e |
| **2.** | a | b | c | **d** | e | **12.** | a | b | c | d | **e** |
| **3.** | a | b | c | d | **e** | **13.** | a | b | **c** | d | e |
| **4.** | a | b | c | **d** | e | **14.** | a | b | c | **d** | e |
| **5.** | a | **b** | c | d | e | **15.** | a | b | **c** | d | e |
| **6.** | a | b | **c** | d | e | **16.** | **a** | b | c | d | e |
| **7.** | a | b | c | d | **e** | **17.** | a | b | c | **d** | e |
| **8.** | a | b | c | d | **e** | **18.** | a | b | c | **d** | e |
| **9.** | a | **b** | c | d | e | **19.** | **a** | b | c | d | e |
| **10.** | a | b | c | d | **e** | **20.** | **a** | b | c | d | e |

**For grading use:**

| **Number Correct** | |
|---|---|
| | **(out of 20 problems)** |

| **Total** | |
|---|---|
| | **(out of 100 points)** |

Please make sure to list the correct section number on the front page of your exam. In case you forgot your section number, consult the following table. Your section number is determined by your recitation time and location.

| Section # | Instructor | Day and Time | Room |
|---|---|---|---|
| 001 | F. Smith | T, 8:00 am - 9:15 am | CB 213 |
| 002 | W. Hough | R, 8:00 am - 9:15 am | CB 213 |
| 003 | D. Akers | T, 12:30 pm - 1:45 pm | CB 342 |
| 004 | W. Hough | R, 9:30 am - 10:45 am | CP 397 |
| 005 | D. Akers | T, 11:00 am - 12:15 pm | TPC 212 |
| 006 | W. Hough | R, 11:00 am - 12:15 pm | TPC 113 |
| 007 | A. Happ | T, 2:00 pm - 3:15 pm | TPC 109 |
| 008 | A. Hubbard | R, 2:00 pm - 3:15 pm | L 108 |
| 009 | A. Happ | T, 11:00 am - 12:15 pm | TPC 113 |
| 010 | A. Hubbard | R, 11:00 am - 12:15 pm | CB 340 |
| 011 | A. Happ | T, 12:30 pm - 1:45 pm | TEB 231 |
| 012 | A. Hubbard | R, 12:30 pm - 1:45 pm | EH 307 |
| 013 | L. Solus | T, 11:00 am - 12:15 pm | CB 340 |
| 014 | D. Akers | R, 11:00 am - 12:15 pm | TPC 101 |
| 015 | L. Solus | T, 12:30 pm - 1:45 pm | OT 0B7 |
| 016 | F. Smith | R, 12:30 pm - 1:45 pm | FB B4 |
| 017 | L. Solus | T, 2:00 pm - 3:15 pm | FB B4 |
| 018 | F. Smith | R, 2:00 pm - 3:15 pm | CB 245 |
| 019 | X. Kong | T, 3:30 pm - 4:45 pm | BH 303 |
| 020 | Q. Liang | R, 3:30 pm - 4:45 pm | EGJ 115 |
| 021 | X. Kong | T, 12:30 pm - 1:45 pm | CB 205 |
| 022 | X. Kong | R, 2:00 pm - 3:15 pm | CB 233 |
| 023 | L. Davidson | T, 9:30 am - 10:45 am | OT 0B7 |
| 024 | L. Davidson | R, 9:30 am - 10:45 am | OT 0B7 |
| 026 | L. Davidson | R, 8:00 am - 9:15 am | CB 243 |
| 027 | Q. Liang | T, 9:30 am - 10:45 am | DH 131 |

**Multiple Choice Questions**

*Show all your work on the page where the question appears. Clearly mark your answer both on the cover page on this exam and in the corresponding questions that follow.*

1. Suppose that

$$\frac{f(x+h)-f(x)}{h} = 8x^2 - 5h + 4$$

and $f(1) = 4$. Find the equation of the tangent line to the graph of $y = f(x)$ at $x = 1$.

$f'(1) = \text{Slope} = \lim_{h \to 0} \frac{f(1+h)-f(1)}{h} = \lim_{h \to 0} 8 \cdot 1^2 - 5 \cdot h + 4$

$= 12 = \text{Slope}$

**Possibilities:** Point $(1, 4)$

(a) $y = 8x + 4$

(b) $y = 12x - 4$

(c) $y = 16x - 5$

(d) $y = 12x - 8$

(e) $y = 16x + 4$

T. Line $y - 4 = 12(x-1)$

$y = 12x - 12 + 4$

$y = 12x - 8$

2. If $f(x) = (x+15)^2$, then

$$\frac{f(x+h)-f(x)}{h} = Ax + Bh + C.$$

Determine the value of $B$.

$\frac{(x+h+15)^2 - (x+15)^2}{h} = \frac{(x+15+h)^2 - (x+15)^2}{h}$

$= \frac{\cancel{(x+15)^2} + 2(x+15)h + h^2 - \cancel{(x+15)^2}}{h}$

**Possibilities:**

(a) $-2$

(b) $-1$

(c) $0$

(d) $1$

(e) $2$

$= \frac{(2(x+15)+h)\cancel{h}}{\cancel{h}} = 2x + 30 + h$

$B = 1$

3. If $H(t) = 2t^3 + 4t^2 + 5t + 3$, find $H'(t)$.

$$H'(t) = 2\cdot 3t^2 + 4\cdot 2t + 5$$
$$= 6t^2 + 8t + 5$$

**Possibilities:**

(a) $6t^2 + 8t + 8$

(b) $6t^3 + 8t^2 + 5t + 3$

(c) 19

(d) 22

(e) $6t^2 + 8t + 5$

---

4. If $Y(s) = \dfrac{1}{3s^5}$, then find $Y'(s)$.

$$Y(s) = \tfrac{1}{3} s^{-5}$$
$$Y'(s) = \tfrac{1}{3}\cdot(-5) s^{-6} = -\tfrac{5}{3} s^{-6}$$

**Possibilities:**

(a) $\dfrac{1}{15s^4}$

(b) $-15s^{-4}$

(c) $-15s^{-6}$

(d) $\dfrac{-5}{3}s^{-6}$

(e) $\dfrac{-5}{3}s^{-4}$

---

5. Suppose $h(x) = x^2 + 7x + 5$, $g(3) = 2$, $g'(3) = -4$, and $F(x) = g(x) \cdot h(x)$. Find $F'(3)$.

$$F'(x) = g'(x)h(x) + g(x)h'(x)$$
$$F'(3) = g'(3)h(3) + g(3)h'(3)$$
$$= -4\cdot(3^2 + 7\cdot 3 + 5) + 2\cdot(2\cdot 3 + 7)$$
$$= -4\cdot 35 + 26 = -140 + 26$$
$$= -114$$

**Possibilities:**

(a) $-52$

(b) $-114$

(c) 166

(d) $-110$

(e) 70

6. If $f(s) = (s^2 + 3s + 8)^8$, find $f'(s)$.

Chain Rule:

$8(s^2+3s+8)^7 \cdot (2s+3)$

**Possibilities:**

(a) $15s^{15} + 3s^7 + 8$
(b) $8(s^2 + 3s + 8)^7$
(c) $8(2s + 3) \cdot (s^2 + 3s + 8)^7$
(d) $16(s^2 + 3s + 8)^7$
(e) $8(2s + 8)^7$

---

7. If $f(x) = (3x + 5)^7$, find the second derivative $f''(x)$.

From Chain

$f'(x) = 7 \cdot (3x+5)^6 \cdot 3$

$= 21(3x+5)^6$

**Possibilities:**

(a) $126x^5$
(b) $42(3x + 5)^5$
(c) $126(3x + 5)^5$
(d) $378x^5$
(e) $378(3x + 5)^5$

$f''(x) = 21 \cdot 6\,(3x+5)^5 \cdot 3$

$= 378(3x+5)^5$

$$\begin{array}{r} 21 \\ 18 \\ \hline 168 \\ 210 \\ \hline 378 \end{array}$$

---

8. Suppose $F(x) = g(h(x))$. If $g(2) = 2$, $g'(2) = 7$, $h(0) = 2$, and $h'(0) = 3$, find $F'(0)$.

Chain: In $= h(x)$ In$' = h'(x)$
Out $= g(--)$ Out$' = g'(--)$

**Possibilities:**

(a) 8
(b) 20
(c) 6
(d) 14
(e) 21

$F'(x) = g'(h(x)) \cdot h'(x)$

$F'(0) = g'(h(0)) \cdot h'(0)$

$= g'(2) \cdot 3 = 7 \cdot 3 = 21$

---

9. Find the 14th derivative, $f^{(14)}(x)$, where

$$f(x) = e^{4x}.$$

**Possibilities:**

(a) 0

(b) $4^{14}e^{4x}$

(c) $14e^{4x}$

(d) $14^4e^{4x}$

(e) $4e^{4x-1}$

$f'(x) = 4e^{4x}$

$f''(x) = 4 \cdot 4e^{4x} = 4^2e^{4x}$

$f'''(x) = 4^2 \cdot 4e^{4x} = 4^3e^{4x}$

$f^{(14)}(x) = 4^{14}e^{4x}$

10. If

$$f(x) = \ln(3x + 11),$$

find $f'(x)$.

**Possibilities:**

(a) $\dfrac{1}{3x+11}$

(b) 3

(c) $e^{3x+11}$

(d) $\dfrac{3}{x} + \dfrac{1}{11}$

(e) $\dfrac{3}{3x+11}$

$f'(x) = \dfrac{(3x+11)'}{3x+11} = \dfrac{3}{3x+11}$

11. Let

$$g(s) = \frac{5s+2}{s^2+3s+17}.$$

Find the derivative $g'(s)$.

**Possibilities:**

(a) $5(s^2+3s+17) - (5s+2)(2s+3)$

(b) $\dfrac{(5s+2)(2s+3) - 5(s^2+3s+17)}{(s^2+3s+17)^2}$

(c) $\dfrac{5s+2}{s^2+3s+17}$

(d) $\dfrac{5(s^2+3s+17) - (5s+2)(2s+3)}{(s^2+3s+17)^2}$

(e) $5(s^2+3s+17) + (5s+2)(2s+3)$

Quotient

$$g'(s) = \frac{\left((5s+2)'(s^2+3s+17) - (5s+2)(s^2+3s+17)'\right)}{(s^2+3s+17)^2}$$

$$= \frac{5(s^2+3s+17) - (5s+2)(2s+3)}{(s^2+3s+17)^2}$$

12. If

$$f(x) = x^5 e^{7x},$$

Product

find the first derivative, $f'(x)$.

$(x^5)' \cdot e^{7x} + x^5 (e^{7x})'$

$= 5x^4 e^{7x} + x^5 \cdot 7e^{7x}$

**Possibilities:**

(a) $5x^4e^{7x} + x^5e^{7x}$

(b) $35x^4e^{7x}$

(c) $5x^4e^{7x} - 7x^5e^{7x}$

(d) $\dfrac{5x^4e^{7x} + 7x^5e^{7x}}{e^{14x}}$

(e) $5x^4e^{7x} + 7x^5e^{7x}$

---

13. The graph of $y = f(x)$ passes through the point $(0, 13)$. The slope of the tangent line to $y = f(x)$ at any point $P$ is 2 times the $y$-coordinate of $P$. Find $f(1)$.

From Exp-Log Notes $\Rightarrow$ f(x) must be

$Ce^{2x} = f(x)$ $\rightarrow$ $f(x) = 13e^{2x}$

Now, $f(0) = 13$

so $Ce^{2 \cdot 0} = 13$

$C \cdot 1 = 13$

$f(1) = 13e^2$

**Possibilities:**

(a) 11

(b) $2e^{13}$

(c) $13e^2$

(d) $26e^2$

(e) 15

---

14. If the number of bacteria in a culture doubles every 3 hours, how many hours will it take before 5 times the original number is present? (HINT: The number of bacteria at time $t$ follows an exponential model, $y(t) = P_0e^{rt}$. You may need to find the value of $r$ before you can solve this problem.)

$\underbrace{y(3) = 2P_0}_{\text{Doubles}} \Rightarrow P_0e^{3r} = 2P_0 \Rightarrow 3r = \ln(2)$

$r = \dfrac{\ln(2)}{3}$

$\underbrace{y(t) = 5P_0}_{\text{When 5 times amount}} \Rightarrow P_0e^{\frac{\ln(2)}{3} \cdot t} = 5P_0$

$e^{\frac{\ln(2)}{3} t} = 5$

$\dfrac{\ln(2)}{3} t = \ln(5) \Rightarrow t = \dfrac{3\ln(5)}{\ln(2)}$

**Possibilities:**

(a) $5/2$

(b) $\dfrac{\ln(3)}{2}$

(c) 4

(d) $\dfrac{3\ln(5)}{\ln(2)}$

(e) $\dfrac{5\ln(3)}{\ln(2)}$

---

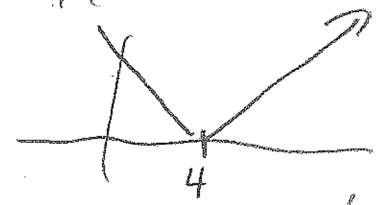
15. Let

$$f(x) = |x-4|.$$

Compute

$$\lim_{h\to 0} \frac{f(4+h)-f(4)}{h}.$$

This is $f'(4)$. But $y = f(x)$ is




So $f'(4)$ DNE, since graph has corner @ $x=4$.

**Possibilities:**

(a) $-4$

(b) 0

(c) This limit does not exist

(d) 1

(e) 4

16. Suppose the equation of the tangent line to the graph of $g(x)$ at $x=2$ is

$$y = 6 + 11(x-2).$$

Find $g(2)$ and $g'(2)$.

$g'(2) = 11$

$g(2) = 6.$

**Possibilities:**

(a) $g(2) = 6$ and $g'(2) = 11$

(b) $g(2) = 2$ and $g'(2) = 6$

(c) $g(2) = 11$ and $g'(2) = 6$

(d) $g(2) = 2$ and $g'(2) = 11$

(e) $g(2) = 6$ and $g'(2) = 2$

17. Find the value of $t$ in the interval $[0,4]$ where $f(t) = 2t^3 - 6t^2 - 18t - 10$ attains its minimum value.

First, find critical points:

$f'(t) = 6t^2 - 12t - 18 = 6(t^2 - 2t - 3) = 0$

$\Rightarrow t^2 - 2t - 3 = 0 \Rightarrow (t-3)(t+1) = 0$

$\Rightarrow t = 3, t = -1$

Discard, not in $[0,4]$

$f(0) = -10$

$f(3) = 2\cdot 3^3 - 6\cdot 3^2 - 18\cdot 3 - 10 = -64$

$f(4) = 2\cdot 4^3 - 6\cdot 4^2 - 18\cdot 4 - 10$

$128 - 96 - 72 - 10$

$128 - 178 = -50.$

Min is $-64$. Occurs at $t=3$.

**Possibilities:**

(a) $t = 0$

(b) $t = -64$

(c) $t = 4$

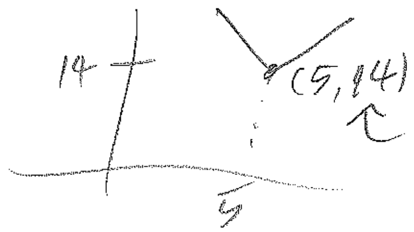
(d) $t = 3$

(e) $t = -1$

18. Find the minimum value of $f(x) = |x-5| + 14$ on the interval $[2, 9]$.

$y = |x|$ $\qquad$ $y = |x-5| + 14$

**Possibilities:**




min. Value = 14

(a) 5
(b) 2
(c) 18
(d) 14
(e) 17

---

19. Let $f(x) = x^2 + 3x - 2$. Find a value $c$ between $x = 0$ and $x = 4$ so that the average rate of change of $f(x)$ from $x = 0$ to $x = 4$ is equal to the instantaneous rate of change of $f(x)$ at $x = c$.

$$ARoC = \frac{f(4) - f(0)}{4 - 0} = \frac{(4^2 + 3\cdot 4 - 2) - (0^2 + 3\cdot 0 - 2)}{4} = 7$$

**Possibilities:**

(a) 2
(b) 26
(c) 11
(d) 7
(e) 12

Inst. RoC $= f'(x) = 2x + 3$

Set $2x + 3 = 7$

$\Rightarrow 2x = 4$

$x = 2.$

---

20. In the previous problem, you were given a function $f(x)$ which was differentiable on an interval $[a, b]$, and you found some number $c$ satisfying $a < c < b$ so that the instantaneous rate of change of $f(x)$ at $x = c$ was equal to the average rate of change from $x = a$ to $x = b$. Which theorem states that the above problem must have a solution?

**Possibilities:**

(a) Mean value theorem
(b) Extreme value theorem
(c) Fermat's theorem
(d) Pythagorean theorem
(e) Fundamental theorem of calculus