MA 123 — Elem. Calculus   Fall 2010   Name: _____   Sec.: _____

EXAM 3   11/17/2010

Do not remove this answer page — you will turn in the entire exam. You have two hours to do this exam. No books or notes may be used. You may use a graphing calculator during the exam, but NO calculator with a Computer Algebra System (CAS) or a QWERTY keyboard is permitted. Absolutely no cell phone use during the exam is allowed.

The exam consists of multiple choice questions. Record your answers on this page. For each multiple choice question, you will need to fill in the box corresponding to the correct answer. For example, if (b) is correct, you must write

a   **b**   c   d   e

Do not circle answers on this page, but please circle the letter of each correct response in the body of the exam. It is your responsibility to make it CLEAR which response has been chosen. You will not get credit unless the correct answer has been marked on both this page and in the body of the exam.

GOOD LUCK!

1. a b c d e   11. a b c d e

2. a b c d e   12. a b c d e

3. a b c d e   13. a b c d e

4. a b c d e   14. a b c d e

5. a b c d e   15. a b c d e

6. a b c d e   16. a b c d e

7. a b c d e   17. a b c d e

8. a b c d e   18. a b c d e

9. a b c d e   19. a b c d e

10. a b c d e   20. a b c d e

For grading use:

Number Correct	
	(out of 20 problems)

Total	
	(out of 100 points)

Please make sure to list the correct section number on the front page of your exam. In case you forgot your section number, consult the following table. If you are enrolled in a lecture with recitation, then your section number is determined by your recitation time and location.

Section #	Instructor	Lectures
001	T. Chapman	MWF 8:00 am - 8:50 am, CB 118
002	D. Leep	MWF 12:00 pm - 12:50 pm, KAS 213
003	M. Shaw	TR 8:00 am - 9:15 am, CP 155
004	J. Schmidt	TR 12:30 am- 1:45 am, CP 155
005	M. Music	T 3:30 pm - 4:45 pm, CP 345
006	M. Music	R 3:30 pm - 4:45 pm, CP 208
007	W. Robinson	T 3:30 pm - 4:45 pm, CP 208
008	W. Robinson	R 3:30 pm - 4:45 pm, CB 204
009	M. Music	T 12:30 pm - 1:45 pm, NURS 214
010	W. Robinson	R 12:30 pm - 1:45 pm, NURS 504
011	S. Taylor	T 9:30 am - 10:45 am, BE 248
012	S. Taylor	R 9:30 am - 10:45 am, CB 214
013	B. Fox	T 9:30 am - 10:45 am, MMRB 243
014	B. Fox	T 9:30 am - 10:45 am, FB B3
015	C. Taylor	T 11:00 am - 12:15 pm, CB 347
016	B. Fox	T 11:00 am - 12:15 pm, CB 243
017	C. Taylor	T 2:00 pm - 3:15 pm, NURS 511
018	C. Taylor	R 2:00 pm - 3:15 pm, DH 323
019	G. Tiser	T 2:00 pm - 3:15 pm, CB 213
020	S. Taylor	R 2:00 pm - 3:15 pm, FB B8
021	G. Tiser	T 12:30 pm - 1:45 pm, FPAT 255
022	G. Tiser	R 12:30 pm - 1:45 pm, DH 323
401	S. Foege	TR 6:00 pm-7:15 pm, CB 347
402	S. Foege	TR 7:30 pm-8:45 pm, CB 347

Show all your work on the page where the question appears.
Clearly mark your answer both on the cover page on this exam
and in the corresponding questions that follow.

1. Suppose that the derivative $f'(x) > 0$ for all x in the interval $(2, 8)$. Which statement is definitely true?

   Possibilities:

   (a) $f(x)$ is decreasing on the interval $(2, 8)$.

   (b) $f(x)$ is concave down on the interval $(2, 8)$.

   (c) $f(x)$ is concave up on the interval $(2, 8)$.

   (d) $f(x)$ is increasing on the interval $(2, 8)$.

   (e) The graph of $f(x)$ must be above the x-axis on the interval $(2, 8)$.

2. Suppose that the derivative of $g(t)$ is $g'(t) = (t - 5)(t - 9)(t - 13)$. Find the value of t in the interval $[5, 13]$ where $g(t)$ has its maximum.

   Possibilities:

   (a) $t = 9$

   (b) $t = 7$

   (c) $t = 6$

   (d) $t = 5$

   (e) $t = 13$

3. Find the largest interval on which $f(x) = x^4 - 6x^3 - 60x^2 - x + 5$ is concave down.

   Possibilities:

   (a) $(-2, 7)$

   (b) $(-2, 6)$

   (c) $(-2, 5)$

   (d) $(-2, 8)$

   (e) $(-2, 4)$

4. Determine the x-coordinate of the inflection point of

$$f(x) = x^3 - 12x^2 - 4x + 5$$

Possibilities:

(a) 4

(b) 5

(c) 6

(d) 7

(e) 8

5. Suppose the first and second derivatives of $f(x)$ are given by

$$f'(x) = (1 - 3x) e^{-3x} \qquad \text{and} \qquad f''(x) = 3(3x - 2) e^{-3x}$$

Find the largest interval on which $f(x)$ is concave down.

Possibilities:
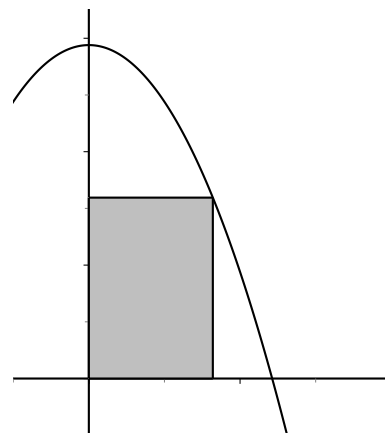
(a) $(-\infty, 2/3)$

(b) $(1/3, \infty)$

(c) $(2/3, \infty)$

(d) $(3, \infty)$

(e) $(-\infty, 3)$

6. Two positive real numbers, x and y, satisfy $xy = 18$. What is the minimum value of the expression $2x + y$?
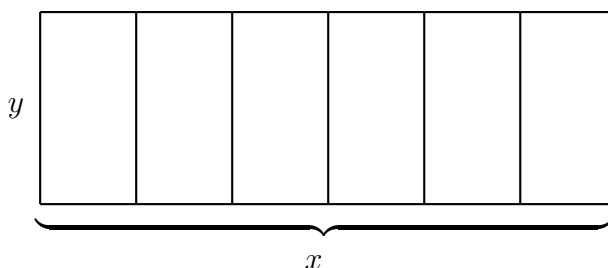
Possibilities:

(a) 10

(b) 11

(c) 12

(d) 13

(e) 14

7. Find the area of the largest rectangle with one corner at the origin, the opposite corner in the first quadrant on the graph of the curve $f(x) = 12 - x^2$. (See the graph, but the graph is not to scale.)

Possibilities:

(a) 14

(b) 15

(c) 16

(d) 17

(e) 18

8. A rectangle is to be constructed with 6 vertical partitions (i.e., 7 vertical walls and 2 horizontal walls) as in the figure below. The rectangle is to be constructed with 3500 feet of material. Let x denote the length of the horizontal wall and y the length of the vertical wall. Which optimization problem needs to be solved in order to determine how to enclose the largest area?

Possibilities:

(a) Maximize $A = xy$, given that $2x + 6y = 3500$.

(b) Maximize $A = 2x + 5y$, given that $xy = 3500$.

(c) Maximize $A = 2x + 6y$, given that $xy = 3500$.

(d) Maximize $A = xy$, given that $2x + 7y = 3500$.

(e) Maximize $A = 2x + 7y$, given that $xy = 3500$.

9. A train is traveling over a bridge at 48 miles per hour. A man on the train is walking toward the back of the train at 4 miles per hour. How fast is the man traveling across the bridge in miles per hour?

Possibilities:

(a) 42

(b) 44

(c) 46

(d) 48

(e) 50

10. A ladder of length 10 feet rests against a wall. The bottom of the ladder is being pulled away from the wall at a rate of 3 feet per second. How fast is the top of the ladder sliding down the wall when the bottom of the ladder is 8 feet from the wall? (Just give the numeric value of the answer. Do not worry about the plus or minus sign.)

Possibilities:

(a) 2 feet per second

(b) 3 feet per second

(c) 4 feet per second

(d) 5 feet per second

(e) 6 feet per second

11. A stock is increasing at a rate of 16 dollars per share per year. An investor is buying stock at a rate of 14 shares per year. How fast is the value of the investor's stock growing when the price of the stock is 53 dollars per share and the investor owns 40 shares of the stock? (Hint: Write down an expression for the total value, V, of the stock owned by the investor.)

Possibilities:

(a) $\$1382$ per year.

(b) $\$224$ per year.

(c) $\$1408$ per year.

(d) $\$2120$ per year.

(e) $\$640$ per year.

12. Estimate the area under the graph of $f(x) = x^2 + 5x$ for x between 0 and 2. Use a partition that consists of 4 equal subintervals of $[0, 2]$ and use the left endpoint of each subinterval as the sample point.

Possibilities:

(a) $37/4$

(b) $633/50$

(c) $65/4$

(d) 6

(e) 20

13. Suppose that the integral $\displaystyle\int_{38}^{52} f(x)\ dx$ is estimated by the sum $\displaystyle\sum_{k=1}^{N} f(38 + k\,\Delta x) \cdot \Delta x$. The terms in the sum equal areas of rectangles obtained using right endpoints of the subintervals of length Δx as sample points. If $N = 700$ equal subintervals are used, what is the value of Δx ?

Possibilities:

(a) $\Delta x = 0.02$

(b) $\Delta x = 0.03$

(c) $\Delta x = 0.04$

(d) $\Delta x = 0.05$

(e) $\Delta x = 0.06$

14. Suppose that the integral $\displaystyle\int_{4}^{24} x^2\ dx$ is estimated by the sum $\displaystyle\sum_{k=1}^{N}(4 + k\,\Delta x)^2 \cdot \Delta x$. The terms in the sum equal areas of rectangles obtained using right endpoints of the subintervals of length Δx as sample points. If $N = 40$ equal subintervals are used, what is area of the second rectangle?

Possibilities:

(a) $81/4$

(b) $81/8$

(c) $25/2$

(d) 8

(e) 25

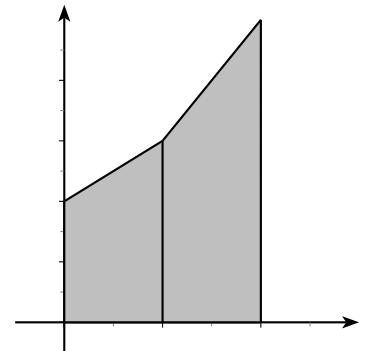15. Suppose you are given the data points for a function $f(x)$:

x	0	1	2
$f(x)$	14	19	23

If $f(x)$ is a linear function on each interval between the given points, find

$$\int_0^2 f(x)\,dx$$

Possibilities:

(a) 33

(b) 75

(c) 37/2

(d) 56

(e) 75/2

(Not drawn to scale)

16. Suppose that you estimate the area under the graph of $f(x) = 1/x$ for x between 1 and 42 in two different ways. First, you divide $[1, 42]$ into 41 equal subintervals and use the left endpoint of each interval as the sample point. Next, you divide $[1, 42]$ into 41 equal subintervals and use the right endpoint of each interval as the sample point. Find the difference between the two estimates. (Left endpoint estimate minus right endpoint estimate). (HINT: It is possible to compute the difference between the two estimates without computing either estimate. Drawing pictures may help you to see how.)

Possibilities:

(a) 40/41

(b) 41/42

(c) 42/43

(d) 43/44

(e) 44/45

8

17. Evaluate the sum

$$\sum_{k=5}^{7} \left(4\,k^2 + k\right)$$

Possibilities:

(a) 457
(b) 458
(c) 459
(d) 460
(e) 461

18. Evaluate the sum

$$\sum_{k=1}^{45} \left(k^2 + k\right)$$

Possibilities:

(a) 32420
(b) 32430
(c) 32440
(d) 32450
(e) 32460

19. Evaluate the sum

$$5 + 10 + 15 + 20 + \ldots + 195 + 200$$

Possibilities:

(a) 4095
(b) 4100
(c) 4105
(d) 4110
(e) 4115

20. Which one of the expressions below is the same as the sum

$$\sum_{k=109}^{154} k \quad ?$$

Possibilities:

(a) $\sum_{k=1}^{154} k \quad - \quad \sum_{k=1}^{108} k$

(b) $\sum_{k=1}^{153} k \quad - \quad \sum_{k=1}^{108} k$

(c) $\sum_{k=1}^{154} k \quad - \quad \sum_{k=1}^{109} k$

(d) $\sum_{k=1}^{153} k \quad - \quad \sum_{k=1}^{109} k$

(e) $\sum_{k=1}^{154} k \quad - \quad \sum_{k=1}^{110} k$

Some Formulas

1. **Summation formulas:**

$$\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$$

$$\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$$

2. **Areas:**

   (a) Triangle $\quad A = \dfrac{bh}{2}$

   (b) Circle $\quad A = \pi r^2$

   (c) Rectangle $\quad A = lw$

   (d) Trapezoid $\quad A = \dfrac{b_1 + b_2}{2} h$

3. **Volumes:**

   (a) Rectangular Solid $\quad V = lwh$

   (b) Sphere $\quad V = \dfrac{4}{3}\pi r^3$

   (c) Cylinder $\quad V = \pi r^2 h$

   (d) Cone $\quad V = \dfrac{1}{3}\pi r^2 h$