

Do not remove this answer page — you will turn in the entire exam. You have two hours to do this exam. No books or notes may be used. You may use a graphing calculator during the exam, but NO calculator with a Computer Algebra System (CAS) or a QWERTY keyboard is permitted. Absolutely no cell phone use during the exam is allowed.

The exam consists of multiple choice questions. Record your answers on this page. For each multiple choice question, you will need to fill in the box corresponding to the correct answer. For example, if (a) is correct, you must write

☒ a ☐ b ☐ c ☐ d ☐ e

Do not circle answers on this page, but please circle the letter of each correct response in the body of the exam. It is your responsibility to make it CLEAR which response has been chosen. You will not get credit unless the correct answer has been marked on both this page and in the body of the exam.

GOOD LUCK!

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For grading use:

|                   |                      |
|-------------------|----------------------|
| Number<br>Correct |                      |
|                   | (out of 20 problems) |

|       |                     |
|-------|---------------------|
| Total |                     |
|       | (out of 100 points) |

Please make sure to list the correct section number on the front page of your exam. In case you forgot your section number, consult the following table. Your section number is determined by your recitation time and location.

| Section # | Instructor        | Day | Time                | Room      |
|-----------|-------------------|-----|---------------------|-----------|
| 001-006   | Jack Schmidt      | MWF | 08:00 am - 08:50 am | KAS 213   |
| 001       | Jinping Zhuge     | T   | 8:00 am - 9:15 am   | FB B9     |
| 002       | Yiyuan Wu         | T   | 9:30 am - 10:45 am  | NURS 501B |
| 003       | Devin Willmott    | T   | 8:00 am - 9:15 am   | CB 235    |
| 004       | Tefjol Pllaha     | T   | 8:00 am - 9:15 am   | CB 237    |
| 005       | Tefjol Pllaha     | T   | 2:00 pm - 3:15 pm   | CB 347    |
| 006       | Tefjol Pllaha     | T   | 3:30 pm - 4:45 pm   | CB 347    |
| 007-012   | Jack Schmidt      | MWF | 09:00 am - 09:50 am | KAS 213   |
| 007       | Yiyuan Wu         | R   | 8:00 am - 9:15 am   | CB 217    |
| 008       | Jinping Zhuge     | R   | 9:30 am - 10:45 am  | DH 323    |
| 009       | Yiyuan Wu         | R   | 11:00 am - 12:15 pm | EH 202    |
| 010       | Jinping Zhuge     | R   | 12:30 pm - 1:45 pm  | DH 323    |
| 011       | Dharma Maharjan   | R   | 2:00 pm - 3:15 pm   | CB 347    |
| 012       | Dharma Maharjan   | R   | 3:30 pm - 4:45 pm   | CB 347    |
| 013-018   | Paul Koester      | MWF | 1:00 pm - 1:50 pm   | BS 116    |
| 013       | Carolyn Troha     | T   | 8:00 am - 9:15 am   | CB 345    |
| 014       | Carolyn Troha     | T   | 9:30 am - 10:45 am  | NURS 214  |
| 015       | Morgan Schreffler | T   | 11:00 am - 12:15 pm | EH 202    |
| 016       | Carolyn Troha     | T   | 12:30 pm - 1:45 pm  | MMRB 243  |
| 017       | Morgan Schreffler | T   | 2:00 pm - 3:15 pm   | BH 301    |
| 018       | Morgan Schreffler | T   | 3:30 pm - 4:45 pm   | CB 235    |
| 025-030   | Paul Koester      | MWF | 2:00 pm - 2:50 pm   | BS 107    |
| 025       | Sarah Orchard     | T   | 12:30 pm - 1:45 pm  | TPC 212   |
| 026       | Marie Meyer       | R   | 8:00 am - 9:15 am   | CB 240    |
| 027       | Marie Meyer       | T   | 2:00 pm - 3:15 pm   | DH 331    |
| 028       | Marie Meyer       | R   | 2:00 pm - 3:15 pm   | EH 304    |
| 029       | Sarah Orchard     | T   | 3:30 pm - 4:45 pm   | OT OB7    |
| 030       | Sarah Orchard     | R   | 3:30 pm - 4:45 pm   | OT OB7    |
| 401       | Brad Schwer       | MTR | 5:30 pm - 6:45 pm   | CB 337    |

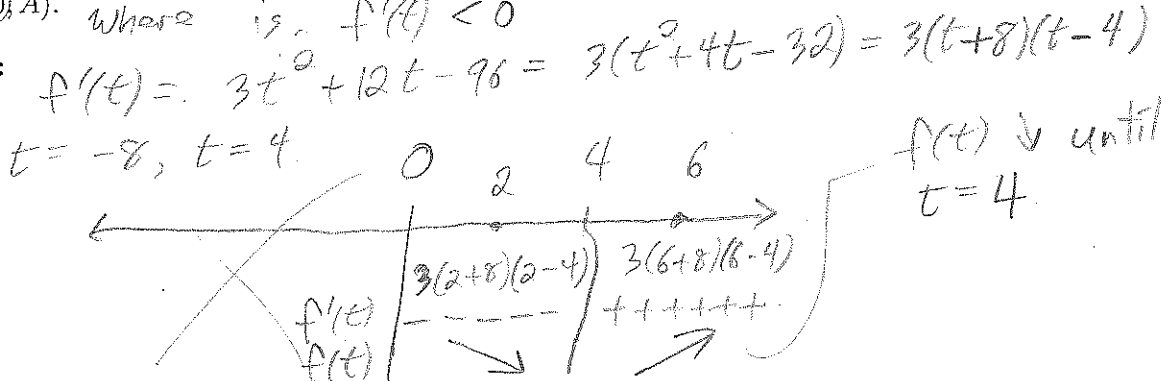
### Multiple Choice Questions

Show all your work on the page where the question appears.  
Clearly mark your answer both on the cover page on this exam  
and in the corresponding questions that follow.

1. Find the largest value of  $A$  such that the function  $f(t) = t^3 + 6t^2 - 96t - 2$  is decreasing for all  $t$  in the interval  $(0, A)$ . Where is  $f'(t) < 0$

Possibilities:

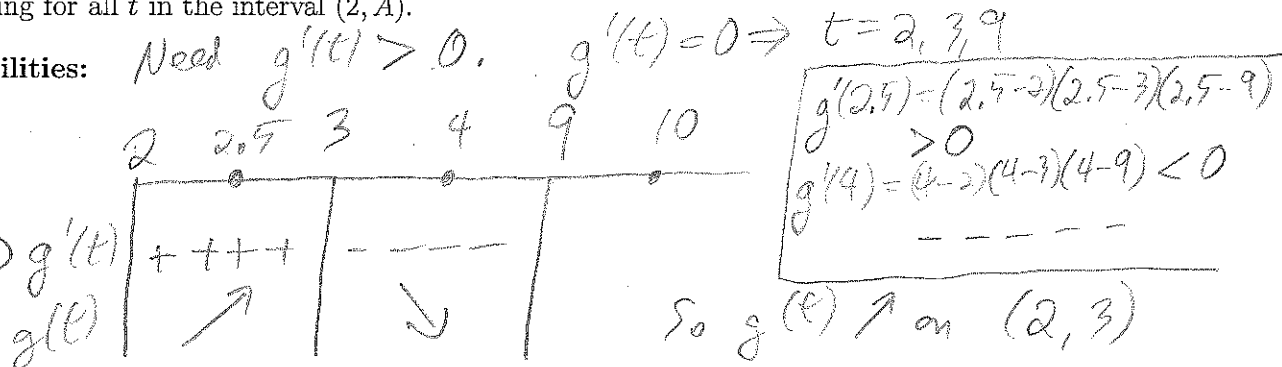
- (a) -2  
(b) 8  
(c)  $\infty$   
(d) -8  
(e) 4



2. Suppose  $g'(t) = (t-2)(t-3)(t-9)$ . Find the largest value of  $A$  such that the function  $g(t)$  is increasing for all  $t$  in the interval  $(2, A)$ .

Possibilities:

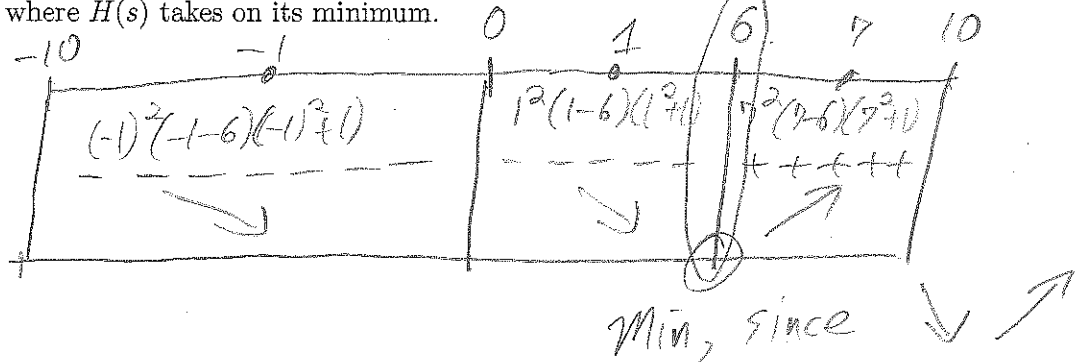
- (a)  $\infty$   
(b) 2  
(c) 54  
(d) 3  
(e) 9



3. Suppose the derivative of  $H(s)$  is given by  $H'(s) = s^2(s-6)(s^2+1)$ . Find the value of  $s$  in the interval  $[-10, 10]$  where  $H(s)$  takes on its minimum.

Possibilities:

- (a) 7  
(b) 1  
(c) 0  
(d) 6  
(e) -1



$H'(s) = 0 \Rightarrow s^2 = 0$  OR  $s-6=0$  OR  $s^2+1=0$   
So  $s=0$  OR  $s=6$

4. If  $f(x) = xe^{2x}$ , find the largest interval on which  $f(x)$  is concave upward. If we write the interval as  $(a, \infty)$ , then what is  $a$ ? Need  $f''(x)$ .

Possibilities:

(a) -1

(b) 3

(c)  $-\frac{1}{2}$

(d) 2

(e) 1

$$f'(x) = x^1 e^{2x} + x(e^{2x})' = e^{2x} + x \cdot 2e^{2x} = (1+2x)e^{2x}$$

$$f''(x) = (1+2x)'e^{2x} + (1+2x)(e^{2x})' = 2e^{2x} + (1+2x) \cdot 2e^{2x}$$

$$= 2e^{2x} + (2+4x)e^{2x} = 2e^{2x} + 2e^{2x} + 4xe^{2x} = (4+4x)e^{2x}$$

Now set  $f''(x) = 0 \Rightarrow (4+4x)e^{2x} = 0 \Rightarrow 4+4x = 0 \Rightarrow 4 = -4x \Rightarrow x = -1$

Also,  $e^{2x}$  Never 0.

| $x$      | $f''(x)$                         | Concavity    |
|----------|----------------------------------|--------------|
| $x < -1$ | $(4-2 \cdot 4)e^{-4} = -4e^{-4}$ | Concave Down |
| $x > -1$ | $(4+0)e^0 = 4$                   | Concave Up   |

Concavity changes @  $x = -1$

5. Suppose the derivative of  $h(x)$  is given by  $h'(x) = (x-4)(x-8)$ . If  $h(x)$  is concave upward on the interval  $(a, \infty)$ , what is  $a$ ?

Possibilities:

(a) 6

(b) 12

(c) 4

(d)  $-\infty$

(e) 8

$$h'(x) = x^2 - 12x + 32, \quad h''(x) = 2x - 12$$

$$h''(x) = 0 \Rightarrow x = 6$$

| $x$     | $h''(x)$              | Concavity |
|---------|-----------------------|-----------|
| $x < 6$ | $2 \cdot 5 - 12 = -4$ | C.D       |
| $x > 6$ | $2 \cdot 7 - 12 = 2$  | C. Up     |

C. Up on  $(6, \infty)$

6. The following is the graph of the derivative,  $f'(x)$ , of the function  $f(x)$ . The zeroes, local extrema, and points of inflection of  $f'(x)$  are marked. Where is  $f(x)$  increasing?

Possibilities:

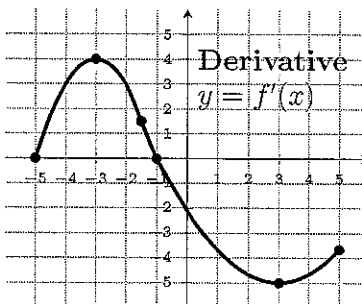
(a) between -5 and -1.5

(b) between -1 and 5

(c) between -5 and -3, also between 3 and 5

(d) between -3 and 3

(e) between -5 and -1



$f(x) \nearrow$  where  $f'(x) > 0$ .  
 $\Rightarrow$  graph of  $f'(x)$  above  $x$ -axis  
 $-5$  to  $-1$ .

7. Find the area of the largest rectangle whose sides are parallel to the coordinate axes, whose bottom-left corner is at  $(0,0)$  and whose top-right corner is on the graph of  $y = 12x - x^2$ .

Possibilities:  $A = xy = x(12x - x^2) = 12x^2 - x^3$

(a) 0

(b) 256

(c) 216

(d) 132

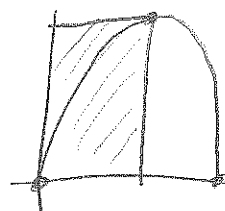
(e) 6

$$A' = 24x - 3x^2 = 0$$

$$\Rightarrow 24x = 3x^2 \Rightarrow x = 0 \text{ or } x = 8$$

$$A(0) = 0 \cdot (12 \cdot 0 - 0^2) = 0$$

$$A(8) = 8(12 \cdot 8 - 8^2) = 8(96 - 64) = 8 \cdot 32 = 256$$



8. Find the point in the first quadrant that lies on the hyperbola  $y^2 - x^2 = 3$  and is closest to the point  $(2,0)$ .

Minimize Distance

Possibilities:

(a)  $(2, \sqrt{7})$

(b)  $(6, \sqrt{39})$

(c)  $(0, \sqrt{3})$

(d)  $(1, 2)$

(e)  $(7, 2\sqrt{13})$

$$D = \sqrt{(x-2)^2 + (y-0)^2} = \sqrt{(x-2)^2 + y^2}$$

$$= \sqrt{(x-2)^2 + 3 + x^2}$$

$$\frac{dD}{dx} = \frac{2(x-2) + 2x}{2\sqrt{(x-2)^2 + 3 + x^2}} \Rightarrow$$

$$\text{Set } \frac{dD}{dx} = 0$$

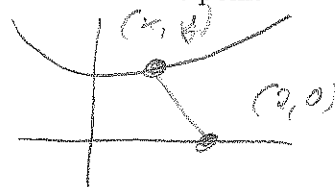
$$2(x-2) + 2x = 0$$

$$2x - 4 + 2x = 0$$

$$4x = 4 \Rightarrow x = 1$$

$$\rightarrow y^2 = 3 + x^2 = 3 + 1^2 = 4$$

$$\text{so } y = \sqrt{4} = 2$$



9. A farmer builds a rectangular pen with 5 vertical partitions (i.e. 6 vertical sides) using 700 feet of fencing. What is the maximum possible total area of the pen?

Possibilities:

(a) 175

(b)  $\frac{30625}{3}$

(c) 700

(d) 30625

(e)  $\frac{175}{3}$

$$P = 2l + 6w = 700$$

$$l + 3w = 350$$

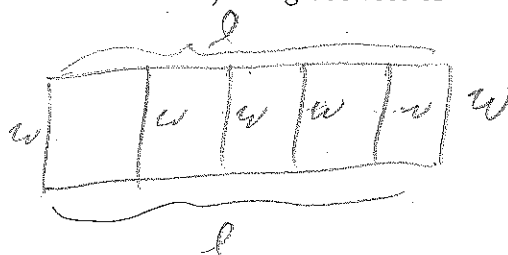
$$l = 350 - 3w$$

$$A = lw = (350 - 3w)w$$

$$= 350w - 3w^2$$

$$A' = 350 - 6w$$

$$\text{Set } A' = 0 \Rightarrow 350 = 6w \Rightarrow w = \frac{350}{6} = \frac{175}{3}$$



$$A = \left(350 - 3 \cdot \frac{175}{3}\right) \cdot \frac{175}{3} = \frac{175 \cdot 175}{3} = \frac{30625}{3}$$

10. The surface area of a sphere of radius  $r$  is given by the formula  $4\pi r^2$ . A certain sphere's radius is growing at a constant speed of .1 meters per year. How fast is the surface area of this sphere changing when the radius is 1000 meters?

Possibilities:

- (a) .1256637062 square meters per year  
 (b) 2513.274123 square meters per year  
 (c) 1256637.062 square meters per year  
 (d) 12566370.62 square meters per year  
 (e) 25132.74123 square meters per year

$$S = 4\pi r^2$$

Given:  $\frac{dr}{dt} = 0.1$ , want  $\frac{dS}{dt}$

$$\text{So } \frac{dS}{dt} = 4\pi \cdot 2r \cdot \frac{dr}{dt}$$

$$= 4\pi \cdot 2 \cdot 1000 \cdot 0.1 = 800\pi$$

$$\approx 2513.27 \dots$$

11. A ladder 30 feet long rests against a vertical wall. If the bottom of the ladder slides away from the wall at a rate of 11 feet per second, how fast is the top of the ladder sliding down the wall (in feet per second) when the bottom of the ladder is 24 feet from the wall? (answer should be positive)

Possibilities:

- (a) 4  
 (b)  $\frac{33}{4}$   
 (c)  $\frac{44}{3}$   
 (d)  $\frac{88}{3}$   
 (e) 11



$$x^2 + y^2 = 30^2$$

$$\text{So } 2x \frac{dx}{dt} + 2y \frac{dy}{dt} = 0$$

$$11 = \frac{dx}{dt}$$

$$x = 24, \text{ so } y^2 = 30^2 - 24^2$$

$$y = 18$$

$$2 \cdot 24 \cdot 11 + 2 \cdot 18 \cdot \frac{dy}{dt} = 0$$

$$528 + 36 \cdot \frac{dy}{dt} = 0$$

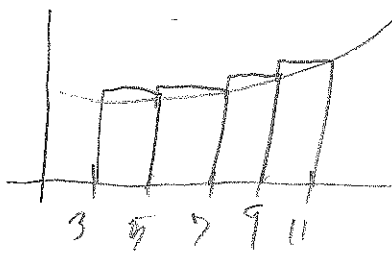
$$36 \frac{dy}{dt} = -528 \Rightarrow \frac{dy}{dt} = \frac{-528}{36} = -\frac{44}{3}$$

Since the question asks for a positive answer, the rate is  $\frac{44}{3}$ .

12. Estimate the area under the graph of  $x^2 - 5x$  for  $x$  between 3 and 11, by using a partition that consists of 4 equal subintervals of  $[3, 11]$  and use the right endpoint of each subinterval as a sample point.

Possibilities:

- (a)  $\frac{464}{3}$   
 (b) 232  
 (c) 220  
 (d) 88  
 (e) 116



$$A = 2 \cdot f(5) + 2 \cdot f(7) + 2 \cdot f(9) + 2 \cdot f(11)$$

$$= 2 [5^2 - 5 \cdot 5 + 7^2 - 5 \cdot 7 + 9^2 - 5 \cdot 9 + 11^2 - 5 \cdot 11]$$

$$= 232$$

widths: 2

13. A train travels in a straight westward direction along a track. The speed of the train varies, but it is measured at regular time intervals of  $1/10$  hour. The measurements for the first half hour are:

*Trapezoids width = 0.1*

|       |   |    |    |    |    |    |
|-------|---|----|----|----|----|----|
| time  | 0 | .1 | .2 | .3 | .4 | .5 |
| speed | 0 | 6  | 9  | 13 | 20 | 26 |

Estimate the total distance (in miles) traveled by the train during the first half hour by assuming the speed is a linear function of  $t$  on the subintervals. The speed in the table is given in miles per hour. Use all six speed measurements in your estimate.

Possibilities:

- (a) 6.5  
(b) 3.0  
(c) 13.0  
(d) 7.4  
(e) 6.1

$$0.1 \left( \frac{0+6}{2} \right) + 0.1 \left( \frac{6+9}{2} \right) + 0.1 \left( \frac{9+13}{2} \right) + 0.1 \left( \frac{13+20}{2} \right) + 0.1 \left( \frac{20+26}{2} \right) = 6.1$$

14. One way to approximate  $\int_a^b e^{15-2x} dx$  is with the sum  $\sum_{k=1}^{100} ((\Delta x) \cdot (e^{15-2(7+k\Delta x)}))$ . What is the best value of  $\Delta x$  to use?

Possibilities:

- (a)  $\frac{1}{2}$   
(b) 100  
(c) 7  
(d) 1.359140914  
(e) 57

$$\Delta x = \frac{b-a}{n} = \frac{57-7}{100} = \frac{50}{100} = \frac{1}{2}$$

15. Suppose you estimate the area under the graph of  $f(x) = x^3$  from  $x = 7$  to  $x = 27$  by adding the areas of the rectangles as follows: partition the interval into 20 equal subintervals and use the right endpoint of each interval to determine the height of the rectangle. What is the area of the 15th rectangle?

Possibilities:

- (a)  $\frac{39775}{4}$   
(b) 142100  
(c) 9261  
(d) 10648  
(e) 27

Area 15th Right Rect  $f(x_{15}) \cdot \Delta x$

$$\Delta x = \frac{27-7}{20} = 1$$

$$x_k = 7 + k \cdot 1 = 7 + k$$

$$x_{15} = 7 + 15 = 22$$

$$f(22) \cdot 1 = 22^3 = 10,648$$

16. Evaluate the sum

$$\sum_{k=4}^6 (6k^3 + 3)$$

Possibilities:

- (a) 1686
- (b) 1299
- (c) 2439
- (d) 387
- (e) 21

$$\begin{aligned} &= (6 \cdot 4^3 + 3) + (6 \cdot 5^3 + 3) + (6 \cdot 6^3 + 3) \\ &= 6 \cdot (4^3 + 5^3 + 6^3) + (3 + 3 + 3) \end{aligned}$$

17. Evaluate the sum

$$\sum_{k=1}^{13} (6k^2) = 6 \sum_{k=1}^{13} k^2$$

Possibilities:

- (a) 4914
- (b) 1014
- (c) 546
- (d) 819
- (e) 1020

$$= 6 \cdot \frac{13 \cdot (13+1) \cdot (2 \cdot 13 + 1)}{6} = 4914$$

18. Evaluate the sum  $6 + 12 + 18 + 24 + \dots + 600$ .

Possibilities:

- (a) 180300
- (b) 660
- (c) 4
- (d) 5
- (e) 30300

$$\begin{aligned} &= 6 (1 + 2 + 3 + \dots + 100) \\ &= 6 \cdot \frac{100 \cdot 101}{2} = 30300 \end{aligned}$$

19. Evaluate the sum  $\sum_{k=6}^{100} (5 + 3k)$ .  $= \sum_{k=1}^{100} (5 + 3k) - \sum_{k=1}^5 (5 + 3k)$

Possibilities:

(a) 15580

(b) 23

(c) 15155

(d) 15650

(e) 305

$$= 5 \sum_{k=1}^{100} 1 + 3 \sum_{k=1}^{100} k - \left[ (5+3) + (5+6) + (5+9) + (5+12) + (5+15) \right]$$

$$= 5 \cdot 100 + 3 \cdot \frac{100(101)}{2} - 70$$

$$= 15580$$

20. Evaluate the sum  $\sum_{k=1}^{100} (4k^2 - 6k)$ .

Possibilities:

(a) 39400

(b) 39398

(c) 1323100

(d) 5050

(e) -2

$$= 4 \sum_{k=1}^{100} k^2 - 6 \sum_{k=1}^{100} k$$

$$= 4 \cdot \frac{100(100+1)(2 \cdot 100+1)}{6} - 6 \cdot \frac{100(100+1)}{2}$$

### Some Formulas

#### 1. Summation formulas:

$$\sum_{k=1}^n k = \frac{n(n+1)}{2}$$

$$\sum_{k=1}^n k^2 = \frac{n(n+1)(2n+1)}{6}$$

#### 2. Areas:

(a) Triangle  $A = \frac{bh}{2}$

(b) Circle  $A = \pi r^2$

(c) Rectangle  $A = lw$

(d) Trapezoid  $A = \frac{h_1 + h_2}{2} b$

#### 3. Volumes:

(a) Rectangular Solid  $V = lwh$

(b) Sphere  $V = \frac{4}{3}\pi r^3$

(c) Cylinder  $V = \pi r^2 h$

(d) Cone  $V = \frac{1}{3}\pi r^2 h$

#### 4. Distance:

(a) Distance between  $(x_1, y_1)$  and  $(x_2, y_2)$

$$D = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$