MA 123 — Elem. Calculus EXAM 3	Fall 2012 11/14/2012	**Name:** ________________________ **Sec.:** _________

Do not remove this answer page — you will turn in the entire exam. You have two hours to do this exam. No books or notes may be used. You may use a graphing calculator during the exam, but NO calculator with a Computer Algebra System (CAS) or a QWERTY keyboard is permitted. Absolutely no cell phone use during the exam is allowed.

The exam consists of multiple choice questions. Record your answers on this page. For each multiple choice question, you will need to fill in the box corresponding to the correct answer. For example, if (b) is correct, you must write

[a] [**b**] [c] [d] [e]

Do not circle answers on this page, but please circle the letter of each correct response in the body of the exam. It is your responsibility to make it CLEAR which response has been chosen. You will not get credit unless the correct answer has been marked on both this page and in the body of the exam.

GOOD LUCK!

1.	a	b	c	d	e	**11.**	a	b	c	d	e
2.	a	b	c	d	e	**12.**	a	b	c	d	e
3.	a	b	c	d	e	**13.**	a	b	c	d	e
4.	a	b	c	d	e	**14.**	a	b	c	d	e
5.	a	b	c	d	e	**15.**	a	b	c	d	e
6.	a	b	c	d	e	**16.**	a	b	c	d	e
7.	a	b	c	d	e	**17.**	a	b	c	d	e
8.	a	b	c	d	e	**18.**	a	b	c	d	e
9.	a	b	c	d	e	**19.**	a	b	c	d	e
10.	a	b	c	d	e	**20.**	a	b	c	d	e

For grading use:

Number Correct		**Total**	
	(out of 20 problems)		**(out of 100 points)**

Please make sure to list the correct section number on the front page of your exam. In case you forgot your section number, consult the following table. Your section number is determined by your recitation time and location.

Section #	Instructor	Day and Time	Room
001	J. Constable	T, 8:00 am-9:15 am	FB B2
002	J. Constable	T, 9:30 am-10:45 am	NURS 501C
003	J. Constable	T, 11:00 am-12:15 pm	TPC 212
004	L. Davidson	T, 12:30 pm-1:45 pm	CP 111
005	L. Davidson	T, 2:00 pm-3:15 pm	CB 219
006	L. Davidson	T, 3:30 pm-4:45 pm	CB 341
007	W. Hough	R, 8:00 am-9:15 am	FB B2
008	W. Hough	R, 9:30 am-10:45 am	MMRB 243
009	W. Hough	R, 11:00 am-12:15 pm	TPC 212
010	X. Kong	R, 12:30 pm-1:45 pm	Laferty 201
011	X. Kong	R, 2:00 pm-3:15 pm	FPAT 257
012	X. Kong	R, 3:30 pm-4:45 pm	CB 341
013	L. Solus	T, 8:00 am-9:15 am	CB 303
014	K. Effinger	T, 8:00 am-9:15 am	CB 233
015	K. Effinger	T, 11:00 am-12:15 pm	CB 347
016	Q. Liang	T, 12:30 pm-1:45 pm	NURS 501C
017	Q. Liang	T, 2:00 pm-3:15 pm	CB 247
018	Q. Liang	T, 3:30 pm-4:45 pm	CB 245
019	K. Effinger	R, 8:00 am-9:15 am	CB 303
020	L. Solus	R, 8:00 am-9:15 am	CB 233
021	L. Solus	R, 3:30 pm-4:45 pm	CB 214
022	A. Happ	R, 12:30 pm-1:45 pm	NURS 501C
023	A. Happ	R, 2:00 pm-3:15 pm	CB 338
024	A. Happ	R, 3:30 pm-4:45 pm	CB 245
025	F. Smith	T, 12:30 pm-1:45 pm	FPAT 263
026	D. Akers	R, 12:30 pm-1:45 pm	FPAT 263
027	F. Smith	T, 2:00 pm-3:15 pm	FPAT 259
028	D. Akers	R, 2:00 pm-3:15 pm	FPAT 259
029	F. Smith	T, 3:30 pm-4:45 pm	CB 205
030	D. Akers	R, 3:30 pm-4:45 pm	CB 205

Multiple Choice Questions

Show all your work on the page where the question appears.
Clearly mark your answer both on the cover page on this exam
and in the corresponding questions that follow.

1. Suppose $g(t) = t^4 - 6t^2 - 8$. Find the smallest value of A so that $g(t)$ is decreasing on the entire interval $(0, A)$.

   Possibilities:

   (a) $A = \sqrt{3}$

   (b) $A = \sqrt{6}$

   (c) $A = \sqrt{(7/2)}$

   (d) $A = \sqrt{7}$

   (e) $A = \sqrt{(5/2)}$

2. Suppose the derivative of $g(t)$ is $g'(t) = (t-9)(t-5)$. Determine the largest interval or collection of intervals on which $g(t)$ is decreasing.

   Possibilities:

   (a) $(-\infty, 5)$

   (b) $(-\infty, 5)$ and $(9, \infty)$

   (c) $(5, 9)$

   (d) $(9, \infty)$

   (e) $(5, \infty)$

3. Determine the x coordinate of the inflection point of $f(x) = x^2 - 2e^{-9x}$.

   Possibilities:

   (a) $\frac{\ln(63)}{9} = 0.4603$

   (b) $\frac{\ln(72)}{9} = 0.4752$

   (c) $\frac{\ln(81)}{9} = 0.4883$

   (d) $\frac{\ln(90)}{9} = 0.5000$

   (e) $\frac{\ln(99)}{9} = 0.5106$

4. Determine the interval or collection of intervals on which $y = f(x)$ is increasing. Please note the graph is of $y = f'(x)$. (Round the endpoints of the intervals to the nearest ± 0.5)

Possibilities:

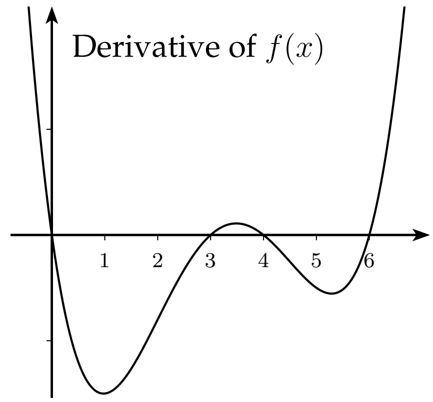
(a) $(-\infty, 0), (3, 4)$, and $(6, \infty)$

(b) $f(x)$ is never increasing

(c) $(-\infty, 1)$ and $(3.5, 5)$

(d) $(0, 3)$ and $(4, 6)$

(e) $(1, 3.5)$ and $(5, \infty)$



5. Determine the interval or collection of intervals on which $y = f(x)$ is concave up. Please note the graph is of $y = f'(x)$. (Round the endpoints of the intervals to the nearest ± 0.5)

Possibilities:

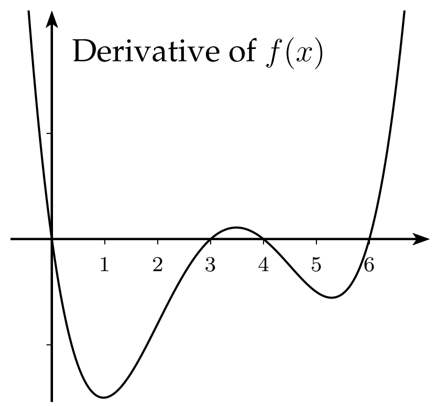
(a) $(1, 3.5)$ and $(5, \infty)$

(b) $(-\infty, 0), (3, 4)$, and $(6, \infty)$

(c) $f(x)$ is never concave up

(d) $(-\infty, 1)$ and $(3.5, 5)$

(e) $(0, 3)$ and $(4, 6)$



6. How many inflection points does the function $g(x) = \ln(x^2 + 1)$ have?
HINT:

$$g'(x) = \frac{2x}{x^2 + 1}$$

Possibilities:

(a) 0

(b) 1

(c) 2

(d) 3

(e) 4

7. Two positive real numbers, x and y, satisfy $xy = 270$. What is the minimum value of the expression $5x + 6y$?

Possibilities:

(a) 180
(b) 181
(c) 182
(d) 183
(e) 184

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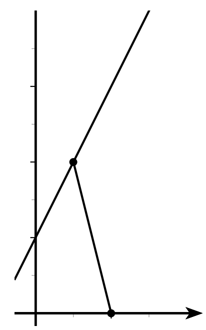
8. Let (a, b) be the point on the line $y = x + 1$ that is closest to the point $(10, 0)$.
Determine a. (NOTE: Graph is not drawn to scale.)



Possibilities:

(a) $1/2$
(b) $5/2$
(c) $9/2$
(d) $13/2$
(e) $17/2$

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9. The area of a circle is increasing at a rate of 12 square inches per minute. Determine the rate at which the radius of the circle is increasing when the radius of the circle is 4

Possibilities:

(a) $\frac{1}{2\pi}$ inches per minute
(b) $\frac{3}{2\pi}$ inches per minute
(c) $\frac{5}{2\pi}$ inches per minute
(d) $\frac{7}{2\pi}$ inches per minute
(e) $\frac{9}{2\pi}$ inches per minute

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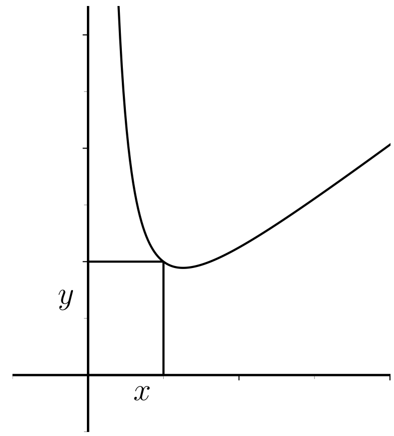
10. Find the horizontal width of the rectangle with minimal area, where one corner of the rectangle is at the origin, the opposite corner in the first quadrant on the graph of the curve $y = 5x + x^{-2}$. (See the graph, but the graph is not to scale.)



Possibilities:

(a) $x = \dfrac{2}{\sqrt{5}} = 0.894$

(b) $x = \dfrac{1}{\sqrt{10}} = 0.316$

(c) $x = \dfrac{2}{\sqrt[3]{5}} = 1.170$

(d) $x = \dfrac{1}{\sqrt[3]{10}} = 0.464$

(e) There is no such rectangle with minimal area

11. A cylindrical tank is being filled with water. The cylinder has a circular base with radius 7 feet. At a particular instance, the height of the water in the tank is increasing at a rate of 2 feet per minute. Determine the rate of change of the volume of the tank at that particular instant. (NOTE: This problem is different from the one we did in class, so set it up carefully)

Possibilities:

(a) 98π cubic feet per minute

(b) $\dfrac{49}{2\pi}$ cubic feet per minute

(c) $\dfrac{2\pi}{49}$ cubic feet per minute

(d) $\dfrac{49\pi}{2}$ cubic feet per minute

(e) $\dfrac{2}{49\pi}$ cubic feet per minute

12. A point moves along the curve $y = x^2$. At the moment when $x = 3$ meters, the rate of change of x with respect to t is $\frac{dx}{dt} = 6$ meters per minute. Determine the rate of change of y with respect to t at that moment. That is, find $\frac{dy}{dt}$ at that moment.

Possibilities:

(a) 18 meters per minute
(b) 7 meters per minute
(c) 36 meters per minute
(d) 12 meters per minute
(e) 6 meters per minute

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13. Determine the sum

$$\sum_{k=5}^{200} 20$$

Possibilities:

(a) 3920
(b) 3880
(c) 3940
(d) 3960
(e) 3900

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14. Determine the value of the sum

$$\sum_{k=4}^{5} \frac{k}{2^k}$$

Possibilities:

(a) 5/32
(b) 13/32
(c) 13/64
(d) 1/4
(e) The sum cannot be determined

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15. Determine the value of the sum

$$\sum_{k=1}^{100}(k^2 - 6k + 3)$$

(NOTE: Formulas for several special sums appear on the back page of the exam.)

Possibilities:

(a) 308341
(b) 308344
(c) 308347
(d) 308350
(e) 308353

16. Estimate the area under the graph of $f(x) = \frac{x^2}{2}$ for x between 0 and 8. Use a partition that consists of 4 equal subintervals of $[0, 8]$ and use the left endpoint of each subinterval to determine the height of the rectangle.

Possibilities:

(a) 160
(b) 120
(c) 0
(d) 32
(e) 56

17. The definite integral

$$\int_2^b 3x^2\, dx$$

is estimated by a sum of areas of rectangles of equal widths,

$$\sum_{k=1}^{N} 3(2 + k\Delta x)^2 \Delta x$$

Suppose that the width of each rectangle is $\Delta x = 0.2$ and suppose we use 50 rectangles. Determine the value of b, the upper limit of integration.

Possibilities:

(a) 9
(b) 10
(c) 11
(d) 12
(e) 13

18. Suppose that the integral $\int_6^{16} x^2\,dx$ is estimated by the sum $\sum_{k=1}^{N}(6+k\,\Delta x)^2 \cdot \Delta x$. The terms in the sum equal areas of rectangles obtained using right endpoints of the subintervals of length Δx as sample points. If $N = 20$ equal subintervals are used, what is area of the first (leftmost) rectangle?

Possibilities:

(a) $49/2$

(b) 49

(c) $169/8$

(d) $169/4$

(e) 18

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19. The area under the curve $y = 1/x$ from $x = 1$ to $x = 67$ is estimated in two ways. First, the area is approximated by subdividing $[1, 67]$ into 66 subintervals of equal width, and the heights of the rectangles are determined by the left endpoints of the subintervals. Then the area is approximated by using the same subdivisions, but the heights of the rectangles are determined by the right endpoint of each subinterval. Determine the difference between the two estimates.

Possibilities:

(a) $1 - \frac{1}{65}$

(b) $1 - \frac{1}{66}$

(c) $1 - \frac{1}{64}$

(d) $1 - \frac{1}{67}$

(e) $1 - \frac{1}{68}$

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20. Suppose that $g(x)$ is decreasing for all x. Which one of the following must also be decreasing for all x? (HINT: what must be true about the derivatives of $g(x)$ and $f(x)$ if the functions are decreasing?)

Possibilities:

(a) $f(x) = x^2 g(x)$

(b) $f(x) = -g(x)$

(c) $f(x) = x \cdot g(x)$

(d) $f(x) = (g(x))^2$

(e) $f(x) = (g(x))^3$

Some Formulas

1. Summation formulas:

$$\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$$

$$\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$$

2. Areas:

(a) Triangle $A = \frac{bh}{2}$

(b) Circle $A = \pi r^2$

(c) Rectangle $A = lw$

(d) Trapezoid $A = \frac{b_1 + b_2}{2} h$

3. Volumes:

(a) Rectangular Solid $V = lwh$

(b) Sphere $V = \frac{4}{3}\pi r^3$

(c) Cylinder $V = \pi r^2 h$

(d) Cone $V = \frac{1}{3}\pi r^2 h$

4. Distance:

(a) Distance between (x_1, y_1) and (x_2, y_2)

$$D = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$