MA123 — Elem. Calculus Spring 2015 Name: SOLUTIONS Sec.: ________

Exam 2 2015-03-12

Do not remove this answer page — you will turn in the entire exam. You have two hours to do this exam. No books or notes may be used. You may use an ACT-approved calculator during the exam, but NO calculator with a Computer Algebra System (CAS), networking, or camera is permitted. Absolutely no cell phone use during the exam is allowed.

The exam consists of two short answer questions and eighteen multiple choice questions. Answer the short answer questions on the back of this page, and record your answers to the multiple choice questions on this page. For each multiple choice question, you will need to fill in the circle corresponding to the correct answer. For example, if (a) is correct, you must write

(a) (b) (c) (d) (e)

Do not circle answers on this page, but please circle the letter of each correct response in the body of the exam. It is your responsibility to make it CLEAR which response has been chosen. You will not get credit unless the correct answer has been marked on both this page and in the body of the exam.

GOOD LUCK!

3. (a) (b) (c) (d) (e)
4. (a) (b) (c) (d) (e)
5. (a) (b) (c) (d) (e)
6. (a) (b) (c) (d) (e)
7. (a) (b) (c) (d) (e)
8. (a) (b) (c) (d) (e)
9. (a) (b) (c) (d) (e)
10. (a) (b) (c) (d) (e)
11. (a) (b) (c) (d) (e)
12. (a) (b) (c) (d) (e)
13. (a) (b) (c) (d) (e)
14. (a) (b) (c) (d) (e)
15. (a) (b) (c) (d) (e)
16. (a) (b) (c) (d) (e)
17. (a) (b) (c) (d) (e)
18. (a) (b) (c) (d) (e)
19. (a) (b) (c) (d) (e)
20. (a) (b) (c) (d) (e)

For grading use:

Multiple Choice		Short Answer
(number right)	(5 points each)	(out of 10 points)

Total	
	(out of 100 points)

Short Answer Questions

Write your answers on this page.

You must show proper, logical, sensible and legible work to be sure you will get full credit.

1. Find the **derivative** of $f(x) = e^{\sqrt{3x+14}}$. You do **not** need to simplify your answer.

$$f(x) = e^{(3x+14)^{\frac{1}{2}}}$$

$$f'(x) = \underbrace{e^{(3x+14)^{\frac{1}{2}}}}_{e\ \text{rule}} \cdot \underbrace{\tfrac{1}{2}(3x+14)^{-\frac{1}{2}}}_{\text{power rule}} \cdot \underset{\text{deriv. of inside}}{3}$$

Final answer: $e^{\sqrt{3x+14}} \cdot \frac{1}{2}(3x+14)^{-\frac{1}{2}} \cdot 3$

2. Let $f(x) = (x+4)^2 \cdot g(x)$. If $g(-1) = -2$ and $g'(-1) = 4$, find $f'(-1)$.

$$f'(x) = \underset{\text{copy}}{(x+4)^2}\ \underset{\text{deriv.}}{g'(x)} + \underset{\text{copy}}{g(x)} \cdot \underbrace{2(x+4)(1)}_{\text{deriv}}$$

$$f'(-1) = (-1+4)^2 \cdot g'(-1) + g(-1) \cdot 2(-1+4)$$

$$= 3^2 \cdot 4 + (-2) \cdot 2(3)$$

$$= 36 - 12$$

Final answer: 24

Name: SOLUTIONS

Multiple Choice Questions

Show all your work on the page where the question appears.
Clearly mark your answer both on the cover page on this exam
and in the corresponding questions that follow.

3. Find the derivative, $f'(x)$, if $f(x) = \sqrt{4x^3 + 5x^2 + 6x + 2}$. $= (4x^3 + 5x^2 + 6x + 2)^{\frac{1}{2}}$

Possibilities:

(a) $(1/2)(4x^3 + 5x^2 + 6x + 2)(12x^2 + 10x + 6)$

(b) $(1/2)(4x^3 + 5x^2 + 6x + 2)^{-1/2}(12x^2 + 10x + 6)$

(c) $(1/2)(4x^3 + 5x^2 + 6x + 2)^{1/2}$

(d) $\sqrt{12x^2 + 10x + 6}$

(e) $\dfrac{\sqrt{12x^2 + 10x + 6}}{\sqrt{4x^3 + 5x^2 + 6x + 2}}$

power rule 1st, then deriv. of inside.

4. Find the derivative, $f'(x)$, if $f(x) = e^{2x^3+6x^2+7x}$.

Possibilities:

(a) $\dfrac{6x^2 + 12x + 7}{2x^3 + 6x^2 + 7x}$

(b) $\ln(2x^3 + 6x^2 + 7x)$

(c) $(6x^2 + 12x + 7)e^{2x^3+6x^2+7x}$

(d) $(6x^2 + 12x + 7)e^x$

(e) $e^{6x^2+12x+7}$

e rule 1st, then deriv. of exponent

5. For the function $f(x) = 2x^3 + 4x^2 + 3x + 1$, find the equation of the tangent line to graph of f at $x = 3$.

at $x = 3$, $y = f(3) = 2 \cdot 3^3 + 4 \cdot 3^2 + 3 \cdot 3 + 1 = 100$

Possibilities:

(a) $y = x^3 + 17$

(b) $y = 100$

(c) $y = 100x - 219$

(d) $y = 81x + 100$

(e) $y = 81x - 143$

$f'(x) = 6x^2 + 8x + 3$

$m = f'(3) = 6 \cdot 3^2 + 8(3) + 3 = 81$

$y - y_1 = m(x - x_1) \Rightarrow y - 100 = 81(x-3)$

$y - 100 = 81x - 243$ $\quad y = 81x - 243 + 100$

6. Suppose $F(x) = g(x) \cdot h(x+2)$. If $g(0) = 5$, $g'(0) = 6$, $h(0) = 8$, $h'(0) = 9$, $h(2) = 3$, and $h'(2) = 7$, find $F'(0)$.

$$F'(x) = \underbrace{g(x)}_{\text{copy}} \cdot \underbrace{h'(x+2)\cdot 1}_{\text{deriv.}} + \underbrace{h(x+2)}_{\text{copy}}\ \underbrace{g'(x)}_{\text{deriv.}}$$

$$F'(0) = g(0) \cdot h'(0+2) + h(0+2) \cdot g'(0)$$
$$= 5 \cdot 7 + 3 \cdot 6$$
$$= 35 + 18$$

Possibilities:

(a) 88

(b) 102

(c) 38

(d) 53

(e) 174

7. Suppose $g(5) = -9$ and $g'(5) = 7$. Find $F'(5)$ if

$$F(x) = \frac{g(x)}{x^2}$$

$$F'(x) = \frac{x^2 \cdot g'(x) - g(x) \cdot 2x}{(x^2)^2}$$

$$F'(5) = \frac{5^2 \cdot g'(5) - g(5) \cdot 2(5)}{25^2}$$

$$= \frac{25(7) - (-9)(10)}{625} = \frac{265}{625} = \frac{53}{125}$$

Possibilities:

(a) $\frac{53}{25}$

(b) $\frac{53}{5}$

(c) $\frac{53}{125}$

(d) $\frac{7}{5}$

(e) $-\frac{53}{125}$

8. Suppose $F(x) = (g(x))^5 + 7$. If $g(2) = 9$, $g'(2) = 13$, and $g''(2) = 3$, then find $F'(2)$.

$$F'(x) = \underbrace{5 \cdot (g(x))^4}_{\text{power rule}} \cdot \underbrace{g'(x)}_{\text{deriv. of inside}} + \underbrace{0}_{\text{deriv. of constant}}$$

$$F'(2) = 5 \cdot (g(2))^4 \cdot g'(2)$$
$$= 5 \cdot 9^4 \cdot 13$$

Possibilities:

(a) $(5)(9^4) + 7$

(b) 3

(c) $13^5 + 7$

(d) $(5)(9^4)(13)$

(e) $9^5 + 7$

9. If $f(x) = \dfrac{8}{x+5}$ then choose the simplified form of $\frac{f(x+h)-f(x)}{h}$:

Possibilities:

(a) $\dfrac{16x+80+8h}{(x+h+5)(x+5)(2x+h)}$

(b) $\dfrac{hx^2+10hx+25h-8}{(x+5)^2}$

(c) $-\dfrac{8}{(x+h+5)(x+5)}$

(d) $\dfrac{8}{(x+h+5)(x+5)}$

(e) $-\dfrac{8}{(x+h+5)^2}$

$$\frac{\dfrac{8}{x+h+5} - \dfrac{8}{x+5}}{h}$$

$$= \frac{\dfrac{x+5}{x+5}\cdot\dfrac{8}{x+h+5} - \dfrac{x+h+5}{x+h+5}\cdot\dfrac{8}{x+5}}{\dfrac{h}{1}}$$

think of h as $h/1$

reciprocal

$$= \frac{8(x+5) - 8(x+h+5)}{(x+5)(x+h+5)} \cdot \frac{1}{h}$$

cancel these h

$$= \frac{8x+40-8x-8h-40}{(x+5)(x+h+5)\cdot h} = \frac{-8h}{(x+5)(x+h+5)h}$$

10. Find the derivative, $f'(x)$, if $f(x) = (6+9x)e^{2+9x}$.

Possibilities:

(a) $(63+81x)e^{2+9x}$

(b) $(9)e^9$

(c) $(81)e^9$

(d) $\dfrac{9}{2+9x}$

(e) $(9)e^{2+9x}$

$$f'(x) = \underbrace{(6+9x)}_{\text{copy}} \cdot \underbrace{e^{2+9x}\cdot 9}_{\text{der. of } e^{2+9x}} + \underbrace{e^{2+9x}}_{\text{copy}} \cdot \underbrace{(9)}_{\text{deriv. of } 6+9x}$$

$$= e^{2+9x}\left[(6+9x)9 + 9\right]$$

pull out the common factor, e^{2+9x}

$$= e^{2+9x}\left[54+81x+9\right] = e^{2+9x}(81x+63)$$

11. Suppose $F(x) = \ln(g(x))$. If $g(2) = 5$, $g'(2) = 3$, and $g''(2) = 11$, then find $F'(2)$.

Possibilities:

(a) $\ln(11)$

(b) $5/\ln(3)$

(c) $\ln(5)/3$

(d) $5/3$

(e) $3/5$

$$F'(x) = \underbrace{\frac{1}{g(x)}}_{\ln\text{ rule}} \cdot \underbrace{g'(x)}_{\text{deriv. of inside}} = \frac{g'(x)}{g(x)}$$

$$F'(2) = \frac{g'(2)}{g(2)} = \frac{3}{5}$$

12. For the function $f(x) = \begin{cases} x^2 - 9 & x < 10 \\ \sqrt{x+7} & 10 \le x < 20 \\ x^3 - 5 & 20 \le x \end{cases}$, find the equation of the tangent line to the graph of f at $x = 18$.

$x = 18$ is between 10 and 20, so only the middle line is relevant.

Possibilities:

(a) $y = 5x - \frac{199}{10}$

(b) $y = 36x - 333$

(c) $y = 315x - 1224$

(d) $y = \frac{1}{10}x + \frac{16}{5}$

(e) $y = 972x - 11669$

$y = f(18) = \sqrt{18+7} = \sqrt{25} = 5.$

For $10 < x < 20$, $f'(x) = \frac{1}{2}(x+7)^{-\frac{1}{2}} \cdot 1$

$f'(18) = \frac{1}{2}(18+7)^{-\frac{1}{2}} = \frac{1}{2} \cdot \frac{1}{\sqrt{25}} = \frac{1}{2} \cdot \frac{1}{5} = \frac{1}{10}$

$y - 5 = \frac{1}{10}(x - 18)$

$y - 5 = \frac{1}{10}x - \frac{18}{10}$

$y = \frac{1}{10}x - \frac{18}{10} + 5$

$y = \frac{1}{10}x - \frac{18}{10} + \frac{50}{10}$

$y = \frac{1}{10}x + \frac{32}{10}$

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13. Find the derivative, $f'(x)$, if $f(x) = (4 + 7x)\ln(8 + 5x)$.

Possibilities:

(a) $\frac{7}{8+5x}$

(b) $7 + \frac{5}{8+5x}$

(c) $1/x$

(d) $\frac{12}{8+5x}$

(e) $(7)\ln(8+5x) + \frac{20+35x}{8+5x}$

$f'(x) = \underbrace{(4+7x)}_{\text{copy}} \cdot \underbrace{\frac{1}{8+5x} \cdot 5}_{\text{deriv.}} + \underbrace{\ln(8+5x)}_{\text{copy}} \cdot \underbrace{7}_{\text{deriv}}$

$= \frac{5(4+7x)}{8+5x} + 7\ln(8+5x)$

$= \frac{20+35x}{8+5x} + 7\ln(8+5x)$

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14. For the function $f(x) = \ln(4x^2 + 7x + 6)$, find the equation of the tangent line to graph of f at $x = 0$.

Possibilities:

(a) $y = 3x + \ln(6)$

(b) $y = x^3 + 17$

(c) $y = \ln(6)$

(d) $y = \frac{7}{6}x + \ln(2) + \ln(3)$

(e) $y = x\ln(2) + x\ln(3) + \frac{7}{6}$

$y = f(0) = \ln(0+0+6) = \ln 6$

$f'(x) = \frac{1}{4x^2+7x+6} \cdot 8x + 7$

$m = f'(0) = \frac{1}{0+0+6} \cdot (0+7) = \frac{7}{6}$

$y - \ln 6 = \frac{7}{6}(x - 0)$

$y = \frac{7}{6}x + \ln 6$

Note: only choice (d) has the correct slope. Properties of logarithms yield $\ln 2 + \ln 3 = \ln(2 \cdot 3) = \ln 6$

15. If $f(x) = 7x^2 + 9x$ then find the second derivative $f''(x)$:

Possibilities:

(a) 6
(b) 14
(c) $28x^2$
(d) $14x + 16$
(e) $14x + 9$

$f'(x) = 14x + 9$

$f''(x) = 14$

16. If $f(x) = (16x + 35)^{28}$ then $f''(x) =$

Possibilities:

(a) $28(27)(16x + 35)^{26}(16)^2$
(b) $28^2(16)^{28}(16x + 35)$
(c) $28(27)16^{26}$
(d) 0
(e) $28(16x + 35)^{27}$

$f'(x) = 28(16x + 35)^{27} \cdot 16$

$= 16 \cdot 28(16x + 35)^{27}$

$f''(x) = \underline{16} \cdot 28 \cdot 27(16x + 35)^{26} \cdot \underline{16}$

17. Find the derivative, $f'(x)$, of $f(x) = \frac{1}{x^8} = x^{-8}$

Possibilities:

(a) $-8x^{-7}$
(b) $1/(8x^7)$
(c) $-8x^{-9}$
(d) $8x^7$
(e) $1/(8x^9)$

$f'(x) = -8x^{-8-1}$

$= -8x^{-9}$

18. If an amount of x dollars is invested at 2% interest compounded continuously, and at the end of
years the value of the investment is \$3000, find x. five

Possibilities:

(a) \$2714.51
(b) \$3315.51
(c) \$300
(d) \$2000
(e) \$588.11

$$P = P_0 e^{rt}$$

$$3000 = P_0 e^{.02(5)}$$

$$P_0 = \frac{3000}{e^{.1}}$$

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19. The half-life of cadmium-109 is 1.267 years. If a sample has a mass of 800 g, find the mass (in g) that remains after 2 years.

Possibilities:

(a) 241.07 g
(b) 515.69 g
(c) 572.99 g
(d) 2389.33 g
(e) 267.86 g

Find r using the half-life:

$$\frac{1}{2} = e^{1.267r}$$

$$1.267r = \ln \frac{1}{2}$$

$$r = \frac{\ln \frac{1}{2}}{1.267} \approx -.5471$$

Now find the amount left after 2 years:

$$P = P_0 e^{rt}$$

$$P = \boxed{800 e^{-.5471(2)}}$$

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20. Find the maximum of $g(t) = (t-3)^2 + 4$ on the interval $[0, 5]$

Possibilities:

(a) 13
(b) 3
(c) 8
(d) 4
(e) 16

$g(t)$ is continuous on the closed interval $[0, 5]$.

$g'(t) = 2(t-3)(1) + 0 = 2t - 6$

$g'(t) = 0$ when $2t - 6 = 0 \Rightarrow t = 3$

Test endpoints and crit. # in original function:

$g(0) = (0-3)^2 + 4 = 13$ $\quad g(3) = (3-3)^2 + 4 = 4$

$g(5) = (5-3)^2 + 4 = 8,$ max value is 13.